XXXII St.Petersburg Summer Meeting in Mathematical Analysis

Abstracts

G. Amosov (Steklov Mathematical Institute)

On informationally complete operator-valued measures on locally compact groups

Let $\mathfrak{G}$ be a locally compact Abelian group with a Haar measure μ and let $\mathfrak{B}(\mathfrak{G})$ be the algebra of all measurable sets on $\mathfrak{G}$. Let $B(H)_+$ denote the cone of positive linear operators acting in a separable Hilbert space H. A map $\mathfrak{M} : \mathfrak{B}(\mathfrak{G}) \to B(H)_+$ is said to be a positive operator-valued measure (POVM) on $\mathfrak{G}$ if

$$\mathfrak{M}(\cup_j B_j) = \sum_j \mathfrak{M}(B_j), \quad B_j \cap B_k = \emptyset, \quad B_j \in \mathfrak{B}(\mathfrak{G}), \;\; j \neq k,$$

$$\mathfrak{M}(\emptyset) = 0, \quad \mathfrak{M}(\mathfrak{G}) = \mathrm{I}.$$

Let $\mathfrak{G} = \hat{G} \times G$ be the locally compact Abelian group generated by a locally compact Abelian group (G, ν) and let $\mu = \hat{\nu} \times \nu$ be the Haar measure on $\mathfrak{G}$. Then the projective unitary representation $(\chi, g) \to U_{\chi,g}$ of $\mathfrak{G}$ in $H = L^2(G, \nu)$ defined by the formula

$$[U_{\chi,g}\psi](h) = [\chi(g)]^{1/2}\chi(h)\psi(h+g), \quad g, h \in G, \quad \chi \in \hat{G}, \quad \psi \in H,$$

is irreducible; $\mathfrak{M}(B) = \int\limits_B U_{\chi,g} P_0 U_{\chi,g}^* d\mu(\chi, g)$, where P_0 is an arbitrary one-dimensional projection on the subspace $\{\mathbb{C}\xi_0\}$, is a POVM on $\mathfrak{G}$ being covariant in the following sense (see [1-2]):

$$U_{\chi,g}\mathfrak{M}(B)U_{\chi,g}^* = \mathfrak{M}(B + (\chi, g)), \quad (\chi, g) \in \mathfrak{G}, \quad B \in \mathfrak{B}(\mathfrak{G}).$$

Given a state (a positive unit-trace operator) ρ, the function F_ρ defined on $\mathfrak{G}$ as

$$F_\rho(\chi, g) = (U_{\chi,g}\xi_0, \rho U_{\chi,g}\xi_0) \qquad (1)$$

is the density of the probability distribution $\{\mathrm{Tr}(\rho\mathfrak{M}(B)),\ B \in \mathfrak{M}(\mathfrak{G})\}$. The POVM $\mathfrak{M} = \{\mathfrak{M}(B),\ B \in \mathfrak{M}(\mathfrak{G})\}$ is said to be informationally complete if (1) can be converted. We show that (1) is informationally complete (see [3]). To prove this fact, we investigate the set of contractions $T_{\chi,g}$ generated by the POVM $\mathfrak{M}$.

References

[1] G.G. Amosov, On quantum tomography on locally compact groups, Physics Letters A, 431 (2022), 128002, 7 pp.

[2] G.G. Amosov, On quantum channels generated by covariant positive operator-valued measures on a locally compact group, Quantum Information Processing, 21 (2022), 81, 15 pp.

[3] G.G. Amosov, On constructing informationally complete covariant positive operator-valued measures, arXiv:2301.12492

S. Astashkin (Samara National Research University)

Calderón-Mityagin and Arazy-Cwikel type interpolation properties for quasi-Banach spaces

We plan to discuss some recent results related to Calderón-Mityagin and Arazy-Cwikel interpolation properties in the class of quasi-Banach spaces (see [1-3]). As an example, we will consider the classical family of couples (ℓ^p, ℓ^q), $0 \le p < q \le \infty$, focusing on an identification of interpolation orbits of elements with respect to such a couple for any p and q, $0 \le p < q \le \infty$, and on a result, showing that (ℓ^p, ℓ^q) is a Calderón-Mityagin couple if and only if $q \ge 1$.

