The use of a bi-nonlinear Volterra integral equation to model an earthquake

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Abstract

In this work, we construct a modern scientific demonstrate to bargain with seismic tremor wonder. the originality of our study, is that we use a nonconstant contact resistance. Our earthquake model is depicted by a bi-nonlinear Volterra equation. We demonstrate that this equation admits a unique solution through a few reasonable conditions. Numerical tests appear that our study is practical and approach superbly this physical phenomena.

Introduction

An earthquake is the shaking of the surface of the Earth resulting from a sudden release of energy in the Earth's lithosphere that creates seismic waves.

In its most general sense, the word earthquake is used to describe any seismic event—whether natural or caused by humans—that generates seismic waves. Earthquakes are caused mostly by rupture of geological faults but also by other events such as volcanic activity, landslides, mine blasts, and nuclear tests. An earthquake's point of initial rupture is called its hypocenter or focus.

The epicenter is the point at ground level directly above the hypocenter. Earthquakes can range in intensity, from those that are so weak that they cannot be felt, to those violent enough to propel objects and people into the air and wreak destruction across entire cities.

Main Objective

Our interest in this work is to model the seismic that results from the movement of tectonic plates.

Preface

The Incorporated Reasearch Institutions for Seismology provided a demo model to give an explanation for the earthquake model. This demonstration is offered in figure 1.

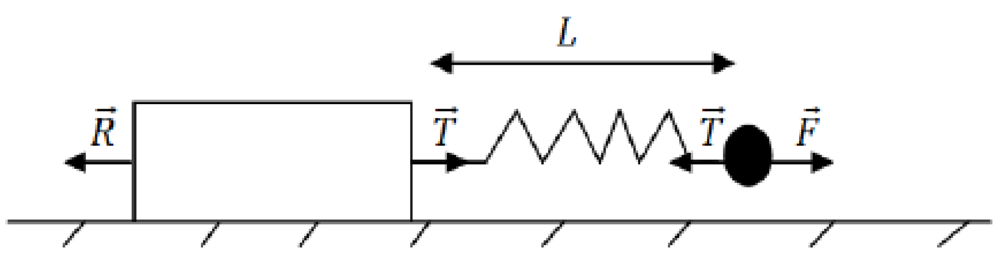

Figure 1: Earthquake Machine

Lately, Salah and al in [1] have provided a new nonlinear Volterra integro-differential equation for modeling simplified seismic phenomena. This equation was derived from the mechanical laws applied to the seismic process in two dimensions (time and position).

In this model, they assume that the resistence force resulting from the earthquake shock is constant over time and position. In order to express the physical phenomenon realistically, it is assumed that motion can be divided into two states, a rest state and a motion state.

To describe the rest state they managed to modifie Newton's law using a Heaviside function

$$(\tau) = \begin{cases} 1 & \tau \geq 0, \\ 0 & \tau < 0. \end{cases}$$

and the state of motion was modelled using the classical version of this law which represents violence to the laws of physics. Finally they obtained the following nonlinear integro-differential equation of Volterra of the second kind that is given in form:

$$u(t) = \int_0^t (t-s)(-\rho(\frac{1}{m}+\frac{1}{M})u(s) - \frac{\rho}{2M}\delta(-\frac{1}{\rho}u'(s) + \frac{1}{m}\int_0^s (F(\alpha) - R - u(\alpha))d\alpha(|u(s)| - u(s)))ds + f(t) \tag{1}$$

where, δ is the kronecker index given in the following way

$$\delta(\tau) = \begin{cases} 1 & \tau = 0, \\ 0 & \tau \neq 0. \end{cases} \tag{2}$$

Because of the discontinuity of δ, they proceed to approximate the problem (1) using δ_ε

$$\delta_\varepsilon(\tau) = \begin{cases} (1-(\frac{\tau}{\varepsilon})^2)^2 & \tau \in [-\varepsilon, \varepsilon], \\ 0 & \tau \in]-\infty, -\varepsilon] \cup [\varepsilon, +\infty[. \end{cases} \tag{3}$$

to obtain the following equation

$$u(t) = \int_0^t (t-s)(-\rho(\frac{1}{m}+\frac{1}{M})u(s) - \frac{\rho}{2M}\delta_\varepsilon(-\frac{1}{\rho}u'(s) + \frac{1}{m}\int_0^s (F(\alpha) - R - u(\alpha))d\alpha(|u(s)| - u(s)))ds + f(t) \tag{4}$$

making with that another approximation of reality.

