# Book of Abstracts

## String equation with singular weight: Asymptotic properties of eigenvalues

Mon, Jun 19
12:50–13:15

Grigoriy Agafonkin

*Moscow State University*

We consider the boundary-value problem

$$\begin{cases} -y'' = \lambda P' y, \\ y(0) = y(1) = 0, \end{cases} \tag{1}$$

where the function $P$ satisfies the similarity equation

$$P(x) = \beta_1 \chi_{[0,1-a)}(x) + \left( d \cdot P\left(\frac{x-(a-1)}{a}\right) + \beta_2 \right) \chi_{(1-a,1]}(x)$$

for $\beta_1, \beta_2 \in \mathbb{R}$, $a \in (0,1)$. It is known that if $0 < ad < 1$ then the spectrum of (1) is purely discrete while if $ad = 1$ it is continuous and equals to the closed interval

$$I = \left[ \frac{1}{d\beta_1 + \beta_2 - \beta_1} \frac{(1-\sqrt{a})^2}{a},\ \frac{1}{d\beta_1 + \beta_2 - \beta_1} \frac{(1+\sqrt{a})^2}{a} \right].$$

We study the case $ad = 1 - \varepsilon$ and the asymptotic behavior of the eigenvalues $\lambda_n = \lambda_n(\varepsilon)$ as $\varepsilon \to 0+$. We prove that $\frac{d\lambda_n}{d\varepsilon} \geq 0$ and provide some basic estimations of the number $N = N(\varepsilon)$ of eigenvalues belonging to the interval $I$.

Support from the Russian Science Foundation grant No. 20-11-20261 is acknowledged.

## Dynamics of eigenvalues under changing gyroscopic forces in mechanical systems

Tue, Jun 20
15:50–16:15

Aleksandr Arakcheev

*Moscow State University*

The report will present results on the dynamics of the eigenvalues for the quadratic pencil

$$\mathcal{L}(\lambda) = \lambda^2 R + \lambda\, \omega\, iB + C$$

with increasing parameter $\omega$. This pencil corresponds to the description of the mechanical system, where matrix $\omega iB$ corresponds to gyroscopic forces, and matrix $C$ to potential forces. Under the conditions $C < 0$, $R > 0$, we will show that for large $\omega$ the problem is stable if and only if $\mathrm{Ker}(B) = 0$. In the general case, we will show that for large $\omega$ the instability index coincides with $\dim(\mathrm{Ker}(B))$. This theorem generalizes the Kozlov–Karapetyan result obtained in the case of $\dim(\mathrm{Ker}(B)) = 1$ provided the matrices $R$, $B$, and $C$ are real. The main result of the report is that the comprehensive dynamics of the eigenvalues $\lambda_j(\omega)$ of the pencil $\mathcal{L}(\lambda)$ will be described as the parameter $\omega$ changes. In particular, the asymptotic behavior of $\lambda_j(\omega)$ is determined by the eigenvalues of three linear pencils composed of the coefficients of the original quadratic pencil, namely $\alpha C + B$, $\mu R + B$, and $P(\gamma R + C)P|_{\mathrm{Ker}(B)}$, where $P = P^*$ is the orthoprojector onto the kernel $\mathrm{Ker}(B)$.

Support from the Russian Science Foundation grant No. 20-11-20261 is acknowledged.

Fri, Jun 23
10:00–10:45

## Inverse problem for 3rd-order operators with small coefficients under 3-point Dirichlet conditions

Andrey Badanin

*Saint Petersburg State University*

We solve an inverse problem for a third-order differential operator under the 3-point Dirichlet conditions. The third-order operator is an $L$-operator in the Lax pair for the good Boussinesq equation and the inverse results for the 3-point problem are important for the integration of the Boussinesq on the circle. We restrict ourselves to the case of small coefficients, which corresponds to small initial data for the Boussinesq equation. We construct the mapping from the set of the coefficients to the set of spectral data and prove that this mapping is an analytic bijection on a neighborhood of 0.

This is joint work with E. Korotyaev.