References

[1] S.V. Astashkin, M. Cwikel and P.G. Nilsson, Arazy-Cwikel and Calderón-Mityagin type properties of the couples (ℓ^p, ℓ^q), $0 \le p < q \le \infty$, Ann. Mat. Pura Appl. (4) 202 (2023), no. 4, 1643–1678.

[2] S.V. Astashkin and P.G. Nilsson, Arazy-Cwikel property for quasi-Banach couples, Positivity 26 (2022), no. 4, paper no. 72.

[3] S.V. Astashkin and P.G. Nilsson, A description of interpolation spaces for quasi-Banach couples by real K-method, Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat. 117 (2023), paper no. 35.

F. Avkhadiev (Kazan Federal University)

Generalizations of Rellich and Birman integral inequalities

We consider integral inequalities for functions $f \in C_0^\infty(\Omega)$ on domains Ω of the Euclidean space $\mathbb{R}^n$, $n \in \mathbb{N}$. There are several inequalities connected with the following two basic results. In 1954, F. Rellich proved that

$$\int_{\mathbb{R}^n} |\Delta f|^2 \, dx \geq \frac{n^2(n-4)^2}{16} \int_{\mathbb{R}^n} \frac{|f|^2}{|x|^4} dx, \qquad \forall f \in C_0^\infty(\mathbb{R}^n \setminus \{0\}) \ (n \geq 3),$$

where Δf is the Laplacian of the function $f : \mathbb{R}^n \setminus \{0\} \to \mathbb{C}$. In 1961, M. Sh. Birman proved that

$$\int_0^\infty |f^{(k)}(t)|^2 dt \geq \left(\frac{(2k-1)!!}{2^k}\right)^2 \int_0^\infty \frac{|f(t)|^2}{t^{2k}} dt, \quad \forall f \in C_0^\infty(0,\infty) \ (k \in \mathbb{N}).$$

We will describe direct generalizations of the Rellich and Birman inequalities. In addition, using the polyharmonic operator defined by

$$\Delta^{k/2} f(x) := \begin{cases} \Delta^j f(x), & \text{if } k = 2j, \\ \nabla \Delta^{j-1} f(x), & \text{if } k = 2j-1, \end{cases}$$

where $j \in \mathbb{N}$, $k \in \mathbb{N}$, $f \in C_0^\infty(\Omega)$, $\Omega \subset \mathbb{R}^n$, we describe results about the inequality

$$\int_\Omega |\Delta^{k/2} f(x)|^2 dx \geq A_k(\Omega) \int_\Omega \frac{|f(x)|^2}{\mathrm{dist}^{2k}(x, \partial\Omega)} dx, \quad \forall f \in C_0^\infty(\Omega).$$

T. Batenev (St.Petersburg University)

Representing systems of reproducing kernels in spaces of analytic functions

We give an elementary construction of reproducing kernels for Hardy spaces H^p, $p \in [1,\infty)$, in the polydisc, ball and in the half-plane. Also, we construct representing systems of reproducing kernels in some weighted Hardy spaces in disc (e.g., in the Dirichlet space).

A. Bufetov (Steklov Mathematical Institute of Russian Academy of Sciences)

Gaussian multiplicative chaos and excess one for the sine-process

The main result of the talk is that the random entire function associated to the sine-process converges to the Gaussian multiplicative chaos under suitable normalization. As a corollary of the developed formalism, we obtain that almost every realization of the sine-process, after removing one particle, becomes a complete minimal set for the Paley-Wiener space.

E. Doubtsov (St. Petersburg Department of V. A. Steklov Mathematical Institute)

Dominant sets for model spaces in several variables

Let $H^2 = H^2(B_n)$ denote the standard Hardy space on the open unit ball B_n of $\mathbb{C}^n$, $n \geq 2$, and let σ denote the normalized Lebesgue measure on the unit sphere ∂B_n. Given an inner function I, a Lebesgue measurable set $E \subset \partial B_n$ is said to be dominant for the large model space $H^2 \ominus IH^2$ if $\sigma(E) < 1$ and $\|f\|_{H^2}^2 \leq C \int_E |f|^2 \, d\sigma$ for all $f \in H^2 \ominus IH^2$. We use Clark measures to construct dominant sets for an arbitrary inner function. The talk is based on joint work with A. B. Aleksandrov.