They use the Nyström method to study their equation numerically. Their numerical results show that this type of modelling is efficient and compatible with the mechanism of the earthquake machine.

Main Objective

In our work,we seek to bring a better physical vision of the model, where we do not make any changes on Newton's law and we use a resistance force exactly as it was described by Coulomb.

New resistance

The thing that makes our study new and different is the fact that our resitance here is not any more concedered as a constant in our model, but it depends of the force that is applied to the object.

Expriments that were made gave us the figure below, that shows us the values of the resistence R in each time that we apply a force F on the object.

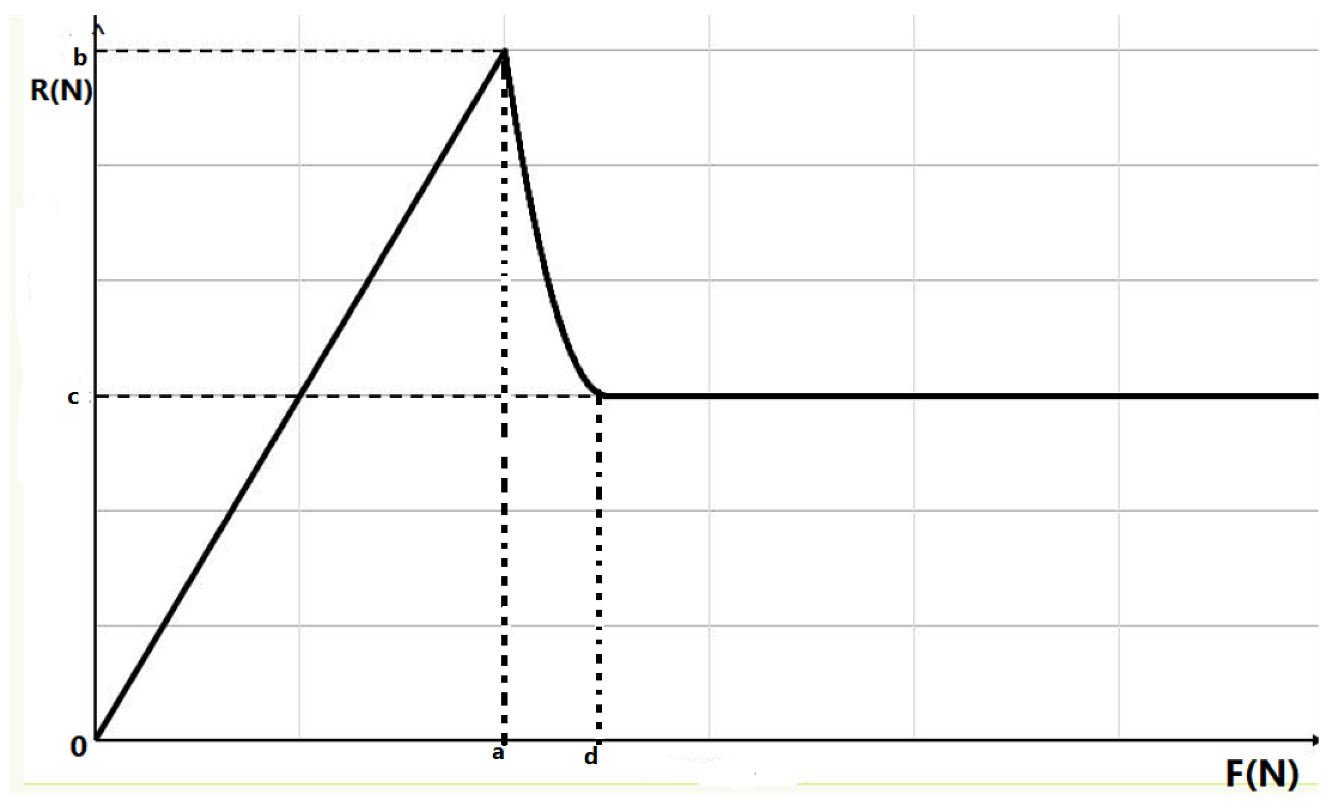

Figure 2: The resistance depending of the force applied to a stationary object.

The graph above demonstrates what happens when a stationary object is set in motion. At first, the frictional force is static, because the object is stationary. As soon as the applied tensile force becomes greater than the static frictional force, the object starts to move. The frictional force decreases rapidly (a phase called adhesive slip) and then becomes constant. The frictional force, now kinetic, is equal to the applied tensile force, thus maintaining the object's state of motion.

New earthquake model

We will follow the following path, applying the Newton's second low to objects in figure 1, we obtain the following system[?, ?, ?].

