Mini-workshop "Algebraic groups: the White Nights season II"

Book of abstracts

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Homogeneous algebraic varieties and transitivity degree

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Abstract

Let X be an algebraic variety such that the group $\mathrm{Aut}(X)$ acts on X transitively. We define the transitivity degree of X as a maximal number m such that the action of $\mathrm{Aut}(X)$ on X is m-transitive. If the action of $\mathrm{Aut}(X)$ is m-transitive for all m, the transitivity degree is infinite. We compute the transitivity degree for all quasi-affine toric varieties and for many homogeneous spaces of algebraic groups. Also we discuss a conjecture and open questions related to this invariant.

The talk is based on joint work [2] with Kirill Shakhmatov and Yulia Zaitseva. Our aim is to develop earlier results on multiple transitivity of Lie groups and algebraic groups [4, 5, 6] and on infinite transitivity of the special automorphism group [1, 2].

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Keywords— Algebraic variety, automorphism group, algebraic group, homogeneous space, quasi-affine variety, transitivity degree, infinite transitivity, toric variety, unirationality

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Existence results for B-root subgroups on affine spherical varieties

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Abstract

In the study of automorphism groups of toric varieties, a key role is played by one-parameter additive groups normalized by the acting torus. Such subgroups are called root subgroups and each of them is uniquely determined by its weight, called a Demazure root of the corresponding toric variety. Moreover, the set of all Demazure roots admits an explicit combinatorial description in terms of the fan defining the toric variety. For an affine toric T-variety X, an important property states that every T-stable prime divisor in X can be connected with the open T-orbit via the action of an appropriate root subgroup.

In the setting of arbitrary connected reductive groups acting on algebraic varieties, a natural generalization of toric varieties is given by spherical varieties. A spherical variety is an algebraic variety X equipped with an action of a connected reductive group G in such a way that a Borel subgroup B of G has an open orbit in X. It was proposed in [1] that a proper generalization of root subgroups for spherical varieties is given by one-parameter additive groups normalized by B, which are called B-root subgroups. At present, a complete description of all B-root subgroups on affine spherical varieties remains an open problem.

In this talk, we shall discuss results of [2, 3] where some sufficient conditions for the existence of B-root subgroups on a given affine spherical G-variety X are found. Here a key role is played by the so-called local structure theorem. As an application, it turns out that every G-stable prime divisor in X can be connected with the open G-orbit via the action of an appropriate B-root subgroup, which generalizes the above-mentioned result in the toric case.

This research is supported by the Russian Science Foundation; grant no. 22-41-02019.

Keywords— additive group action, spherical variety, Demazure root, locally nilpotent derivation

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[3] R. Avdeev, V. Zhgoon, in preparation.

On commuting locally nilpotent derivations and isotropy subgroups

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Abstract

Let k be an algebraically closed field of characteristic zero. In this talk, we shall discuss properties of commuting locally nilpotent derivations on the polynomial rings $k[X,Y]$ and $k[X,Y,Z]$ respectively. We will describe an algorithm to construct an arbitrary irreducible locally nilpotent derivation on $k[X,Y]$ in finitely many steps starting from the partial derivatives $\frac{\partial}{\partial X}$ and $\frac{\partial}{\partial Y}$, using commuting locally nilpotent derivations. We shall also give necessary and sufficient conditions on a locally nilpotent derivation (not necessary irreducible) of $k[X,Y,Z]$ to possess a non-equivalent commuting locally nilpotent derivation and describe all locally nilpotent derivations which commute with it.

The isotropy subgroup of a locally nilpotent derivation of an affine k-domain B is defined to be the subgroup of algebraic automorphisms of B which keep the derivation invariant under the natural action by conjugation. Exponents of locally nilpotent derivations which commute with a fixed locally nilpotent derivation are always in its isotropy subgroup. We shall describe the structure of the isotropy subgroups of some important classes of locally nilpotent derivations on $k[X,Y,Z]$.

This talk is based on an ongoing joint work with S. Gayfullin.

Toric degeneration of semi-infinite Grassmannians

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Abstract

Toric degenerations of the classical Grassmannians are well studied. The subject attracted attention of many experts due to numerous applications in combinatorial and geometric representation theory. Semi-infinite Grassmannians are obtained by replacing the base field by the ring of series in one variable. We will describe the basic properties of the semi-infinite Grassmannians and explain how to construct their toric degenerations. Toric varieties in question are constructed using combinatorics of certain posets.

This a joint work with Igor Makhlin and Alexander Popkovich.

Keywords— Semi-infinite Grassmannians, toric degenerations

Orbits of automorphism groups of trinomial hypersurfaces

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Abstract

Let $\mathbb{K}$ be an algebraically closed field of characteristic zero. We consider affine hypersurfaces given in $\mathbb{K}^{n_0+n_1+n_2}$ by equations of the form

$$T_{01}^{l_{01}}\dots T_{0n_0}^{l_{0n_0}}+T_{11}^{l_{11}}\dots T_{1n_1}^{l_{1n_1}}+T_{21}^{l_{21}}\dots T_{2n_2}^{l_{2n_2}}=0.$$

Such hypersurfaces are called trinomial hypersurfaces. We are interested in orbits of the natural action of group of regular automorphisms $\mathrm{Aut}(X)$, where X is a trinomial hypersurface. In [2] there are criterium for trinomial hypersurfaces to be rigid and a sufficient condition to be flexible. In [1] the group $\mathrm{Aut}(X)$ on a rigid trinomial hypersurface are described. So, in rigid case we know the description of $\mathrm{Aut}(X)$-orbits.

In nonrigid case it is more convenient to consider the neutral component $\mathrm{Aut}(X)^0$. In [3] we prove that each nonrigid trinomial hypersurface admits finite number of $\mathrm{Aut}(X)^0$-orbits. For some subclasses of trinomial hypersurfaces we obtain a complete description of $\mathrm{Aut}(X)^0$-orbits.