This research was supported by Russian Science Foundation (grant No. 19-11-00058).

Z. Fazullin (Ufa University of Science and Technology)

Regularized sums for eigenvalues of bounded perturbations of model second-order partial derivative operators

We are going to analyse computation of regularized trace formulas for perturbed model second-order partial derivative operators under assumption that the perturbation is performed by the operator of multiplication by a function. In particular, we will formulate the corresponding results for the Laplace-Beltrami operator on S^2, for the harmonic oscillator acting in the whole plane $\mathbb{R}^2$ or in a strip, for the Schrödinger operator in a homogeneous magnetic field. We will also discuss certain approaches and results for the Dirichlet problem on a square.

K. Fedorovskiy (Lomonosov Moscow State University)

Uniform approximation by polynomial solutions of certain elliptic PDE and systems

We plan to discuss the problems on uniform approximation of functions on compact sets in the complex plane by elements of polynomial modules of polyanalytic type generated by antiholomorphic functions and by polynomial solutions to general second-order elliptic but not strongly elliptic systems of PDEs with constant coefficients. We will present some known facts and recent results on this topis and we will explore the special analytic characteristics of planar simply connected domains in terms of which the obtained approximation criteria are stated (the concept of a g-Nevanlinna domain, where g is the generator of the module by elements of which the approximation is carried out, and the concept of an L-special domain, where L is the elliptic differential operator that determines the corresponding system).

The talk is based on the results obtained in frameworks of the project 22-11-00071 by the Russian Science Foundation.

A. Gaisin (Institute of Mathematics with Computing Centre, Ufa Federal Research Centre of RAS)

Regular growth of Dirichlet series. Applications

The report will focus on theorems of the minimum modulus type and their applications in the theory of the distribution of values.

For entire transcendental functions of finite order having the form $f(z) = \sum_n a_n z^{p_n}$, $p_n \in \mathbb{N}$, G. Polya showed that if the density of the sequence $\{p_n\}$ is equal to zero, then for any curve γ going to infinity, there is an unbounded sequence $\{\xi_n\} \subset \gamma$, such that

$$\ln M_f(|\xi_n|) \sim \ln |f(\xi_n)| \quad \text{as } \xi_n \to \infty,$$

where $M_f(r)$ denotes the maximum of the modulus of f on the circle of radius r. Later, these results were extended by I. D. Latypov to the entire Dirichlet series of finite order and finite lower order by Ritt. A further generalization was obtained by N. N. Yusupova and Aitkuzhina for more general classes $D(\Phi)$ and $\underline{D}(\Phi)$ defined by a convex majorant Φ. In 2022, we obtained necessary and sufficient conditions for the exponents λ_n: under this conditions, the logarithm of the modulus of the sum of any Dirichlet series from this classes on the curve γ with a bounded K-slope is equivalent to the logarithm of the maximum term for $\sigma = \operatorname{Re} s \to +\infty$ over an asymptotic set whose upper density is not less than $\frac{1}{\sqrt{K^2+1}}$. It will be shown that this result can be amplified.

The results about iterations of entire transcendental functions with a regular behavior of the modulus minimum will be presented in the report.

G. Gaisina (Ufa University of Science and Technology)

Exponential series with a given growth majorant near the boundary

We will talk about the behavior of series of exponents near the boundary in the cases, where the domain of regularity of the sum of the series is the entire plane, a half-plane, a bounded convex domain. Next, we will discuss the representation of functions analytic in the right half-plane by series of exponents, taking into account a given convex majorant of growth.

R. Gaisin (Institute of Mathematics with Computing Centre, Ufa Federal Research Centre of RAS)

Nontriviality of Siddiqi class and nonspanning systems of exponentials on curves

We study the problem of noncompleteness for a system of exponentials $\{e^{\pm\lambda_n z}\}$ ($\lambda_n > 0$) in the space of continuous functions on an arc of bounded slope. In terms of the minorant sequence, we obtain new equivalent condition, which is sufficient for noncompleteness of the given system of exponentials. We also prove that Siddiqi class is nontrivial if the sequence $\{M_n^c\}$ satisfies the A-condition.