$$\begin{cases} \rho l(t) - R(Mx''(t)) = Mx''(t) \\ F - \rho l(t) = my''(t) \\ l(t) = (y(t) - x(t)) - L \end{cases} \tag{5}$$

with $x''(t)$ and $y''(t)$ are the acceleration of masses M and m respectively.

Using

$$l(t) = \frac{Mx''(t) + R(Mx''(t))}{\rho} \tag{6}$$

and

$$y(t) = L + l(t) + x(t) \tag{7}$$

after simplifications and some changes of variables we obtain the following bi-nonlinear Volterra integral equation

$$G(z(t) = -\frac{\rho}{M}\int_0^t (t-s)[z(s) + \frac{M}{m}z(s) + \frac{1}{m}R(Mz(s))]ds + g(t) \tag{8}$$

where $z(t) = x''(t)$ and $G(z(t)$ is a function depending of a,b,c,d and z.

Assumption 0.1. While

1. $b > c > 0$
2. $d > a > 0$

then

- $G(z) = y$ has a unique solution for all z and,for all positive scalar y and for all $0 \leq s \leq t \leq T$.
- There exists a contant $\theta > 0$ such that $|G(z) - G(\tilde{z})| \geq \theta|z - \tilde{z}|$ for all $0 \leq s \leq t \leq T$.
- The function $R(Mz(s))$ satisfies a Lipschitz condition for all $0 \leq s \leq t \leq T$.

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Analytical study

the idea of our analytical study is to use Picard's method that leans on the construction of a successive sequence $z_n(t)_{n\in\mathbb{N}}$, which is similar to the techniques used by Linz in [...].

This sequence is given by

$$G(z_n(t) = -\frac{\rho}{M}\int_0^t (t-s)[z_{n-1}(s) + \frac{M}{m}z_{n-1}(s) + \frac{1}{m}R(Mz_{n-1}(s))]ds + g(t) \tag{9}$$

Theorem 0.1. Suppose the assumptions (1) and (2) are verified, and then, (8) has a unique solution.