In the talk we will discuss description of $\mathrm{Aut}(X)^0$-orbits when it is known, conjectures and difficulties in cases when the answer is still open.

Keywords— Automorphism, orbits, affine variety, trinomial hypersurface

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Rota—Baxter operators on the polynomial algebra

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Abstract

A linear operator R defined on an algebra A over a field F is called a *Rota–Baxter operator* of weight λ, where $\lambda \in F$, if the identity

$$R(f)R(g) = R(R(f)g + fR(g) + \lambda fg) \tag{1}$$

holds for all $f, g \in A$. When $\lambda = 0$, such operators generalize of integration by parts formula for the integral operator. When $\lambda \neq 0$, Rota–Baxter operators extend decompositions of an algebra into a direct vector space sum of two subalgebras. Rota–Baxter operators have been studied since 1950s from algebraic, combinatorial, topological, physical and many other points of view, see [2].

The intensive study of Rota—Baxter operators on the polynomial algebras was started from the work [5] of S.H. Zheng, L. Guo, and M. Rosenkranz (2015). They described ompletely all injective monomial Rota–Baxter operators of weight zero on $F[x]$; an operator on $F[x]$ is called *monomial* if it sends each monomial to a monomial with some coefficient.

Consider two operators on $F[x]$: an operator l_r of multiplication on a fixed polynomial $r \in F[x]$ and the formal integration J_a at a point $a \in F$. For the general case, S.H. Zheng, L. Guo, and M. Rosenkranz made a significant progress toward the following

Conjecture. Every injective Rota–Baxter operator of weight 0 on $\mathbb{R}[x]$ equals $J_a \circ l_r$ for some nonzero polynomial $r \in \mathbb{R}[x]$ and $a \in \mathbb{R}$.

In [1] the conjecture of S.H. Zheng, L. Guo, and M. Rosenkranz [5] was proved over any field of characteristic zero.

All monomial Rota—Baxter operators of arbitrary weight on $F[x]$ were classified in [4] (2016).

In [3], a systematic study of monomial Rota—Baxter operators on $F[x, y]$ was initiated.

Given an algebra A, an operator T on A is called a *homomorphic averaging operator*, if $T(a)T(b) = T(T(a)b) = T(aT(b)) = T(ab)$ holds for all $a, b \in A$. One can show [3] that except trivial cases we have exactly two families of monomial homomorphic averaging operators on $F[x, y]$:

(1) $A_r(x^n y^m) = x^{rm} y^m$, $r \in \mathbb{N}$,

(2) $B_r(x^n y^m) = y^{m+rn}$, $r \in \mathbb{N}$.

The idea is to find monomial Rota—Baxter operators R of nonzero weight on $F[x, y]$ in the form $R(x^n y^m) = \alpha_{n,m} T(x^n y^m)$, where $\alpha_{n,m} \in F$ and T is a monomial homomorphic averaging operator. We find all such Rota—Baxter operators when $T = A_r$, $T = B_0$, and $T = B_1$.

Keywords— Rota—Baxter operator, polynomial algebra

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Coadjoint orbits of low dimension

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Abstract

Let G be a simple algebraic group over an algebraically closed field $\mathbb{F}$ of characteristic zero or large enough, and N be a maximal nilpotent subgroup of G. The orbit method plays the crucial role in representation theory of N. Let $\mathfrak{n}$ be the Lie algebra of N, and $\mathfrak{n}$ be the dual space. The group N acts on $\mathfrak{n}^*$ be the adjoint action; the dual action of N on the space $\mathfrak{n}^*$ is called coadjoint.

In general, a complete description of coadjoint orbits is a wild problem. In my talk, I will present a classification of orbits of dimension ≤ 6 for a classical group G. It turns out that such a classification can be given in terms of so-called rook placements in the root system of G.

A long-standing I.M. Isaacs's conjecture [2] claims that, given $d \geq 0$, the number of irreducible complex characters of the group $N(q)$ of $\mathbb{F}_q$-points of N of the degree q^d is a polynomial in $q-1$ with nonnegative integer coefficients. As a corollary of our main result, we obtain a proof of this conjecture for all classical groups for $d \leq 3$.

The talk is based on my joint work with A.V. Petukhov. The work is supported by the Russian Science Foundation under grant 22–11–00081.

Keywords— codjoint orbit, nilpotent Lie group, Isaacs's conjecture

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Mitosis in Schubert calculus

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Abstract

Mitosis operations in type A were invented by Allen Knutson and Ezra Miller about twenty years ago [M]. These operations can be thought of as combinatorial counterparts of divided difference (or Demazure) operators in the cohomology rings of complete flag varieties. In the last decade, several geometric versions of mitosis operations were defined using convex geometric approach to Schubert calculus. Recently, Naoki Fujita constructed mitosis operations on faces of Gelfand–Zetlin polytopes in types A and C using representation theory [F].

In my talk, I survey various geometric incarnations of mitosis operations and their applications to Schubert calculus. I also define simple geometric operations on faces of polytopes using ideas of [K]. For Gelfand–Zetlin polytopes, these operations are closely related to Knutson–Miller mitosis in type A and Fujita mitosis in type C.

Keywords— Schubert calculus, mitosis, divided difference operators

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Noncommutative Novikov algebras

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Abstract

The variety of Novikov algebras appeared in the paper [1] devoted to the study of Poisson brackets of hydrodynamic type, though it emerged earlier in [2] as a tool for constructing Hamiltonian operators in formal variational calculus. The axioms of Novikov algebras appear in [1] as necessary and sufficient conditions for the local algebra of a formal Poisson bracket to meet the Jacobi identity.

