

## QUASIPERIODIC OPERATORS WITH MONOTONE POTENTIALS: SOME LITERATURE RECOMMENDATIONS

General spectral theory (spectral theorem, etc.):

- G. Teschl, *Mathematical Methods in Quantum Mechanics*.
- M. Reed, B. Simon, *Methods of Modern Mathematical Physics, Volume 1*.
- M. Birman, M. Solomyak, *Spectral Theory of Self-Adjoint Operators in Hilbert Space*.

Schnol's theorem (specific version for our purposes)

- R. Han, *Schnol's theorem and the spectrum of long range operators*, Proc. Amer. Math. Soc. **147** (2019), 2887 – 2897. See references for the original papers by Schnol.

Relation between spectral type and quantum dynamics:

- Y. Last, *Quantum dynamics and decompositions of singular continuous spectra*, J. Funct. Anal. **142**, 406 – 445 (1996).
- For further reading relating Fourier transforms and dimensions, see P. Mattila, *Fourier Analysis and Hausdorff Dimension*, Cambridge University Press, 2015.

Introduction to ergodic Schrödinger operators

- H. Cycon, R. Froese, W. Kirsch, B. Simon, *Schrödinger operators, with application to quantum mechanics and global geometry*, Texts and Monographs in Physics. Berlin etc. Springer-Verlag (1987). Chapters 9,10; also Chapter 6 (RAGE theorem).
- Book by D. Damanik and J. Fillman is in progress.
- D. Damanik, *Lyapunov exponents and spectral analysis of ergodic Schrödinger operators: a survey of Kotani theory and its applications*. Spectral theory and mathematical physics: a Festschrift in honor of Barry Simon's 60th birthday, 539 – 563, Proc. Sympos. Pure Math. **76**, Part 2, Amer. Math. Soc., Providence, RI, 2007.
- P. Deift, B. Simon, *Almost periodic Schrödinger operators III. The absolutely continuous spectrum in one dimension*, Comm. Math. Phys. **90** (1983), 389 – 411.

Ergodic theorems:

- A. Furman, *On the multiplicative ergodic theorem for uniquely ergodic systems*, Ann. Inst. H. Poincaré Probab. Statist. **33** (1997), no. 6, 797 – 815.

Anderson model: there are many proofs of localization in 1D, including two recent papers which contain a lot of references. I also include the standard book on the multi-dimensional case. Make sure to check references therein for full history.

- V. Bucaj, D. Damanik, J. Fillman, V. Gerbuz, T. VandenBoom, F. Wang, Z. Zhang, *Localization for the one-dimensional Anderson model via positivity and large deviations for the Lyapunov exponent*, Trans. Amer. Math. Soc. 372 (2019), 3619 – 3667.
- S. Jitomirskaya, X. Zhu, *Large Deviations of the Lyapunov Exponent and Localization for the 1D Anderson Model*, Comm. Math. Phys. 370 (2019), 311 – 324.
- M. Aizenman, S. Warzel, *Random Operators: Disorder Effects on Quantum Spectra and Dynamics*, AMS, 2016.

There are probably 100 or so papers on the almost Mathieu operator. The following papers contain most of references and state-of-the-art results:

- A. Avila, *The absolutely continuous spectrum of the almost Mathieu operator*, preprint, <https://arxiv.org/abs/0810.2965>, 2008.
- S. Jitomirskaya, *Metal-insulator transition for the almost Mathieu operator*, Ann. of Math. **150**, (1999), 1159 – 1175.
- S. Jitomirskaya and W. Liu, *Universal hierarchical structure of quasiperiodic eigenfunctions*, Ann. Math. **187** (2018), no. 3, 721 – 776.

Maryland model:

- A. Figotin, L. Pastur, *An exactly solvable model of a multidimensional incommensurate structure*, Comm. Math. Phys. 95 (1984), no. 4, 401 – 425.
- H. Cycon, R. Froese, W. Kirsch, B. Simon, see above (Section 10.3).
- D. Grempel, S. Fishman, and R. Prange, *Localization in an incommensurate potential: An exactly solvable model*, Phys. Rev. Lett. 49 (1982), 833 – 836.
- B. Simon, *Almost periodic Schrödinger operators. IV. The Maryland model*, Ann. Physics 159 (1985), no. 1, 157 – 183.
- S. Jitomirskaya and W. Liu, *Arithmetic spectral transitions for the Maryland model*, Comm. Pure Appl. Math. 70 (2017), no. 6, 1025 – 1051.
- A. Fedotov, F. Sandomirskiy, *An exact renormalization formula for the Maryland model*, Comm. Math. Phys. 334 (2015), no. 2, 1083 – 1099.

Monotone quasiperiodic operators:

- J. Bellissard, R. Lima, and E. Scoppola, *Localization in  $\nu$ -dimensional incommensurate structures*, Comm. Math. Phys. 88 (1983), no. 4, 465 – 477.
- S. Jitomirskaya and I. Kachkovskiy, *All couplings localization for quasiperiodic operators with lipschitz monotone potentials*, J. Eur. Math. Soc. 21 (2019), no. 3, 777 – 795.
- I. Kachkovskiy, *Localization for quasiperiodic operators with unbounded monotone potentials*, J. Funct. Anal. 277 (2019), no. 10, 3467 – 3490.
- I. Kachkovskiy, L. Parnowski, R. Shterenberg, *Convergence of perturbation series for unbounded monotone quasiperiodic operators*, <https://arxiv.org/abs/2006.00346v1>.