Proof. See [2] □

Numerical results

The results of our numerical calculus are illustrated in

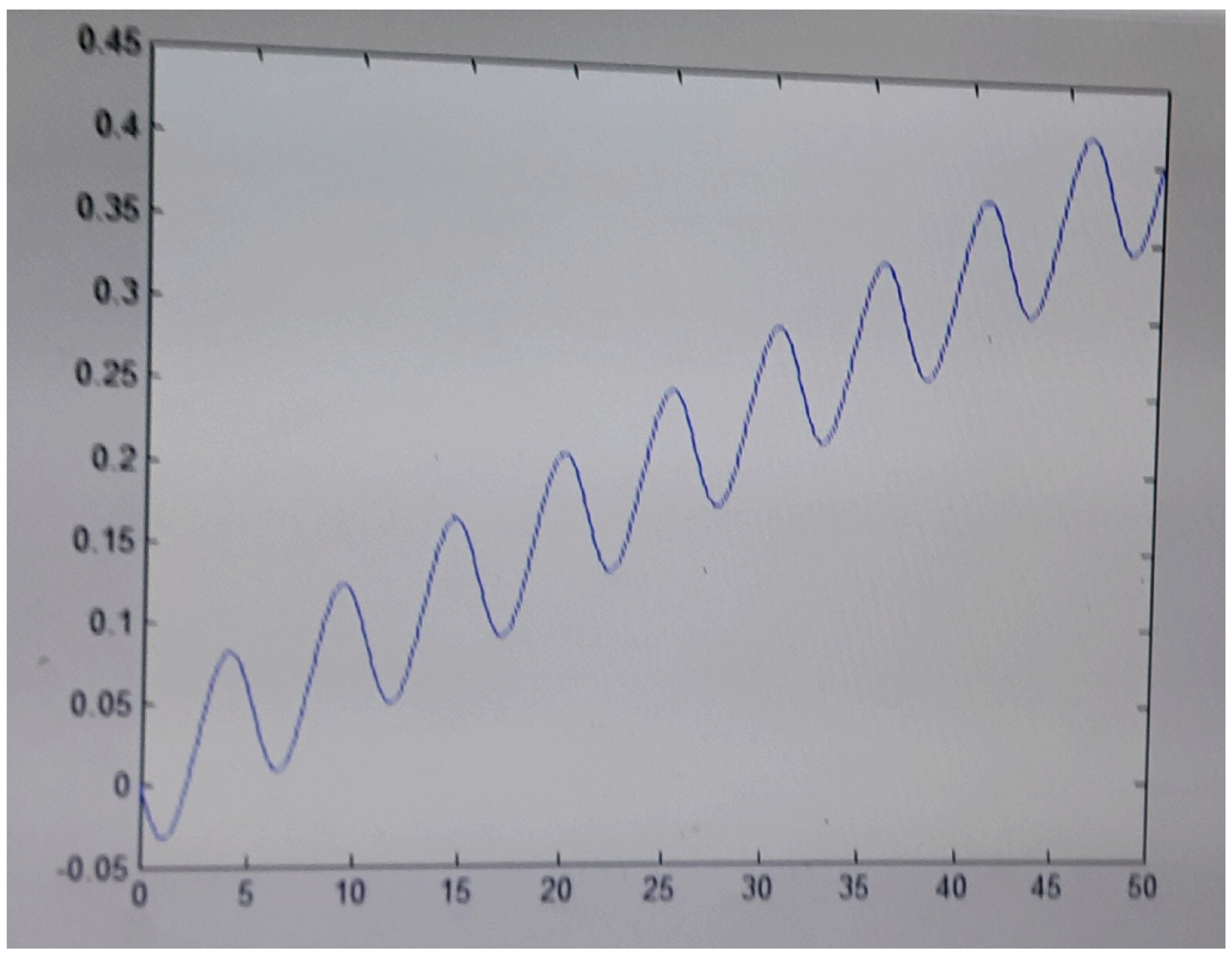

Figure 3: The seismic function

Conclusion

In this work, we build a new mathematical model to deal with earthquake phenomenon. What makes our study original, more efficient and realistic compared to other studies developed in this area is the use of a nonconstant friction resistance in opposite to old works. Our earthquake model is described by a bi-nonlinear Volterra equation. We prove that this equation has a unique solution through some realistic conditions. Numerical results show that our analytical and numerical visions are realistic and approach perfectly this physical phenomenon.

References

[1] Selma S. Solution of an integro-differential nonlinear equation of Volterra arising of earthquake model // J Bol. Soc. Paran. Mat 2022. https://doi.org/10.5269/bspm.48018

[2] Linz P.Analytical and numerical methods for Volterra equations (SIAM Studies in Applied Mathematics, Philadelphia, 1985). https://doi.org/10.1137/1.9781611970852

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