A series of examples of Novikov algebras may be constructed as follows [2]. For an associative and commutative algebra V with a derivation d, let $u \circ v = ud(v)$, for $u, v \in V$. Then $(V, \circ)$ is a Novikov algebra. This construction is known to be generic [3], i.e., every Novikov algebra embeds into an appropriate commutative algebra with a derivation. The proof of this statement in [3] is based on the Gröbner–Shirshov bases theory for Novikov algebras, the latter essentially uses the fundamental result of [4], where it was shown that the free Novikov algebra $\mathrm{Nov}(X)$ generated by a set X embeds into the algebra of differential polynomials in X. However, modulo this fact from [4], the embedding of an arbitrary Novikov algebra into a commutative differential algebra may be proved in a shorter way (see [5]). Therefore, the result of [4] (which is mostly combinatorial) plays a key role in the theory of Novikov algebras.

Our purpose is to find a straightforward way to prove the embedding of a Novikov-type algebra with two operations satisfying the identities found by J.-L. Loday [7],

$$\begin{gathered} x \succ (y \prec z) = (x \succ y) \prec z, \\ (x \prec y) \succ z - x \succ (y \succ z) = x \prec (y \succ z) - (x \prec y) \prec z, \end{gathered} \tag{1}$$

into an appropriate associative differential algebra by means of the (differential) Gröbner–Shirshov bases theory. On the one hand, Gröbner and Gröbner–Shirshov bases are the tools that are especially designed for solving such embedding problems. On the other hand, the explicit calculation of the Gröbner basis (relative to a chosen order of monomials) for the ideal generated by defining identities highly depends on the particular multiplication table.

We present a way how to overcome this problem and prove that every algebra that meets the identities (1) embeds into an associative differential algebra.

This is a joint work with B. Sartayev.

Keywords— Derivation, Novikov algebra, Gröbner basis

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On unitary Nil K_1-groups

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Abstract

In the talk we will introduce several Nil-subgroups of the unitary Bass' nilpotent K_1-group of a unitary ring and present some properties of these Nil-groups. These properties are unitary analogues well-known properties of the Bass' nilpotent K_1-group of a ring in algebraic $K-$theory.

Plane Curves: a Linear Algebra Approach

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Abstract

By a *generic plane curve* $\gamma : S^1 \to \mathbb{R}^2$ we mean an immersion of an (oriented) circle S^1 into a plane $\mathbb{R}^2$ having only transversal double points of self-intersection. Any generic plane curve γ may be encoded by its *Gauss diagram* $\mathfrak{G}(\gamma)$. The Gauss diagram is the immersing circle S^1 with the preimages of each double point connected with a chord.

A Gauss diagram $\mathfrak{G}$ is called *realizable* if there is a generic plane curve $\gamma : S^1 \to \mathbb{R}^2$ such that $\mathfrak{G} = \mathfrak{G}(\gamma)$.

The problem concerning which Gauss diagrams can be realized by a plane curve is an old one and has been solved in several ways.

We aim to show that a rialization of Gauss diagram can be also obtained by using the famous Jordan curve theorem. In fact we introduce a notation of Jordan curve in terms of Guass diagram and prove that Gauss diagram is ralizable if and only if any such Jordan curve diviedes it into two regions.

We also show that the criteria of realizability of Gauss diagrams can be reformulated in a more useful and practical way; to check whether a Gauss diagram is realizable it is enough to solve a system of linear equations over the field $\mathsf{GF}(2)$.

Given a Guass diagram $\mathfrak{G}$, we then get its adjacency matrix $M = (m_{i,j})_{1\le i,j\le n} \in \mathrm{Mat}_{n\times n}(\mathsf{GF}(2))$, and hence $M^2 = (\langle m_i, m_j\rangle)_{1\le i,j\le n}$, where

$$\langle m_i, m_j\rangle := m_{i,1}m_{j,1} + \cdots + m_{i,n}m_{j,n}, \tag{2}$$

and $m_k := (m_{k,1}, \ldots, m_{k,n})$ is the kth row of the M.

Theorem. *A Guass diagram $\mathfrak{G}$ is realizable if and only if the following system of equations*

$$\Big\{(\alpha_i + \alpha_j)m_{i,j} = \langle m_i, m_j\rangle + m_{i,j}, \quad 1 \le i,j \le n \tag{3}$$

has a solution over the field $\mathsf{GF}(2)$.

We then deduce a needed and sufficient conditions for a graph which is correspondents to a meander *i.e.,* in fact we give a bijection between meanders an a special sort of Gauss diagrams which aroused form the Thurston generators of braid groups. It allows us to give an algorithm to construct such diagrams and to code meanders by matrices which are exactly incident matrices of the corresponding adjacency graphs of the diagrams. Finally we show that these matrices are idempotent over the field $\mathsf{GF}(2)$.

Keywords— plane curves, Gauss diagrams, meanders.

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The Golod property of face rings from the topological viewpoint

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Abstract

In 1950s J.-P.Serre proved that Poincaré series of a commutative local Noetherian ring is bounded by a certain rational function depending on the Betti numbers of the Koszul complex and the minimal number of generators in the maximal ideal. In 1962 E.S.Golod showed that Serre's inequality turns into equality if and only if multiplication and all Massey products in Koszul homology of a local ring are trivial. J.Backelin proved in 1982 that Poincaré series of monomial rings are rational; among monomial rings there is the well-known class of Stanley-Reisner rings (or, face rings) of simplicial complexes.

In this talk we will discuss how toric topology enables us to establish combinatorial, algebraic and topological conditions equivalent to Golodness and minimal non-Golodness of a face ring of a simplicial complex over any field. We will describe these two classes of Stanley-Reisner rings in terms of their Poincaré series, Koszul homology, and the Lie algebra structure on the loop homology of the corresponding moment-angle-complexes. We will see how the theory of spaces with compact torus actions allows us to obtain topological interpretations of algebraic properties of Poincaré series and Koszul homology of Stanley-Reisner rings as well as to get new results.

The talk is based in part on a joint work with Taras Panov.

