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**Lecture 3. Percolation and other stories**

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1. Show that the radii of convergence of the growth series  $\sum_{n=0}^{\infty} \beta(n)z^n$  and of the spherical growth series  $\sum_{n=0}^{\infty} \sigma(n)z^n$  coincide.

2. Show that the augmented growth series  $\Gamma(z)$  of the free group  $F_2$  is rational as a formal power series and compute the corresponding rational function with coefficients in  $\mathbb{Z}[F_2]$ . Use its interpretation as  $\Gamma^{\lambda_{F_2}}(z)$  to find the spectral radius of the adjacency operator on the 4-regular tree (that you have already computed by a different method in one of the previous exercises).

3. Show that for the infinite  $d$ -regular tree  $T_d$ ,

$$p_c(T_d) = \frac{1}{d-1}$$

and that for any infinite  $d$ -regular graph  $\Gamma$ ,

$$p_c(\Gamma) \geq p_c(T_d) \quad \text{and} \quad \rho(\Gamma) \geq \rho(T_d).$$

4. Prove that the function  $|C(0)|$  defined on  $\Omega = \{0, 1\}^{E(\Gamma)}$  is measurable.