Thu, Jun 22
11:40–12:25

## On perturbations of internal thresholds in essential spectrum and violation of preserving total multiplicity

Denis I. Borisov

*Institute of Mathematics, Ufa Federal Research Center RAS*

We consider general perturbations of an elliptic operator

$$\mathcal{H} = -\sum_{i,j=1}^{n-1} \frac{\partial}{\partial x_i} A_{ij}(x') \frac{\partial}{\partial x_j} - \frac{\partial^2}{\partial x_n^2} + \sum_{j=1}^{n-1} A_j(x') \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j} \overline{A_j}(x') + A_0(x')$$

in a cylindrical domain $\Omega \coloneqq \omega \times \mathbb{R}$ subject to some classical boundary condition, where $\omega \subseteq \mathbb{R}^{n-1}$ is an arbitrary domain with a sufficiently smooth boundary. The perturbation is described by a general abstract operator, which is localized in a certain sense. The essential spectrum of the considered perturbed operator possesses certain thresholds generated by isolated eigenvalues of the operator

$$\mathcal{H}' = -\sum_{i,j=1}^{n-1} \frac{\partial}{\partial x_i} A_{ij}(x') \frac{\partial}{\partial x_j} + \sum_{j=1}^{n-1} \Big( A_j(x') \frac{\partial}{\partial x_j} + \frac{\partial}{\partial x_j} \overline{A_j}(x') \Big) + A_0(x')$$

on $\omega$. We show that under the perturbation each such threshold bifurcates into eigenvalues and resonances and their total multiplicity is twice greater than the multiplicity of the unperturbed threshold. Such situation is due to the presence of two local meromorphic continuations of the perturbed resolvent in the vicinity of internal thresholds, each continuation possesses poles, which correspond to the eigenvalues and the resonances. The total number of such poles counting their orders coincides with the multiplicity of the unpertrubed threshold and this explains the doubling of the multiplicity under the perturbation. However, in the particular case when the unperturbed operator is the two-dimensional Schrödinger operator $\mathcal{H} = -\Delta + V_1(x_1) + V_2(x_2)$ on $\mathbb{R}^2$ and the potentials $V_1$ and $V_2$ are such that the eigenvalues corresponding to one potential coincides in certain sense with the virtual level and spectral singularity of the other potential, then the corresponding thresholds again bifurcate into twice more eigenvalues and poles. At the same time, now the associated meromorphic continuations have different number of poles, in other words, one continuation borrows a pole from the other continuation.

## Toda without Hamiltonian structure. The work of R. Leite, N. Saldanha, D. Torres and C. Tomei

Wed, Jun 21
17:30–18:15

Percy Deift

*Courant Institute, NYU*

The speaker will describe the work of R. Leite et al on an approach to the Toda lattice that is based on elementary linear algebraic operations and does not invoke symplectic structure. The work greatly extends the scope of Toda theory.

## Operator pencils and the linear problem of generation of long waves on the surface of a liquid layer by localized sources in an elastic base

Wed, Jun 21
12:00–12:45

Sergey Dobrokhotov

*Ishlinsky Institute for Problems in Mechanics RAS*

The problem under consideration in the framework of the so-called Podyapolsky model is described by functions given in a liquid layer (Laplace equation) and in an infinite half-space (Lame equations), connected by boundary and coupling conditions. Some mathematical nonstandardness of the problem consists in the fact that time derivatives are present in the Lame equations and are absent in the Laplace equation, in addition, some of the boundary conditions include time derivatives. Assuming that the elastic base is homogeneous and the interface between the elastic base and the liquid is a horizontal plane, the problem using the Fourier transform is reduced to studying the (complicated) spectrum of the operator pencil in a vertical variable. This, in turn, makes it possible to correctly formulate the Cauchy problem for appropriate functions and present, after a number of useful transformations ("tricks"), its solution with initial data localized in an elastic half-space, in a simple constructive form that clearly reflects the influence of the problem parameters, such as sizes, depth and direction of action of the source, the propagation velocity of elastic waves, the density of the liquid and the elastic medium and the depth of the liquid layer.

The talk is based on the results obtained in joint work with R. Griniv, H. Ilyasov, O. Tolstova, I. Chudinovich and A. Shkalikov. The work is partially supported by the Russian Science Foundation, project 21-11-00341.