Equivariant cohomology of moment-angle complexes with respect to coordinate subtori

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Abstract

Given a simplicial complex $\mathcal{K}$ on $[m] = \{1, \dots, m\}$, the moment-angle complex $\mathcal{Z}_\mathcal{K}$ is the polyhedral product $(D^2, S^1)^\mathcal{K}$, a complex with a compact torus action composed of products of discs and circles indexed by the faces of $\mathcal{K}$.

There is an equivariant deformation retraction $U(\mathcal{K}) \to \mathcal{Z}_\mathcal{K}$, where $U(\mathcal{K})$ is the universal toric space (the complement of a coordinate subspace arrangement)

$$U(\mathcal{K}) = (\mathbb{C}, \mathbb{C}^\times)^\mathcal{K} = \mathbb{C}^m \setminus \bigcup_{\{j_1,\dots,j_k\}\notin\mathcal{K}} \{z_{j_1} = \dots = z_{j_k} = 0\}$$

in the Batyrev–Cox quotient construction of toric varieties.

It is well known that the T^m-equivariant cohomology ring of $\mathcal{Z}_\mathcal{K}$ (or $U(\mathcal{K})$) is isomorphic to $\mathbb{Z}[\mathcal{K}]$, the face ring of $\mathcal{K}$ (the Stanley–Reisner ring).

We consider equivariant cohomology of $\mathcal{Z}_\mathcal{K}$ with respect to the action of coordinate subtori $T_I \subset T^m$, where $I = \{i_1, \dots, i_k\} \subset [m]$. Using a polyhedral product descomposition of the Borel construction $ET_I \times_{T_I} \mathcal{Z}_\mathcal{K}$ we construct the following commutative integral dga model for $H^*_{T_I}(\mathcal{Z}_\mathcal{K})$:

$$\big(\Lambda[u_i\colon i \notin I] \otimes \mathbb{Z}[\mathcal{K}], d\big), \quad du_i = v_i,\ dv_i = 0,$$

where $\Lambda[u_i\colon i \notin I]$ is the exterior algebra on degree-one generators u_i. This leads to ring isomorphisms

$$H^*_{T_I}(\mathcal{Z}_\mathcal{K}) \cong H\big(\Lambda[u_i\colon i \notin I] \otimes \mathbb{Z}[\mathcal{K}], d\big) \cong \mathrm{Tor}_{\mathbb{Z}[v_1,\dots,v_m]}\big(\mathbb{Z}[v_i\colon i \in I], \mathbb{Z}[\mathcal{K}]\big).$$

Next, we study the equivariant formality of $\mathcal{Z}_\mathcal{K}$, that is, whether $H^*_{T_I}(\mathcal{Z}_\mathcal{K})$ is a free module over the polynomial ring $H^*_{T_I}(pt) = H^*(BT_I) = \mathbb{Z}[v_i\colon i \in I]$. We prove

Theorem. *Let $\mathcal{K}$ be a simplicial complex on a finite set V. The following conditions are equivalent:*

(a) For any $I \in \mathcal{K}$, the equivariant cohomology $H^*_{T_I}(\mathcal{Z}_\mathcal{K})$ is a free module over $H^*(BT_I)$.

(b) There is a partition $V = V_1 \sqcup \dots \sqcup V_p \sqcup U$ such that

$$\mathcal{K} = \partial\Delta(V_1) * \dots * \partial\Delta(V_p) * \Delta(U),$$

where $\Delta(U)$ denotes a full simplex on U, and $\partial\Delta(V_i)$ denotes the boundary of a simplex on V_i.

(c) The rational face ring $\mathbb{Q}[\mathcal{K}]$ is a complete intersection ring (the quotient of the polynomial ring by an ideal generated by a regular sequence).

This is a joint work with Indira Zeinikesheva.

Keywords— moment-angle complex, equivariant cohomology, equivariant formality, graded modules over polynomial rings

References

[1] Panov, Taras; Zeinikesheva, Indira. *Equivariant cohomology of moment-angle complexes with respect to coordinate subtori.* Preprint (2022), arXiv:2205.14678.

Automorphism groups of affine varieties without non-algebraic elements

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Abstract

Given an affine algebraic variety X over an algebraically closed field $\mathbb{K}$, we study when the neutral component $\mathrm{Aut}^\circ(X)$ of the automorphism group consists of algebraic elements. We conjectured in [1] that the following conditions on $\mathrm{Aut}^\circ(X)$ are equivalent:

- all unipotent elements commute,
- it consists of algebraic elements,
- it is nested, i.e., a direct limit of algebraic subgroups,
- it is a semidirect product of an algebraic torus and an abelian unipotent group.

In [1] we proved the conjecture for the group generated by connected algebraic subgroups instead of $\mathrm{Aut}^\circ(X)$. In this talk we present our further development: we proved that $\mathrm{Aut}^\circ(X)$ consists of algebraic elements if and only if it is nested.

To prove it, we obtained the following fact: if a connected ind-group G contains a closed connected ind-subgroup $H \subset G$ with a geometrically smooth point, and for any $g \in G$ some power of g belongs to H, then $G = H$.

The talk is based on the joint work with Andriy Regeta [2]. The research of the speaker was carried out at the HSE University at the expense of the Russian Science Foundation (project no. 21-71-00062)

Keywords— affine variety, automorphism group, ind-group, nested group

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Geometry of minuscule A_1-subgroups in E_7

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Abstract

This talk is joint with Andrei Semenov.

Let G be a simple algebraic group over a field F of characteristic 0. In [4] the authors introduced a notion of a *minuscule* A_1-subgroup of G: this means that the Dynkin index of the respective embedding is 1 or, what is the same, the adjoint representations of G decomposes into a sum of the adjoint representation of A_1 and several copies of the trivial and the vector representations. All such subgroups are conjugate over an algebraic closure and form F-points of a symmetric space. In the case of groups of type E_7 the respective symmetric space is $EVI = E_7/D_6 + A_1$, and the set of F-points becomes non-empty after passing to an odd degree extension, see [5].

