International Meeting
"Trends in Partial Differential Equations"
dedicated to the 90th birthday
of outstanding mathematician
Vsevolod A. Solonnikov

St. Petersburg (online),
June 9-10, 2023

ABSTRACTS

Organizing Committee:

D.E. Apushkinskaya (*RUDN University, Moscow*)
A.I. Nazarov (*St. Petersburg Dept of the Steklov Institute and St. Petersburg University*)
J.F. Rodrigues (*Universidade de Lisboa*)
N.V. Zalesskaya (*L. Euler International Mathematical Institute, St. Petersburg*)

Schedule of the Meeting

Time*	09.06.2023, Friday	10.06.2023, Saturday
10.45–10.55	*Opening*	
11.00–11.40	Y. Giga	V. Pukhnachev
11.50–12.30	H. Beirão da Veiga	M. Korobkov
12.35–14.00	*Lunch*	*Lunch*
14.00–14.40	G. Grubb	S. Shaposhnikov
14.50–15.30	G. Seregin	A. Fasano
15.35–16.00	*Coffee-break*	*Coffee-break*
16.00–16.40	N. Krylov	K. Pileckas
16.50–	*Virtual party*	*Closing*

* This is St. Petersburg time.

On fully developed space-time periodic viscous flows

HUGO BEIRÃO DA VEIGA

Università di Pisa

We study the motion of a viscous incompressible fluid in an $n+1$-dimensional double-infinite pipe Λ with an L-periodic shape in the $z = x_{n+1}$ direction. Denote by Σ_z the cross-section of the pipe at the level z, and by v_z the $(n+1)-$th component of the velocity. We look for fully developed solutions $\mathbf{v}(x, z, t)$ with a given T-time periodic total flux $g(t) = \int_{\Sigma_z} v_z(x, z, t)\, dx$ which should be simultaneously T-periodic with respect to time and L-space-periodic with respect to z. We prove existence and uniqueness for the above problem. The problem of determining $\mathbf{v}$ and Γ requires to solve a *non-standard parabolic equation involving a non-local term* of the solution.

The new results (which extend those proved by the author in a 2005 paper published in Arch. Ration. Mech. Anal.) were obtained in collaboration with Jiaqi Yang, from the Northwestern Polytechnical University, Xi'an, China, and will appear in the J. of Math. Physics.

Modeling angiogenesis in tumor growth

ANTONIO FASANO

Università degli Studi di Firenze

Angiogenesis is a fundamental process in tumors evolution creating a blood vessels network providing oxygen and nutrients. It occurs through a multi-step mechanism involving chemotaxis and including a random component. The formulated model leads to solutions behaving as travelling waves. The underlying structure of free boundary problem is investigated.

On analyticity of the L^p-Stokes semigroup for some non-Helmholtz domains

YOSHIKAZU GIGA

The University of Tokyo

It is a very fundamental topic whether the Stokes semigroup is analytic in a solenoidal L^p space ($1 < p < \infty$). Starting from the work of V.A. Solonnikov (1977) and Y. Giga (1981), it is by now well known that the Stokes semigroup is analytic in a solenoidal L^p space provided that the domain admits the L^p-Helmholtz decomposition and that it is smooth as shown by M. Geißert, H. Heck, M. Hieber and O. Sawada (2012). This applies to a standard domain like a bounded domain or an exterior domain. However, there is a planar sector-like domain which does not allow the L^p-Helmholtz decomposition as given by M. E. Bogovskiĭ (1986).

In this talk, we prove that the Stokes semigroup is analytic in L^p even if for a sector-like planar domain having a graph boundary which does not allow the L^p-Helmholtz decomposition. For BMO type space, the analyticity of the Stokes semigroup has been established for such a domain by M. Bolkart and Y. Giga (2016). We interpolate this result with the result for L^2 to get a desired result. However, the interpolation of solenoidal spaces is not easy because there is no L^p-Helmholtz decomposition. We establish a non-Helmholtz decomposition based on a solution operator for the divergence equation due to Solonnikov (1983), which is different from the Bogovskiĭ operator. This talk is based on a joint work with M. Bolkart, T.-H. Miura, T. Suzuki and Y. Tsutsui (2017).

Nonlocal evolution equations

GERD GRUBB

Københavns Universitet

In joint works with Vsevolod Solonnikov in the ′80–′90-ies we treated the Navier–Stokes problem in general function spaces by a reduction to a nonlocal parabolic problem with pseudodifferential ingredients. Currently I am studying evolution problems with another type of nonlocality, namely equations $d_t u + Pu = f$, where P is a pseudodifferential operator of noninteger order, and will expose some recent results.