A. Gasparyan (Ailamazyan Program Systems Institute of RAS)

Hypercubic generalizations and refinements of Cauchy-Bunyakovsky-Schwarz inequality

The classical Cauchy-Bunyakovsky-Schwartz inequality is one of the fundamental inequalities of mathematics. This inequality has been generalized and applied by many mathematicians in different directions. A family of new type generalizations for the CBS inequality are presented in the talk. These results are applied to give an infinite sequence of refinements both of the CBS inequality and its generalizations.

D. Gorbachev (Tula State University)

Weighted Bernstein's inequality in L^p on the axis with power weight for $p > 0$

Bernstein's inequality for the derivative of a trigonometric polynomial in the L^p space is a classical inequality of approximation theory. It has been generalized to the case of entire functions of exponential type on the axis. The question of boundedness of the constant remained open in the case of a power weight for $p > 0$. We show that this is true for both the ordinary derivative and the Dunkl differential-difference operator.

K. Isaev (Institute of Mathematics with Computing Centre, Ufa Federal Research Centre of RAS)

On a sufficient condition for the existence of unconditional bases of reproducing kernels in Fock type spaces with nonradial weights

We consider Fock type spaces, the spaces $\mathcal{F}_\varphi$ of entire functions f such that $fe^{-\varphi} \in L_2(\mathbb{C})$, where φ is a subharmonic function. We have obtained new sufficient conditions for the existence of unconditional bases of reproducing kernels (or Riesz bases of normalized reproducing kernels) in Fock type spaces with nonradial weights. The issue on existence of unconditional bases of reproducing kernels is actively studied, in particular, due to the fact that this question is closely related to such classical problems of complex analysis as the problem of representing of functions by exponential series and the problem of interpolation by entire functions.

E. Kalita (Institute of Applied Mathematics and Mechanics)

Nonlinear Riesz transforms in the neighborhood of L_2

Considering the p-harmonic equation $div(|Du|^{p-2}Du) = div\, f$, we let define the p-Riesz transform $T_p : f \mapsto |Du|^{p-2}Du$. It is naturally defined in $L_{p'}(R^n)$. We consider T_p in the domain $L_q \cap L_{p'}$ and establish that the growth of the norm has the second degree of smallness near L_2, precisely $\|T_p\|_{L_q \to L_q} \leq 1+c(|p-2|^2+|q-2|^2)$ for p, q close to 2. The same results are established in the weighted case, as well as for higher order operators.

I. Kayumov (Kazan Federal University)

Rotations of convex harmonic univalent mappings

I am going to describe recent results, obtained jointly with S. Ponnusamy and Le Anh Xuan, about some properties of convex harmonic mappings in the unit disk.

B. Khabibullin (Bashkir State University)

Completeness of exponential systems in terms of mixed areas

We give a scale of completeness conditions for exponential systems in spaces of continuous functions on a compact set of the complex plane that are simultaneously holomorphic inside a compact set, in spaces of holomorphic functions in a domain, and so on. These conditions will be formulated in geometric terms of mixed areas for the convex hull of a compact or domain. We will characterize the distribution of exponents of the exponential system both in terms of their moduli and in terms of arguments using various options for the convexity of functions. The vast majority of known results on the completeness of exponential systems in such spaces turn out to be very special cases from this scale. Our completeness results extend to parameterized systems of entire functions, which are more general than exponential systems. All studies cover several complex variables.

D. Khammatova (Kazan Federal University)

Refinement of powered Bohr inequality

Let B denote the class of all analytic functions mapping the unit disk into itself. We consider the powered Bohr sum of a function from the class B with an additional term including the area of the image of the disk of radius r. In this setting, we prove a counterpart of the Bohr theorem for two different cases.

R. Khasyanov (St. Petersburg State University)

The Bohr radius of a pair of operators

In 1914, H. Bohr, studying Dirichlet series, discovered the following interesting fact in complex analysis, which is now called the Bohr phenomenon:

Theorem. Let $f(z) = \sum_{n\geq 0} a_n z^n$ and $\|f\|_\infty := \sup_{z\in\mathbb{D}} |f(z)|$ in the unit disc $\mathbb{D} = \{|z| < 1\}$. Then $M_r f := \sum_{n\geq 0} |a_n| r^n \leq \|f\|_\infty,\ \ 0 \leq r \leq 1/3$. Moreover, the constant $1/3$ is the best possible.