Moreover, as Boris Rosendfeld noted in [6], one can introduce a structure of an "elliptic plane" on this set: both points and lines are minuscule A_1-subgroups, and a point is incident to a line of the respective subgroups commute. It is not, however, an elliptic plane in the usual sense: two lines in a general position meet at 3 points [7]. More generally, in the case when $F = \mathbb{R}$ Atsuyama in [1] described the intersection set of any two lines; in each case it forms a smaller symmetric space.

We generalize Atsuyama's result to the case of arbitrary field of characteristic 0, provided that the group is anisotropic. We describe all possible mutual positions of two minuscule A_1-subgroups in the table below. The first column contains the type of the subgroup generated by two A_1-subgroup, the second column contains the type of its centralizer in E_7, and the third column is the symmetric space from Atsuyama's list.

A_1	D_6	$SO_{12}/SO_4 \cdot SO_8$
$A_1 + A_1$	$D_4 + A_1$	$SO_8/SO_4 \cdot SO_4 \coprod pt$
A_2	A_5	$SU_6/S(U_4 \cdot U_2)$
B_2	$B_3 + A_1$	$SO_7/SO_4 \cdot SO_3 \coprod pt$
G_2	C_3	$Sp_6/Sp_4 \cdot Sp_2$
A_3	$A_3 + A_1$	$SO_6/SO_4 \cdot SO_2 \coprod pt$
B_3	$B_2 + A_1$	$SO_5/SO_4 \coprod pt$
D_4	$A_1 + A_1 + A_1$	$pt \coprod pt \coprod pt$

The proof essentially relies on the classification of "symplectic ternary algebras" (also known as J-ternary algebras and Freudenthal triple systems) by Faulkner and Ferrar [2]. We hope that our result may help in settling the E_7-case of the famous Serre II Conjecture (see [3]).

Keywords— symmetric spaces, minuscule embeddings, exceptional groups

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Singular Del Pezzo varieties

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Abstract

A del Pezzo variety X is a Fano variety whose anticanonical class has the form

$$-K_X = (n-1)A,$$

where A is an ample line bundle and n is the dimension of X. This is a higher-dimensional analog the notion of del Pezzo surfaces. I am going to discuss biregular and birational classifications of del Pezzo varieties admitting terminal singularities.

The talk is based on a joint work with Alexander Kuznetsov.

Toral varieties

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Abstract

Let X be an affine algebraic variety over algebraically closed field k of characteristic zero. Denote by $\mathbb{G}_a = (k, +)$ the additive group of the field k. We say that X is rigid if X admits no non-trivial actions of $\mathbb{G}_a$.

In some cases the abssence of additive actions allows to describe the group of automorphisms of rigid varieties. It was proven in [1] that if X is a rigid variety then there is a subtorus T in $\mathrm{Aut}(X)$ that contains any other subtorus of $\mathrm{Aut}(X)$.

A variety is called toral if it is isomorphic to a closed subvariety of an algebraic torus. It is equivalent to saying that the algebra of regular functions is generated by invertible functions. Toral varieties are natural examples of rigid varieties. In the talk, we will discuss some properties of toral varieties obtained jointly with Anton Trushin. In particular, we will show the if T is the maximal subtorus in $\mathrm{Aut}(X)$ of toral variety X then X is isomorphic to $Y \times T$ where Y is an affine variety.

Keywords— Rigid varieties, toral varieties

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Birational automorphism groups of surfaces over finite fields

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Abstract

I will survey some results on finite subgroups of birational automorphism groups of surfaces over finite fields. One of them is that the group of birational automorphisms of a plane is Jordan, that is, every finite subgroup contains a normal abelian subgroup whose index is bounded by a universal constant (depending only on the field). Moreover, the relevant constants can be explicitly computed. This is in contrast to the birational automorphism groups of the plane over algebraically closed fields of positive characteristic, and of higher-dimensional projective spaces over finite fields, which lack the Jordan property.

On the $\mathbb{A}^1$-fundamental group of Chevalley groups.

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Abstract

This talk is based on the preprint [2] which is a joint work with A. Lavrenov and E. Voronetsky. The main purpose of [2] is to compute the fundamental group of Chevalley–Demazure group schemes $\mathrm{G}_{sc}(\Phi, -)$, where Φ is an irreducible simply-laced root system.

Let A be a regular Noetherian ring containing an arbitrary field k. Recall that for an A-scheme X the $\mathbb{A}^1$-homotopy group $\pi_n^{\mathbb{A}^1}(X)$ is defined as the sheaf $R \mapsto \mathrm{Hom}_{\mathcal{H}_*^{\mathbb{A}^1}}(S^n \wedge \mathrm{Spec}(R)_+, X)$, where $\mathcal{H}_*^{\mathbb{A}^1}$ is the pointed unstable $\mathbb{A}^1$-homotopy category over A. From the results of [1] it follows that for an arbitrary isotropic reductive group scheme G defined over A the sheaf $\pi_n^{\mathbb{A}^1}(G)$ coincides with the sheaf of Karoubi–Villamayour groups $R \mapsto \mathrm{KV}_{n+1}(G, R)$. The latter groups are, in turn, defined as n-th homotopy groups of the singular simplicial group $\mathrm{Sing}^{\mathbb{A}^1}(G) = G(R[\Delta^\bullet])$.