Fri, Jun 23
12:00–12:45

## Global solutions of the tt*-Toda equations of $A_n$ type

Martin Guest

*Waseda University*

This talk is based on joint work with Alexander Its and Chang-Shou Lin, on a particular real form of the periodic Toda equations (arXiv:2302.04597). The choice of real form is dictated by the theory of toplogical-antitopological fusion of Cecotti and Vafa (Lie-theoretically, it is the split real form of $A_n$ type). We compute the asymptotics and monodromy data of all global solutions of these "tt*-Toda" equations.

The cases $n = 1$ (sinh-Gordon) and $n = 2$ (Tzitzeica, or Bullough–Dodd), have been studied over 30 years ago, and we have treated the case $n = 3$ (in considerable detail) in a series of papers starting over 10 years ago. More recently we have extended our methods to general $n$. Although the details of the general case are somewhat involved, the essential features of the problem and its solution have been clarified. The aim of this talk is to explain these features in a non-technical way.

Fri, Jun 23
12:50–13:15

## Asymptotics of solutions of the Helmholtz equation in a two-layer medium with a localized right-hand side

Alexander Klevin

*Ishlinsky Institute for Problems in Mechanics RAS*

We consider the domain $(x, z) \in \mathbb{R}^2 \times [z_-, z_+]$ separated by a boundary in the form of a graph of the function $z = D(x)$, $z_- < D(x) < z_+$, into two media with $x$-dependent densities $\rho_\pm(x)$. In this domain we consider the Helmholtz equation

$$\Big(\frac{\partial^2}{\partial {x_1}^2} + \frac{\partial^2}{\partial {x_2}^2} + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2(x,z)}\Big)u = -f\Big(\frac{x - x^0}{\mu}, z\Big)$$

with the right-hand side localized in a neighborhood of the set $x = x^0$. The parameter $\mu > 0$ is responsible for the degree of localization of right-hand side defined by a function $f(\xi, z)$ rapidly decreasing as $|\xi| \to +\infty$. The following boundary conditions are considered:

$$u|_{z=z_\pm} = 0, \quad u|_{z=D(x)-0} = u|_{z=D(x)+0}, \quad \frac{1}{\rho_-(x)}\frac{\partial u}{\partial z}\Big|_{z=D(x)-0} = \frac{1}{\rho_+(x)}\frac{\partial u}{\partial z}\Big|_{z=D(x)+0}.$$

We assume that the characteristic scales $l_z$ and $l_x$ of the domain in the variables $z$ and $x$ satisfy the estimates $l_z \sim \mu$ (values of the same order) and $l_z/l_x \ll 1$. This equation arises in problems of acoustics when studying the propagation of sound waves in the ocean (liquid-bottom medium) generated by a localized source. In this paper, the asymptotic solution of this equation with respect to the small parameter $h \equiv l_z/l_x$ is constructed. The solution method is based on the operator method of separation of variables $z$ and $x$ (see [1], [2]) and on a recently developed method for constructing asymptotic solution of equations with a localized right-hand side (see [3]).

This work was supported by the Russian Science Foundation under grant No. 21-11-00341 (`https://rscf.ru/project/21-11-00341`).

References

[1] V. V. Belov, S. Yu. Dobrokhotov, V. P. Maslov, T. Ya. Tudorovskii, *A generalized adiabatic principle for electron dynamics in curved nanostructures*, Phys. Usp., 48 (2005), 962–968.

[2] V. V. Belov, S. Yu. Dobrokhotov, T. Ya. Tudorovskii, *Operator separation of variables for adiabatic problems in quantum and wave mechanics*, J. Eng. Math., 55 (2006), 183–237.

[3] A. Yu. Anikin, S. Yu. Dobrokhotov, V. E. Nazaikinskii, M. Rouleux, *Lagrangian manifolds and the construction of asymptotics for (pseudo)differential equations with localized right-hand sides*, Theor. Math. Phys., 214 (2023), 1–23.