We introduce the concept of the Bohr radius of a pair of operators, in terms of which many well-known results related to the Bohr inequality can be formulated. The report will discuss questions about the Bohr radius for the differential operators, Hadamard convolution operators, and other operators. Using the concept of the Bohr radius of a pair of operators, we generalize the theorem of B. Bhowmik and N. Das on the comparison of majorant series of subordinate functions.

E. Korotyaev (St.Petersburg State University)

Isomorphic inverse problems

Consider two inverse problems for Sturm-Liouville problems on the unit interval. It means that there are two corresponding mappings F, f from a Hilbert space of potentials H into their spectral data. They are called isomorphic if F is a composition of f and some isomorphism U of H onto itself. A isomorphic class is a collection of inverse problems isomorphic to each other. We consider basic Sturm-Liouville problems on the unit interval and on the circle and describe their isomorphic classes of inverse problems. For example, we prove that the inverse problems for the case of Dirichlet and Neumann boundary conditions are isomorphic. The proof is based on the non-linear analysis.

A. Kuznetsova (Kazan Federal University)

Blaschke C^*-algebras

The idea of the talk is to bring together the well known Blaschke products and operator algebras. We suggest the notion of the Blaschke C^*-algebra, which is the universal C^*-algebra generated by some relations. The relations are defined by a family of finite Blaschke products. In this way we extend the Coburn's Theorem for a family of non-unitary isometries connected by a family of finite Blaschke products. We show that the Blaschke C^*-algebra is isomorphic to the inductive limit of Toeplitz algebras. The talk is based on joint work with Tamara Grigoryan.

M. Malamud (Peoples Friendship University of Russia)

On a formula for characteristic determinants of boundary value problems for $n \times n$ Dirac type equations and its applications

In this talk, we will discuss spectral properties of boundary value problems for the following first order system of ordinary differential equations

$$Ly = -iB(x)^{-1}\big(y' + Q(x)y\big) = \lambda y, \quad B(x) = B(x)^*, \quad y = \mathrm{col}(y_1, \dots, y_n), \quad x \in [0, \ell],$$

on a finite interval $[0, \ell]$. Here $Q \in L^1([0, \ell]; \mathbb{C}^{n\times n})$ is a potential matrix and $B \in L^\infty([0, \ell]; \mathbb{R}^{n\times n})$ is an invertible self-adjoint diagonal "weight" matrix. If $n = 2m$ and $B(x) = \mathrm{diag}(-I_m, I_m)$, then this equation is equivalent to the classical Dirac equation of order n. We will discuss the spectral properties of boundary value problems associated with the above equation subject to general boundary conditions $U(y) = Cy(0) + Dy(\ell) = 0$ satisfying the maximality condition $\mathrm{rank}(C\ D) = n$.

One of our main results is the formula for the deviation of the characteristic determinants $\Delta(\lambda) - \Delta_0(\lambda)$ of the perturbed and unperturbed (with $Q = 0$) boundary value problems subject to the same boundary conditions $U(y) = 0$. Namely, we show that it is the Fourier transform of a certain integrable function explicitly expressed via kernels of the transformation operators.

In turn, this result is applied to establish asymptotic behavior of eigenvalues as well as sharp evaluation of the reminder.

The talk is based on joint work with Anton Lunyov.

A. Mednykh (Sobolev Institute of Mathematics, Novosibirsk State University)

Spectral invariants of circulant graphs and their application in combinatorial analysis

The purpose of this report is to study the spectral invariants of circulant graphs and their generalizations. Circulant graphs arise as cyclic coverings of a single-vertex graph with a given number of loops. More complex representatives of the family of cyclic coverings are I-, Y-, H-graphs, generalized Petersen graphs, sandwich graphs, discrete tori, and many others.

In the talk, analytical formulas will be given that allow one to calculate the number of rooted spanning forests and trees in cyclic coverings, their asymptotics will be found, and the arithmetic properties of these numbers will be studied. In addition, for circulant graphs, exact formulas for calculating the Kirchhoff index will be indicated and it will be established that, up to an exponentially small remainder term, they are given by polynomials of the third degree.