Let G be an isotropic group of rank at least 2. In her recent work [5] A. Stavrova has shown that the group of connected components $\pi_0^{\mathbb{A}^1}(G)(A)$ coincides with the group of values of the unstable K_1-functor modeled on G. Much less is known about $\mathbb{A}^1$-fundamental groups. For example, in [7] it was shown that the group $\mathrm{KV}_2(G, k)$ coincides with the Schur multiplier $\mathrm{H}_2(G(k), \mathbb{Z})$ provided k is an infinite field. In turn the

Theorem(see [2], Theorem 1.1) *Let A be as above and assume that either $\Phi = \mathsf{A}_\ell$ for $\ell \geq 4$ or $\Phi = \mathsf{D}_\ell$ for $\ell \geq 7$. In the latter case assume additionally that* $\mathrm{char}(k) \neq 2$. *Then the unstable K_2-functor modeled on the Chevalley group $\mathrm{G}_{\mathrm{sc}}(\Phi, R)$ is $\mathbb{A}^1$-invariant, i. e.* $\mathrm{K}_2(\Phi, A) = \mathrm{K}_2(\Phi, A[t])$ *and, moreover, one has $\pi_1^{\mathbb{A}^1}(G)(A) = \mathrm{KV}_2(G_{\mathrm{sc}}(\Phi, -), A) = \mathrm{K}_2(\Phi, A)$.*

In the above statement we denote by $\mathrm{K}_2(\Phi, A)$ the kernel of the natural map from the Steinberg group $\mathrm{St}(\Phi, A)$ to the simply-connected Chevalley group $\mathrm{G}_{\mathrm{sc}}(\Phi, A)$. The case $\Phi = \mathsf{A}_\ell$ of our Theorem is actually a refinement of the main result of [6], whereas the case $\Phi = \mathsf{D}_\ell$ is based on the Horrocks theorem for KO_2, see [3].

The invariance of $\mathrm{K}_2(\Phi, A)$ mentioned above is the analogue of the famous Lindel–Popescu theorem for the functor K_2 (recall that the Lindel–Popescu theorem is a partial answer to the geometric case of Bass–Quillen conjecture). From the $\mathbb{A}^1$-invariance for $\mathrm{K}_2(\Phi, -)$ one can extract presentations a la Steinberg for Chevalley groups over multivariate polynomial rings in terms of generators and relations. Such presentations are a far-reaching generalization of the classical results for polynomial rings in one variable over a field (e. g. [4]). Yet another corollary of our results are the following co: $\mathrm{H}_2(\mathrm{SO}_{2\ell}(R[t]), \mathbb{Z}) = \mathrm{H}_2(\mathrm{SO}_{2\ell}(R), \mathbb{Z})$, $\mathrm{H}_2(\mathrm{O}_{2\ell}(R[t]), \mathbb{Z}) = \mathrm{H}_2(\mathrm{O}_{2\ell}(R), \mathbb{Z})$.

Keywords— Steinberg groups, Chevalley groups, K_2-functor, $\mathbb{A}^1$-homotopy theory

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Danilov-Koshevoy arrays and dual Grothendieck polynomials

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Abstract

The ring of symmetric polynomials has several nice bases; probably the most remarkable of them is formed by Schur polynomials. They appear everywhere: in combinatorics as generating functions for Young tableaux, in representation theory as the characters of $\mathrm{GL}(n)$, in geometry as the cohomology classes of Schubert varieties in Grassmannians. The multiplication of Schur polynomials is given by a quite involved combinatorial rule, called the Littlewood–Richardson rule.

In [1] Danilov and Koshevoy introduced a new combinatorial tool: the arrays. An array is a rectangular board divided into squares (like a chessboard). These squares can contain balls that can be moved according to certain rules. Using arrays provides uniform and simple proofs of various statements about Schur polynomials: the RSK correspondence, the Bender–Knuth involution, the Littlewood–Richardson rule, the crystal operators on Kashiwara crystals etc.

Schur polynomials have numerous generalizations. One of them, the dual stable Grothendieck polynomials, is obtained by replacing the Young tableaux in the combinatorial definition by the so-called reverse plane partitions: tableaux filled by numbers weakly increasing along both rows and columns. They were introduced by T. Lam and P. Pylyavskyy in [2] as a combinatorial gadget for dealing with the K-theory of Grassmannian. I will speak about a generalization of Danilov–Koshevoy arrays that allows us to work with these polynomials: for this, we will put on the board not just single balls, but also balls connected into "strings of beads".

The talk is based on our joint work with Anastasia Sukacheva.

Keywords— symmetric functions, Schur polynomials, Grothendieck polynomials, Grassmannians, K-theory

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Super Yangians and Quantum Loop Superalgebras

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Abstract

We consider the relationship between super Yangians and quantum loop superalgebras. We consider structures of tensor categories on analogs of the category $\mathfrak{O}$ for representations of the super Yangian $Y_\hbar(A(m,n))$ of the special linear Lie superalgebra and the quantum loop superalgebra $U_q(LA(m,n))$, explore the relationship between them. The construction of an isomorphism in the category of Hopf superalgebras between completions of the super Yangian and the quantum loop superalgebra endowed with the so-called "Drinfeld" comultiplications is described. A theorem on the equivalence of the tensor categories of modules of the super Yangian and the quantum loop superalgebra is formulated, which strengthens the previous result. We also describe the relationship between Quasi-Triangular structures and Abelian difference equations, which are determined by the Abelian parts of universal R-matrices. We also define an affine super Yangian $Y_{\epsilon_1,\epsilon_2}(\tilde{sl}(m,n))$ for an arbitrary system of simple roots Π of affine Kac-Moody superalgebra $\tilde{sl}(m,n)$. We introduce two type presentation of super Yangian, namely minimalistic and current presentation. We prove that this two presentations are equivalent. It is proved that the super Yangians of a quantum affine superalgebra $\tilde{sl}(m,n)$ defined by different simple root systems Π and Π_1 are isomorphic as associative superalgebras. Some of these results were obtained in articles [1], [2].