## Boundary value problems with boundary conditions linearly depending on the spectral parameter

Fri, Jun 23
16:20–16:45

Valeriia Kobenko

*Moscow State University*

We study boundary value problems generated by the $n$-th order differential expression and by $n$ boundary conditions, such that $m$ ($m \leq n$) of them depend on the spectral parameter linearly. We construct a linear operator $\mathcal{L}$ associated with such a problem in the Hilbert space $H = L_2[0,1] \oplus \mathbb{C}^m$ and find an explicit expression for the adjoint one, provided some additional assumptions.

Further we define a subclass of regular boundary value problems. We prove that the system of the root functions of the operator $\mathcal{L}$, associated with a regular boundary value problem, form an unconditional basis in the space $H$. It follows from this theorem that the system of the eigen and associated functions of the boundary problem itself form a basis only after taking away exactly $m$ functions. We give a criterion which says what functions have to be taken away for the preserving of the basis property.

The report is based on the joint work with A. A. Shkalikov. Support from the Russian Science Foundation grant No. 20-11-20261 is acknowledged.

## Spectral asymptotics for $n \times n$ systems of ordinary differential equations and applications

Mon, Jun 19
17:50–18:15

Alexey Kosarev

*Moscow State University*

We deal with a $n \times n$ system of differential equations of the form

$$\mathbf{y}' = \lambda A(x)\,\mathbf{y} + B(x)\,\mathbf{y}, \qquad x \in [0,1],$$

where $A(x) = \operatorname{diag}\{a_1(x), \dots, a_n(x)\}$, all elements of matrices $A$ and $B$ are at least summable and complex-valued, $\lambda$ is a large complex parameter. Assuming that

$$a_i(x) \neq a_j(x), \qquad a_i(x), b_{ij}(x) \in W_1^n[0,1],\ n \geq 0,$$

we obtain asymptotic representation

$$Y(x,\lambda) = M(x)\left(I + R^1(x)\lambda^{-1} + \cdots + R^n(x)\lambda^{-n} + o(1)\lambda^{-n}\right)E(x,\lambda)$$

of fundamental matrix $Y(x,\lambda)$ in some sectors of the complex plane, where matrices $M(x)$, $E(x,\lambda)$ are expressed in terms of the entries $A(x)$, $B(x)$. Moreover, we write down explicit formulas for the elements of $R^i(x)$.

In the last part of the talk we shall give applications of these results to boundary value problems for systems, as well as to scalar spectral problems with a nonlinear dependence on the spectral parameter.

The report is based on the joint work with A. A. Shkalikov. Support from the Russian Science Foundation grant No. 20-11-20261 is acknowledged.

Fri, Jun 23
15:00–15:45

## Universality in structures related to the XXZ spin-1/2 chain at low-temperature

Karol K. Kozlowski

*Laboratoire de Physique, ENS de Lyon, UMR 5672 du CNRS*

The quantum transfer matrix is an auxiliary tool allowing one to significantly simplify the problem of effectively calculating the *per site* free energy as well as the correlation functions of a one-dimensional quantum spin chain model at finite temperature. It is conjectured that certain universal features arising in the long-distance asymptotic behaviour of multi-point functions of critical one-dimensional quantum spin chains directly at zero temperature also manifest themselves on the level of the low-temperature behaviour of various quantities related with the associated quantum transfer matrix. In particular, if a given conformal field theory captures the long distance behaviour in the model at zero temperature, then the spectrum of this conformal field theory should arise in the low-temperature behaviour of the spectrum of the quantum transfer matrix.

In the case of the XXZ spin-1/2 chain, the quantum transfer matrix may be chosen to be integrable, what allows one, in principle, to study the mentioned universality properties of its spectrum by means of the Bethe Ansatz. In this talk, I will describe how the Bethe Ansatz approach can be put on rigorous grounds for the quantum transfer matrix subordinate to the XXZ chain. Further, I will explain how those results then allow one to access to the universal features of the spectrum of the quantum transfer matrix by showing that a subset thereof explicitly contains, in the low-temperature limit, the spectrum of the $c = 1$ free Boson conformal field theory.

This is a joint work with S. Faulmann and F. Göhmann.