On steady Navier–Stokes equations in $2D$ exterior domains: to make the long story short

MIKHAIL KOROBKOV

Fudan University, Shanghai

The talk is devoted to a review of results on solutions to the steady Navier–Stokes system with a finite Dirichlet integral in the exterior plane domain (= "D-solutions"). Some progress has been made on this problem in the years: the uniform boundedness in the C-norm and the uniform convergence (at spatial infinity) of such solutions, the uniqueness of the solutions to the flow around an obstacle problem in the class of all D-solutions, the nontriviality of the Leray solutions (obtained by the method of "invading domains") in flow around an obstacle problem and their convergence to a given limit at low Reynolds numbers.

It turns out just recently, that all the mentioned results can be deduced easily from some basic estimates for general Navier–Stokes solutions. These estimates have rather simple forms and control the difference between mean values of the velocity over two concentric circles in terms of the Dirichlet integral in the annulus between them. Most of the reviewed results were obtained in our joint papers with Konstantin Pileckas, Remigio Russo, Xiao Ren, and Julien Guillod.

On parabolic equations in Morrey spaces with VMO a and Morrey b, c

NICOLAI KRYLOV

University of Minnesota

We prove existence and uniqueness of solutions in Morrey spaces of functions with mixed norms for second-oder parabolic equations in the whole space with VMO a and Morrey b, c.

Existence of nonstationary Poiseuille type solution under minimal regularity assumptions

KONSTANTIN PILECKAS

Vilniaus Universitetas

The steady-state Poiseuille flow in an infinite straight pipe

$$\Pi = \{x : (x_1, x_2) \in \sigma, x_3 \in (-\infty, +\infty)\}$$

of constant cross-sectional σ was described by Jean Louis Poiseuille in 1841. Today this classical solution of the Navier–Stokes equation seems to be trivial although it is used in numerous studies of fluid motion. The Poiseuille flow is characterised by the fact that the associated velocity field has only one nonzero component $u(x)$ directed along the x_n-axis and it depends only on the variables x' of the cross-sectional σ, while the pressure function $p = p(x_n)$ is a linear function.

The Poiseuille-type solutions can be also defined in the non-steady case. Since in real applications one usually does not have data defined by smooth functions, it is important to study non-stationary Poiseuille-type solutions assuming minimal regularity of data. This is the subject of the present talk. Existence and uniqueness of a very weak solution to the non-stationary Navier–Stokes equations having a prescribed flow rate (flux) in the infinite cylinder Π are proved. It is assumed that the flow rate $F(t)$ is an element of $L^2(0,T)$ and the initial data $u_0 = (0, 0, u_{0n})$ is an element of $L^2(\sigma)$. The non-stationary Poiseuille solution has the form

$$u(x,t) = (0, 0, U(x', t)), \qquad p(x,t) = -q(t)x_n + p_0(t),$$

where $(U(x', t), q(t))$ is a solution of an inverse problem for the heat equation with a specific over-determination condition. Under the above regularity assumptions the solution of this inverse problem does not have the usual for parabolic problems regularity: it is much weaker.

Slip condition in dynamics of incompressible fluid

VLADISLAV PUKHNACHEV

Lavrentyev Institute of Hydrodynamics
and Novosibirsk State University

Slip condition on the fluid boundary moving along solid surface proposed by Navier (1823). This condition is used in dynamics of rarefied gas and polymer melts. Besides, application of the traditional no-slip condition leads to unbounded density of energy dissipation in the problem of the motion fluid-fluid interface along a solid surface (Dussan and Davis, 1973) and three phase moving contact line (Pukhnachev and Solonnikov, 1982). We distinguish partial slip perfect slip. There is a similarity between free boundary problem and problem with perfect slip for the Navier-Stokes equations. Our communication contains the review of results in study of problems to Navier-Stokes equations with slip boundary condition. In particular, compressible viscous fluid motion (Tani, 1986), the problems with unilateral constraints (Baiocchi and Pukhnachev, 1990; Fuijita, 1994), initial boundary value problem with the slip boundary condition for second grade fluid motion (C. le Roux, 1999), the tensorial hydrodynamic slip (Bazant and Vinogradova, 2008), steady motions and their stability (Itoh, Tanaka and Tani, 2009), viscous flow along the superhydrophobic surface (Ageev and Osiptsov, 2022). In conclusion, we formulate open problems in this area.

On potential Type II blowups of solutions to the Navier–Stokes equations

GREGORY A. SEREGIN

PDMI RAS and Oxford University

In the talk, I will explain how to use the Euler scaling to study a certain scenario of potential Type II blowups of solutions to the Navier–Stokes equations.

Nonlinear Fokker–Planck–Kolmogorov equations

STANISLAV SHAPOSHNIKOV

Moscow State University

The talk is devoted to nonlinear Fokker–Planck–Kolmogorov equations with local and nonlocal nonlinear terms. We will present conditions under which solutions converge to the stationary solution. Moreover some examples and counterexamples will be discussed.