Keywords — Super Yangian, Quantum Loop Superalgebra, Hopf superalgebra structure, affine super Yangian, category of representations

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Component group of the real locus of a connected linear algebraic group

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Abstract

For a connected linear algebraic group G defined over the field of real numbers $\mathbb{R}$, the group of real points $G(\mathbb{R})$ is a real Lie group, not necessarily connected; look at $GL_n(\mathbb{R})$ or $SO_{k,l}(\mathbb{R})$ for example. A natural problem is to determine the component group of $G(\mathbb{R})$. In our joint work with Mikhail Borovoi [1] we came to this problem as a by-product of our consideration of Galois cohomology of linear algebraic groups over $\mathbb{R}$. Namely it turns out that the component group $\pi_0 G(\mathbb{R})$ is isomorphic to the kernel of the Galois cohomology map $H^1(\mathbb{R}, \pi_1 G) \to H^1(\mathbb{R}, \widetilde{G})$, where $\pi_1 G$ is the fundamental group of G and $\widetilde{G}$ is the universal cover of G. Based on this observation, we obtained in [1] a certain combinatorial description of $\pi_0 G(\mathbb{R})$ as the stabilizer of a distinguished point in a finite combinatorially defined set acted on by a finite Abeian group.

In this talk I present a much more explicit description for $\pi_0 G(\mathbb{R})$ [2]. Let T_s be a maximal $\mathbb{R}$-split subtorus in G and T a maximal torus in G defined over $\mathbb{R}$ and containing T_s. Let $\mathsf{X}^\vee$ and $\mathsf{X}_s^\vee$ denote the cocharacter lattices of T and T_s, respectively, $\mathsf{Q}^\vee$ the coroot lattice of the quotient of G modulo its radical, and $\mathsf{Q}_s^\vee = \mathsf{Q}^\vee \cap \mathsf{X}_s^\vee$. Let $\widetilde{\mathsf{X}}_s^\vee$ be the image of $\mathsf{X}^\vee$ under projection $\operatorname{Lie} T \to \operatorname{Lie} T_0$. Then

$$\pi_0 G(\mathbb{R}) \simeq \mathsf{X}_s^\vee / (2\widetilde{\mathsf{X}}_s + \mathsf{Q}_s^\vee).$$

To prove this formula, we relate Galois cohomology of $\pi_1 G$ to the maximal split torus T_s instead of maximal anisotropic tori used in [1] for computation of Galois cohomology of G.

Keywords— Real algebraic group, component group, cocharacter lattice, Galois cohomology

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A generating set of the automorphism group of a graded algebra

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Abstract

Let $A = \mathbb{K}[x_1, \ldots, x_n]$ be the polynomial ring in variables $x_1, \ldots, x_n$ over a field $\mathbb{K}$, and let $\mathrm{Aut}A$ be the group of automorphisms of A as an algebra over $\mathbb{K}$. An automorphism $\tau \in \mathrm{Aut}A$ is called *elementary* if it has form

$$\tau : (x_1, \ldots, x_{i-1}, x_i, x_{i+1}, \ldots, x_n) \mapsto (x_1, \ldots, x_{i-1}, \alpha x_i + f, x_{i+1}, \ldots, x_n), \tag{4}$$

where $0 \neq \alpha \in \mathbb{K}, f \in \mathbb{K}[x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n]$. We call an automorphism φ *tame* if it is the composition of elementary automorphisms. An automorphism is said to be wild if it is not tame. In 2004 Shestakov and Umirbaev proved that the Nagata automorphism σ of the polynomial algebra in three variables is wild [1], where

$$\begin{cases} \sigma(x) = x + (x^2 - yz)z; \\ \sigma(y) = y + 2(x^2 - yz)x + (x^2 - yz)^2 z; \\ \sigma(z) = z. \end{cases} \tag{5}$$

We fix a $\mathbb{Z}$-grading on this algebra and consider graded-wild automorphisms, i.e. such automorphisms that can not be decomposed onto elementary automorphisms respecting the grading. The paper [2] gives a classification of gradings that admit graded-wild automorphisms.
Our goal is a system of group-generating automorphisms for such algebras. In the paper [2] graded polynomial algebras in three variables admitting wild automorphisms are reduced to polynomial algebras in two variables. With this technique, we will show that Nagata-type automorphisms generate the entire automorphism group. This report was supported by RSF grant № 22-41-02019.

Keywords— graded algebras, wild automorphisms

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Bounded generation of Chevalley groups

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This talk is based on a series of recent joint works with Boris KUNYAVSKII and Eugene PLOTKIN, see, in particular [1], where one can find a general survey, many further references, precise statements, and proofs.

We state several results on bounded elementary generation and bounded commutator width for Chevalley groups over Dedekind rings of arithmetic type in positive characteristic.

In particular, Chevalley groups of rank ≥ 2 over polynomial rings $\mathbb{F}_q[t]$ and Chevalley groups of rank ≥ 1 over Laurent polynomial $\mathbb{F}_q[t, t^{-1}]$ rings, where $\mathbb{F}_q$ is a finite field of q elements, are boundedly elementarily generated.

We sketch several proofs, which start with reduction to smaller ranks, either via Tavgen rank reduction, or via surjective stability for K_1, with explicit bounds, which oftentimes are better than the known ones even in the number case.

This leaves us with the analysis of rank 1 or rank 2 cases, which depend on rather deep arithmetical results. For the group SL_2 we refer to an old paper by Clifford Queen, the case of $\mathrm{SL}(3, \mathbb{F}_q[t])$ was recently solved by Bogdan Nica [2].

The core of the present work is a similar calculation for $\mathrm{Sp}(4, \mathbb{F}_q[t])$. Our proof follows the same general lines as the proofs by David Carter, Gordon Keller and Oleg Tavgen in the number case, but now we have to redo all calculations using the fancier form of the reciprocity laws, and also the classical calculations by Bass, Milnor, and Serre regarding properties of [the long and short root type] symplectic Mennicke symbols.