Thu, Jun 22
09:40–10:25

## Curvature equation and isomonodromy deformation

Chang-Shou Lin

*National Taiwan University*

$$\Delta u + e^u = 8\pi n\delta_0 \quad \text{on } E_\tau \tag{1}$$

where $n \in \mathbb{N}$ and $\delta_0$ is the Dirac measure at the origin 0 of $E_\tau$, $E_\tau$ is the torus with periods 1 and $\tau \in \mathbb{H}$. The equation (1) possesses the so-called bubbling phenomena, that is, there are no a priori bounds.

**Conjecture.** *There are at most $n$ distinct one-parameter family of solutions.*

This conjecture has been proved for $n = 1$, and is open for $n \geq 2$. Recently, I have given a necessary and sufficient condition for the existence of solutions of (1). In particular, the set $\{\tau \in \mathbb{H} \mid (1) \text{ has a solution}\}/\operatorname{SL}(2,\mathbb{Z})$ is open and connected. Moreover, if $\tau = ib$, $b > 0$, then (1) has no solution.

One of applications is to consider

$$\Delta u + e^u = 8\pi(\delta_0 + \delta_p + \delta_q) \tag{2}$$

where $p \neq -q$, $\wp(p) + \wp(q) = 0$, and $\wp(z)$ is the Weierstrass elliptic function of order 2.

**Theorem** (Kuo–Lin)**.** *Suppose* (1) *with $n = 2$ has a solution. Then* (2) *has a solution provided that* $\tau \not\equiv \dfrac{1+\sqrt{3}i}{2} \mod \operatorname{SL}(2,\mathbb{Z})$.

Equation (1) is related to the classic Lame equation (an ODE of second order). In this lecture, I will talk about the recent works by Ting-Jung Kuo and my recent works with my collaborators Ting-Jung Kuo and Zhijie Chen. If time permits, I will discuss some open problem about Lame equation from the aspects of Floquet theory.

## Exponential decay and continuous monitoring

Mon, Jun 19
12:00–12:45

Konstantin A. Makarov

*University of Missouri*

In this talk I will recall the concept of continuous monitoring of a quantum system, and then discuss the related Quantum Zeno and Exponential Decay scenarios in quantum measurements.

I will show that for a typical initial state of the system, continuous monitoring of massive particles yields complementarity of the quantum Zeno and anti-Zeno effects, while for systems of massless particles, the quantum Zeno and Exponential Decay scenarios are instead complementary. Also, I will provide an example of an unstable pure state of the quantum oscillator that decays exponentially under continuous monitoring, which eventually confirms the conclusions of the phenomenological Weisskopf–Wigner theory of decay.

Our approach is based on the observation that the exponential decay scenario under continuous monitoring can be justified by applying a stable limit theorem.

## Birman's problem for symmetric Schrödinger operators

Mon, Jun 19
10:50–11:35

Mark Malamud

*RUDN University*

In early 2000s M. S. Birman posed the following problem:

**Problem.** *Let $A$ be a closed non-negative symmetric densely defined operator in a Hilbert space $\mathfrak{H}$ and let $\mathfrak{H}_1 := \operatorname{ran}(A+I)$. Assume that the inverse operator $(A+I)^{-1} \colon \mathfrak{H}_1 \to \mathfrak{H}$ is compact. Is it true that the resolvent of the Friedrichs extension $A_F$ of $A$ is also compact?*

First we discuss a complete solution to this problem in abstract setting by showing that the Friedrichs extension $A_F$ might have arbitrary non-negative spectrum, for instance, it might be purely absolutely continuous while $(A+I)^{-1}$ is compact.

The second part of the talk will be devoted to a solution to the Birman problem regarding the Schrödinger operators $H(q) = -\Delta + q \geq 0$ in $\mathbb{R}^n$. Namely, we give an explicit construction of appropriate subsets $Y \subset \mathbb{R}^n$ of zero Lebesgue measure that determine symmetric Dirichlet-type restrictions $A = H_Y(q) \geq 0$ of $H(q)$ to the domain $\operatorname{dom} A = \operatorname{dom}(H_Y(q))$ that consists of functions from the Sobolev space $W^{2,2}(\mathbb{R}^n)$ vanishing on $Y$. These restrictions meet the following properties:

1 The inverse operator $(A+I)^{-1}$ is compact;

2 Under the additional assumption that $Y$ has zero $(1,2)$-capacity and a certain assumption on a potential $q$, the Friedrichs extension $A_F$ of $A$ has continuous (sometimes absolutely continuous) spectrum filling the whole semiaxes $\mathbb{R}_+$.