All these quantities are spectral invariants. They depend on the eigenvalues of the characteristic polynomial of the Laplace matrix. The structure of the polynomial itself for circulant graphs remained unknown. In recent papers [Xiaogang Liu and Sanming Zhou (2012), Xiaogang Liu and Pengli Lu (2016)], it was found that the characteristic polynomials for a number of well-known families of graphs, such as the theta graph, dumbbell graph, and propeller graph, are effectively expressed in terms of Chebyshev polynomials. These results gave the key to understanding the structure of the characteristic polynomial for circulant graphs.

We will show that the characteristic polynomial can be represented as a finite product of algebraic functions calculated in the roots of a linear combination of Chebyshev polynomials. In particular, this will make it possible to establish the periodicity of such polynomials at prescribed integer points, which is of interest from the point of view of discrete topological dynamics.

References

[1] A.D. Mednykh, I.A. Mednykh, Cyclic coverings of graphs. Enumeration of rooted spanning forests and trees, Kirchhoff index and Jacobians. Uspekhi Matematicheskikh Nauk 78 (2023), no. 3(471), 115–160.

I. Musin (Institute of Mathematics with Computing Centre, Ufa Federal Research Centre of RAS)

On Gelfand-Shilov type spaces

Following the scheme of constructing of Gelfand-Shilov spaces of type S, three new spaces of rapidly decreasing infinitely differentiable functions in $\mathbb{R}^n$ are defined with a help of families of separately radial convex weight functions in $\mathbb{R}^n$. Under some mild restrictions on weight functions these spaces are characterized via Fourier transforms. Some interesting particular cases of such spaces corresponding to special families of weight functions are considered. The talk is based on joint work with R.S. Yulmukhametov and A.V. Lutsenko.

R. Nasibullin (Kazan Federal University)

L^p-Hardy-type inequalities for special weight functions

We prove one-dimensional L^p-Hardy inequalities with additional terms and use them for justifying spatial analogues in convex domains with finite volumes. We consider spatial inequalities in arbitrary domains, and we simplify them for convex domains. The constants in the additional terms in these spatial inequalities depend on the volume or on the diameter of the domain.

S. Nasyrov (Kazan Federal University)

The Nuttall decomposition of a three-sheeted torus and the asymptotics of the rational Hermite-Pade approximants

We study the three-sheeted Riemann surface S of genus 1, corresponding to the algebraic function $w = \sqrt[3]{(z-a_1)(z-a_2)(z-a_3)}$ with pairwise distinct complex numbers a_j. There exists an abelian integral G on S which is regular on S, except for three points lying over infinity, where the function $\mathrm{Re}\, G$ has prescribed singularities of logarithmic type.

The harmonic function $\mathrm{Re}\, G$ induces a partition of S into three sheets; it is called the Nuttall decomposition. Such a decomposition plays an important role in investigation of the asymptotics of the rational Hermite-Pade approximants. According to the Nuttall conjecture, the sheet gluing lines and their projections onto the Riemann sphere define the convergence domains for the Hermite-Pade approximants.

The differential-topological structure of the Nuttall decomposition depends essentially on the mutual location of the points a_j. A.I. Aptekarev and D.N. Tulyakov (2016) described the structure for the case, where the triangle with vertices at the points a_j is sufficiently close to the regular one. We consider the general case and completely solve the problem for isosceles triangles with the apex angle less than $\pi/3$. In the talk, we also discuss other cases.

The study was carried out at the expense of the grant of the Russian Science Foundation No.23-11-00066.

V. Nazaikinskii (Institute for Problems in Mechanics RAS, Moscow)

Lattice equations and semiclassical asymptotics

We consider linear equations with shifts of the arguments on the rectangular lattice with small step h in $\mathbb{R}^n$ and construct a version of the canonical operator providing semiclassical asymptotics for such equations. In the case of functions of continuous argument, equations with shifts can be written as h-pseudodifferential equations with symbols 2π-periodic in the momenta. This representation can also be given meaning for functions of a discrete argument, although differentiation operators are not defined for lattice functions. The phase space of such equations is the product $\mathbb{R}^n \times T^n$. Developing Maslov's ideas, we construct a canonical operator on Lagrangian submanifolds of this phase space with values in the space of lattice functions. Compared with the classical version of the canonical operator and the new formulas recently introduced by S.Yu. Dobrokhotov, A.I. Shafarevich, and the author, the construction involves a number of new features.