As a result, we establish rather plausible explicit bounds for the elementary width that depend on the root system alone, and not on q, such as

$$w_E\big(\mathrm{Sp}(4, \mathbb{F}_q[t])\big) \leq 79, \qquad w_E\big(\mathrm{Sp}(6, \mathbb{F}_q[t])\big) \leq 72, \qquad w_E\big(\mathrm{SO}(7, \mathbb{F}_q[t])\big) \leq 65.$$

Using these bounds we can also produce sharp bounds on the commutator width of these groups.

We also mention several immediate applications:

- affine Kac—Moody groups,
- model theoretic and logical applications, such as first order rigidity;

and imminent generalisations:

- strong bounded generation,
- bounded verbal width, etc.

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Presentation of relative Steinberg groups of types BCF

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Abstract

Let $\mathrm{G}(\Phi, K)$ be the simply connected Chevalley group over a commutative ring K associated with a simple root system Φ of rank $\ell \geq 3$. The *Steinberg group* $\mathrm{St}(\Phi, K)$ is the abstract group with the generators $x_\alpha(p)$ corresponding to the root elements $t_\alpha(p) \in \mathrm{G}(\Phi, K)$ and the only "obvious" relations between the root elements, namely,

$$x_\alpha(p)\, x_\alpha(q) = x_\alpha(p+q),$$
$$[x_\alpha(p), x_\beta(q)] = \prod_{\substack{i\alpha+j\beta\in\Phi \\ i,j>0}} x_{i\alpha+j\beta}(N_{\alpha\beta ij} p^i q^j),$$

where in the second identity $\alpha \neq -\beta$ and $N_{\alpha\beta ij} \in \mathbb{Z}$ are the structure constants.

For any ideal $\mathfrak{a} \leq K$ there is an appropriate *relative Steinberg group* $\mathrm{St}(\Phi, K, \mathfrak{a})$ [2, 3] defined as the crossed module over $\mathrm{St}(\Phi, K)$ with the generators $x_\alpha(a)$ for $a \in \mathfrak{a}$ and some relations involving their conjugates by elements of the absolute Steinberg group. It is classically known that $\mathrm{St}(\Phi, K, \mathfrak{a})$ is generated as an abstract group by the "elementary conjugates" $z_\alpha(a, p) = {}^{x_{-\alpha}(p)}x_\alpha(a)$. Recently we found all the relations between these generators if Φ is simply laced [4], i.e. of type ADE.

In this talk we recall the simply laced case and give a complete list of the relations between $z_\alpha(a, p)$ if Φ is doubly laced, i.e. of one of the remaining cases B_ℓ, C_ℓ, or F_4. This result actually generalizes to the relative Steinberg groups parametrized by E. Abe's *admissible pairs* $(\mathfrak{a}, \mathfrak{b})$ [1] instead of single ideals $\mathfrak{a}$.

Keywords— Chevalley groups, Steinberg groups

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Gorenstein local algebras and additive actions on projective hypersurfaces

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Abstract

An additive action on an algebraic variety is an effective regular action with an open orbit of a commutative unipotent linear algebraic group. In other words, we study open equivariant embeddings of vector groups into algebraic varieties. Hassett and Tschinkel established a correspondence between commutative local Artinian unital algebras and additive actions on projective spaces [5]. This approach may be applied to the study of additive actions on projective hypersurfaces [2, 1, 4]. It turns out that the case of non-degenerate hypersurfaces corresponds to Gorenstein local algebras and several results on additive actions may be proved using this technique. In particular, we prove that there is at most one additive action on a non-degenerate projective hypersurface. The talk is based on the joint work with Ivan Arzhantsev [3]. Supported by the RSF grant 19-11-00172.

Keywords— additive action, local algebra, projective space, Gorenstein algebra, projective hypersurface

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On some questions around Berest's conjecture

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Abstract

Berest's conjecture about orbits in the first Weyl algebra states that the number of orbits of solutions of a polynomial equation F(X,Y)=0 in the first Weil algebra, where F is an irreducible polynomial over a field of characteristic zero, is finite, if the arithmetic genus of the corresponding plane curve is > 1, and is infinite otherwise. This conjecture is closely related to the theory of commuting ordinary differential operators, as well as with the well-known Dixmier conjecture on endomorphisms of the first Weyl algebra. Several recent works were devoted to testing this conjecture in some special cases. Although in the various studied examples this conjecture turns out to be false, it is still interesting for further study, especially over the field Q. In my talk I will review the already known as well as recently obtained together with Junho Guo results around this conjecture.

Keywords— Weyl algebra, Dixmier conjecture, commuting ordinary differential operators

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Twisted forms of commutative monoid structures on affine spaces

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Abstract

An affine algebraic monoid is an irreducible affine variety M together with an assosiative multiplication map $\cdot : M \times M \longrightarrow M$, where map $\cdot$ is a morphism of algebraic varieties, and unit $e \in M(K)$ such that $em = me = m$ for all $m \in M$. A monoid M is called commutative, if, in addition, $ab = ba$ for all $a, b \in M$. This talk is based on the paper [2], which stands as a generalization of the paper [1], where affine algebraic commutative monoids were studied over algebraically closed field L of characteristic zero, and classification of such monoids on $\mathbb{A}^2_L$ and $\mathbb{A}^3_L$ was reached. Also it is worth mentioning that same results on algebraic monoids over affine surfaces in the case of algebraically closed field were obtained in [3] and [4]. Our goal is to obtain such classification over an arbitrary field K of characteristic zero, not necessary algebraically closed.

It will be explained that all the monoids studied here are the twisted forms of monoids defined in [1], i.e. they are obtained by Galois descent. Our main results claims that in dimensions 2 and 3 all nontrivial twisted forms can be described in terms of formulae that define multiplication and norm in separable algebras over the base field.

In order to achieve these results we use the idea of the connection between a monoid M and its group of invertible elements $G(M)$. Furthermore, we develop the connection between $Aut(M)$ and $Aut(G(M))$ and use the Galois descent technique applied to $Aut(M)$.

Keywords— Algebraic monoids, Affine spaces, Galois descent, Non-closed fields, Schemes

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