As a byproduct of the above construction of the operators $A = H_Y(q)$ gives surprising explicit examples of symmetric second-order elliptic operators whose squares $A^2$ are densely defined nonnegative symmetric operators with the following properties:

(i) The Friedrichs extension $(A^2)_F$ of the operator $A^2$ is $A^*A$ and its spectrum is discrete, i. e. the inverse operator $((A^2)_F)^{-1}$ is compact;

(ii) The operator $(A_F)^2$ is not discrete. Moreover, its spectrum is

$$\sigma((A_F)^2) = \sigma_{\text{ess}}((A_F)^2) = [0, \infty).$$

The main results of the talk are announced in [1], [2].

References


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[2] M. M. Malamud, *On the Birman problem in the theory of nonnegative symmetric operators with compact inverse*, Func. Anal. Appl., 57 (2023), No. 2, 111–116.


Thu, Jun 22
10:30–11:15

## On the spectra of discrete Hausdorff operators


Adolf R. Mirotin

*Francisk Skorina Gomel State University*


In the talk the general case of normal discrete Hausdorff operators in $L^2(\mathbb{R}^d)$ is investigated. The main result states that under some arithmetic condition the spectrum of such operator is rotationally invariant. The Weyl and the point spectrum and several special cases are also considered.

## Homogenization of a one-dimensional periodic elliptic operator at the edge of a spectral gap: Operator estimates in the energy norm

Tue, Jun 20
12:30–12:55

Arseniy Mishulovich

*Saint Petersburg State University*

In $L_2(\mathbb{R})$, we consider a second-order elliptic differential operator

$$A_\varepsilon = -\frac{d}{dx} g(x/\varepsilon) \frac{d}{dx} + \varepsilon^{-2} p(x/\varepsilon), \qquad \varepsilon > 0,$$

with periodic coefficients. For small $\varepsilon$, we study the behavior of the resolvent of $A_\varepsilon$ in a regular point close to the edge of a spectral gap.

We will discuss the results on approximation of this resolvent in the "energy" norm (i. e., the norm of operators acting from $L_2(\mathbb{R})$ to the Sobolev space $H^1(\mathbb{R})$) with error $O(\varepsilon)$. Approximation is described in terms of the spectral characteristics of the operator $A \coloneqq A_1$ at the edge of the gap.

The talk is based on a joint work with T. A. Suslina and V. A. Sloushch.

## The a priori $\tan\Theta$ theorem and other estimates in the perturbation theory for spectral subspaces

Mon, Jun 19
15:00–15:45

Alexander K. Motovilov

*Bogoliubov Laboratory for Theoretical Physics, JINR*

We start with reviewing known bounds on variation of the spectral subspace of a self-adjoint operator under an additive symmetric perturbation. To this end we first recall the concept of operator angle between two subspaces of a Hilbert space $\mathfrak{H}$ as well as the concept of symmetric norm on a two-sided ideal in the algebra of bounded linear operators on $\mathfrak{H}$. Then we recollect the spectral dispositions for which sharp norm bounds on the variation of the spectral subspace associated with an isolated spectral set have already been established. Starting from the celebrated Davis-Kahan trigonometric theorems in the subspace perturbation theory, all of these bounds have the form of an estimate of a norm of some trigonometric function of the operator angle $\Theta$ between the unperturbed and perturbed spectral subspaces through the same norm of the perturbation operator. The latest known sharp norm estimate called the a priori $\tan\Theta$ theorem serves for the case where the unperturbed spectral set lies in a finite gap of the remainder of the spectrum. Unlike the previous trigonometric estimates that were proven to hold for any symmetric norm, by now the $\tan\Theta$ theorem was only established for the standard operator norm. In the present research, we prove estimates for the operators $\tan\Theta$ and $(\tan\Theta)^2$ that already involve their symmetric norms. We conclude the presentation by mentioning those questions of the subspace perturbation theory that still remain open.