As an example, we give equations on a two-dimensional lattice that arise in quantum theory (the Feynman checkers model) and in the problem on the propagation of wave packets on a homogeneous tree. The talk is based on the results of joint work with V.L. Chernyshev (HSE University, Moscow) and A.V. Tsvetkova (Ishlinsky Institute for Problems in Mechanics RAS, Moscow).

The author's research was supported by the Russian Foundation for Basic Research under project no. 21-51-12006.

N. Osipov (St. Petersburg Department of V. A. Steklov Mathematical Institute)

Rational trading in an efficient market and inequalities in analysis

Trading in an efficient market where the asset price behaves as a martingale leads to a zero expected payoff. However, the problem of how to make such trading as rational as possible remains meaningful and non-trivial: it turns out that there is a certain gap between trading that has zero profit expectation, but is still rational in the basic sense, and completely irrational economic behavior that violates the basic von Neumann–Morgenstern rationality axioms. By solving the problem of describing this gap and finding optimal trading strategies that get into it, we will arrive at the Bellman functions that have previously arisen in solving completely abstract problems about finding sharp constants in inequalities from Analysis. Namely, solving the economic problem in the absolute context, where the strategy to be chosen does not depend on the current wealth of the agent, we will arrive at the Bellman function related to the John–Nirenberg inequality in integral form. Solving the problem in a relative context, where all the agent's actions in the market are considered relative to his current wealth, we will arrive at the Bellman function related to the inequalities that describe the relationship between Gehring classes. Thus, we will obtain a natural economic interpretation for the listed inequalities and the Bellman functions associated with them.

V. Peller (St.Petersburg State University)

Real-valued spectral shift functions for pairs of contractions and pairs of dissipative operators

The talk is based on joint results with M.M. Malamud. I am going to discuss conditions under which a pair of contractions on a Hilbert space with trace class difference has a real integrable spectral shift functions. The case of pairs of dissipative operators will also be discussed.

The work is supported by the Ministry of Education and Science of the Russian Federation within the framework of the state task (project number FSSF-2023-0016).

S. Platonov (Petrozavodsk State University)

Some asymptotic formulas connected with Bessel analysis

In various sections of the classical Fourier Analysis, the problem of the convergence of the integrals

$$\int_0^\infty f(\lambda t)\, g(t)\, dt$$

as $\lambda \to +0$ or $\lambda \to +\infty$ under various assumptions on the functions f and g are considered. In the talk we study some analogues of such problems for weight integrals of the form

$$\int_0^\infty f(\lambda t)\, g(t)\, t^{2\alpha+1}\, dt, \quad \alpha > -1/2,$$

for functions f and g from some weighted functional classes connected with the Fourier–Bessel harmonic analysis.

D. Rutsky (St. Petersburg Department of V. A. Steklov Mathematical Institute)

Bounded BMO-regularity

In 1995, N. Kalton discovered that the stability of complex interpolation $(X_A, Y_A)_\theta = [(X,Y)_\theta]_A$ of a couple of Hardy-type spaces for Banach lattices of measurable functions on the unit circle is characterized in terms of the BMO-regularity property of lattice $X'Y$, that is, the existence of majorants w for arbitrary functions of the lattice satisfying $\log w \in \mathrm{BMO}$ with suitable estimates of the norms. BMO-regularity was studied extensively, its most elegant equivalent definition for couples of lattices is the existence of majorants (u,v) for arbitrary functions in (X,Y) satisfying $\log u/v \in \mathrm{BMO}$ with suitable control on the norms.

A few years ago, the stability of the real interpolation and the K-closedness of Hardy-type spaces were similarly characterized in terms of a weaker BMO-regularity property, namely that a real interpolation space $(\mathrm{L}_1, X'Y)_{\theta,p}$ is BMO-regular. We will explore several alternative characterizations of this property that elucidate its relation to the usual BMO-regularity.