Tue, Jun 20
15:20–15:45

## Uniform convergence for problems with perforation along a manifold and nonlinear Robin boundary condition on boundaries of cavities

Albina I. Mukhametrakhimova

*Bashkir State Pedagogical University named after M. Akhmulla*

We consider a boundary value problem for a second-order elliptic equation with variable coefficients in a bounded domain perforated along a given manifold. The domain can be either bounded or unbounded. We suppose that there is some minimal distance between the cavities, while their shapes and the distribution are arbitrary. On the boundaries of the cavities we impose a nonlinear Robin boundary condition. We consider the case, when the homogenization leads to a nonlinear delta interaction on the reference manifold. Our main result states the convergence of the solution of the perturbed problem to that of the homogenized one in the norm of the space $W_2^1$ uniformly in $L_2$-norm of the right-hand side in the equation. We also provide estimates for the convergence rate.

This is a joint work with D. I. Borisov.

Wed, Jun 21
15:50–16:35

## Uniformization of degenerate equations and semiclassical asymptotics

Vladimir E. Nazaikinskii

*Ishlinsky Institute for Problems in Mechanics RAS*

Let $X$ be a stratified manifold, and let $\widehat{H}$ be a (pseudo)differential operator on $X$ with possible degeneration along lower-dimensional strata. In some cases, the equation $\widehat{H}u = 0$ can be uniformized (i. e., lifted to a (pseudo)differential equation on a smooth manifold, generally speaking, of higher dimension) with the help of the following construction. Assume that $X$ can be represented as the orbit space of a smooth action of a compact Lie group $G$ on a smooth compact manifold $M$. Let us define Sobolev spaces on $X$ as subspaces of $G$-invariant functions in Sobolev spaces on $M$ and pseudodifferential operators on $X$ as the restrictions to these subspaces of $G$-invariant pseudodifferential operators on $M$. If the operator $\widehat{H}$ can be obtained in this way (i. e., it is the restriction of some $G$-invariant pseudodifferential operator $\widehat{P}$), then solving the equation $\widehat{H}u = 0$ is reduced to finding $G$-invariant solutions of the equation $\widehat{P}v = 0$. Semiclassical asymptotic solutions in this approach are given by Maslov's $G$-invariant canonical operator on Lagrangian submanifolds in $T^*M$. It can be interpreted as the canonical operator $K_\Lambda$ on the Lagrangian submanifolds $\Lambda$ of the phase space $\Phi$ obtained from a dense open subset $\tilde{\Phi} \subset T^*M$ of “regular points” by symplectic reduction with respect to the action of $G$ and containing the cotangent bundle $T^*X_0$ of the stratum $X_0$ of maximum dimension as a dense open subset. The talk will give a detailed description of the operator $K_\Lambda$. Here the construction of this operator in a neighborhood of points of $\Lambda \cap (X \setminus X_0)$ is of main interest; it leads to a representation of asymptotic solutions in a neighborhood of the set $X \setminus X_0$ in terms of special functions. Examples of the construction under consideration include linearized shallow water equations describing the run-up of long waves on a shallow beach and pseudodifferential equations on orbifolds.

The report presents the results obtained in the course of work on RSF project No. 21-11-00341 and RFBR project No. 21-51-12006.

## Asymptotic structure of spectra of Dirichlet thin tee beams

Tue, Jun 20
10:30–11:15

Sergei A. Nazarov

*Institute of Mechanical Engineering Problems RAS*

The Dirichlet problem in the T-shaped junction $\Omega^h = \Omega^h_{\|} \cup \Omega^h_{=} \subset \mathbb{R}^3$ of two thin ($h \ll 1$) rectangular plates, vertical and horizontal,

$$\Omega^h_{\|} = \Big\{ x = (x_1, x_2, x_3) \colon |x_1| < a_1/2,\ |x_2| < a_2/2,\ x_3 \in (0, h) \Big\},$$

and

$$\Omega^h_{=} = \Big\{ x \colon |x_1| < a_1/2,\ |x_2| < hH/2,\ x_3 \in (0, a_3) \Big\}$$