This research was supported by Russian Science Foundation (grant No. 23-11-00171).

N. Shirokov (St.Petersburg State University)

B. Ya. Levin function for some sets of segments

Let $\{I_k\}_{k\in\mathbb{Z}}$ be a set of disjoint segments of the real axis, $E = \bigcup_{k\in\mathbb{Z}} I_k$. A function $f_{E,\sigma}(z)$, $\sigma > 0$, is called a B. Ya. Levin one if the following conditions are satisfied:

(1) $f_{E,\sigma}(z)$ is subharmonic on the complex plane $\mathbb{C}$ and harmonic on $\mathbb{C}\setminus E$;
(2) $f_{E,\sigma}(z) = 0$, $x \in E$; $f_{E,\sigma}(z) > 0$, $z \in \mathbb{C}\setminus E$;
(3) $\limsup_{z\to\infty} \frac{f_{E,\sigma}(z)}{|z|} = \sigma$, $f_{E,\sigma}(\bar z) = f_{E,\sigma}(z)$;
(4) if g is subharmonic on $\mathbb{C}$, $g(x) \le 0$, $x \in E$, $\limsup_{z\to\infty} \frac{g(z)}{|z|} \le \sigma$, then $g(z) \le f_{E,\sigma}(z)$.

We construct B. Ya. Levin functions for some sets of segments such that $|I_k|_{|k|\to\infty} \to 0$.

This research was supported by Russian Science Foundation (grant No. 23-11-00171).

D. Stolyarov (St.Petersburg State University)

Hardy spaces of fractional order

The space of measures of bounded total variation and L_1 quite often lack good properties. Usually this is related to the unboundedness of the maximal function on L_1. The interest in these spaces is justified not only by the simplicity of their norms, but also by direct connections with the geometric measure theory, and, thus, geometry. In harmonic analysis, we sometimes replace L_1 with a narrower space H_1. Here H_1 is the real Hardy class, it behaves "better". However, we lose the relationship with the geometric measure theory when passing to H_1: by "the Riesz brothers' theorem" there are no analogues of the space of measures whose norm resembles that of H_1. We will try to suggest a scale of spaces that interpolates L_1 and H_1. These intermediate spaces do contain singular measures of fractional dimension and also posses some properties of the Hardy class; for example, some "trace" inequalities for fractional integration operators are valid for measures in these spaces. We do not have complete proofs yet and are also slightly unsure whether our definitions are best possible. A larger part of work is done in a martingale model that simplifies the classical setting of Euclidean spaces. Nevertheless, the story seems worth telling. For example, the aforementioned scale extends the definition of lower Hausdorff dimension to arbitrary distributions.

Work in progress with Daniel Spector, Taiwan

A. Tsikh (Siberian Federal University)

Hypergeometry in the theory of algebraic functions

We will discuss universal algebraic functions, i.e., polynomial equations (or systems of such equations) with independent variable coefficients. It is known that for such equations the algebraic functions have hypergeometric type (Mellin 1921, Birkeland 1927, Stepanenko 2015).

For functions of hypergeometric type, we prove an analogue of Gorn-Kapranov theorem (1889-1991) that states that the singularities of such functions coincide with the discriminant sets of the corresponding systems. The results obtained make it possible to calculate the convergence domains for series of hypergeometric type in the language of functional inequalities for modules of variables of power series. Existence of such inequalities can be interpreted as an appearance of "quantum entanglement" in the theory of hypergeometric structures of Feynman integrals.

I. Vasilyev (Université Paris Saclay and PDMI RAS)

"Good" multipliers in higher dimensions

In this talk, we will show how to construct "good" Fourier multipliers in higher dimensions. If time permits, we will also show how to apply these multipliers to some modern problems of the uncertainty principle in harmonic analysis.

V. Yashin (MIPT, Steklov Mathematical Institute of RAS, Russian Quantum Center)

Extensions of conditionally completely positive maps between operator systems

In noncommutative probability theory, conditionally completely positive maps arise as the generators of unital completely positive semigroups. We will propose a generalized definition for conditionally completely positive maps between (possibly different) operator systems and in the finite dimensional case we will prove Arveson-type theorem on the extension of conditionally completely positive maps.