is examined for the Laplace and Lamè operators. Here, the lengths $a_j > 0$ are fixed while the parameters $h > 0$ and $H > 0$ are respectively small and variable. For both problems, the values $H_* \in (1, 2)$ and $H_\bullet \in (1, H_*)$ are found with the following properties. In the case $H \in (0, H_\bullet)$ the eigenfunctions are localized near the junction set $\omega^h = \Omega^h_{\|} \cap \Omega^h_{=}$ and decay as $O(e^{-\delta \operatorname{dist}(x, \omega^h)/h})$, $\delta > 0$, at a distance from it. On the other hand, for $H > H_*$, the eigenfunctions are concentrated at the wall $\Omega^h_{\|} \setminus \Omega^h_{=}$ but again decay exponentially in the base $\Omega^h_{=} \setminus \Omega^h_{\|}$. The limit problems describing behaviour of the eigenpairs of the original problem in $\Omega^h$ become the Dirichlet problems in the interval $(-a_1/2, a_1)$ for $H < H_\bullet$ and in the rectangle $(-a_1/2, a_1) \times (0, a_3)$ for $H > H_*$.

The localization effects provide opening of gaps in the spectrum of other, infinite in the direction $x_2$, periodic waveguides with the periodicity cell $\Omega^h$ while the number of opened spectral gaps grows unboundedly as $h \to +0$.

Asymptotic structures are crucially based on the spectral analysis of the Dirichlet problem in the two-dimensional T-shaped quantum and elastic waveguides composed from the unit strip and the semi-infinite strip of width $H$. However, many particular questions on their spectra remain unsolved that, in particular, does not allow to make conclusions on the spectral characteristics for walls of the relative thickness $H \in [H_\bullet, H_*]$.

The talk will focus on the scalar problem because, although the results look very similar and are obtained by resembling approaches, the vector problem in elasticity theory requires much more cumbersome calculations and demonstrations.

Support from the Russian Science Foundation grant 22-11-00046 is acknowledged.

## Einstein relation in homogenization theory

Tue, Jun 20
09:40–10:25

Andrey Piatnitski

*The Arctic University of Norway, UiT, campus Narvik & Institute for Information Transmission Problems RAS*

In the beginning of 20th century A. Einstein formulated a relation between the so-called “mobility” and “diffusivity” of a non-homogeneous medium. In the framework of homogenization theory this means that under a small constant perturbation of the convection terms of operator the effective drift can be expressed in terms of the effective diffusion of the non-perturbed operator. The talk will focus on Einstein relation for second-order elliptic operators in periodic and random media and for convolution type operators in periodic media.

Mon, Jun 19
15:50–16:35

## Weyl asymptotics for Poincaré–Steklov eigenvalues in a domain with Lipschitz boundary

Grigori Rozenblum

*Chalmers University & Saint Petersburg State University & Euler International Mathematical Institute*

The Weyl type asymptotic formula for eigenvalues of the Poincaré–Steklov problem, the one with spectral parameter in the boundary condition, in a domain with Lipschitz boundary is justified for second-order divergence elliptic operators with very week conditions imposed on the coefficients. The perturbation approach with roots in the fundamental papers by M. Sh. Birman and M. Z. Solomyak is used.

### References

[1] G. Rozenblum, *Weyl asymptotics for Poincaré–Steklov eigenvalues in a domain with Lipschitz boundary*, arXiv:2304.04047 [math.SP].

Wed, Jun 21
15:00–15:45

## $\eta$-invariants of boundary value problems

Anton Yu. Savin

*RUDN University*

Atiyah–Patodi–Singer [1] introduced $\eta$-invariants $\eta(A)$ of elliptic self-adjoint operators $A$ on closed manifolds as a regularized number of positive eigenvalues minus the number of negative eigenvalues. The regularization is defined in terms of analytic continuation of so-called $\eta$-function of the operator defined in terms of the eigenvalues. This invariant is a spectral invariant and has numerous applications and generalizations. For instance, it appeared as a contribution of the boundary in index formulas for Dirac operators on manifolds with boundary.

Melrose [2] introduced $\eta$-invariants $\eta(D(p))$ for elliptic parameter-dependent families of operators $D(p)$ on closed manifolds as a regularization of the winding number of the family. This invariant is a generalization of the Atiyah–Patodi–Singer $\eta$-invariant and also has applications. For instance, it appears as a contribution of the conical point for general elliptic operators on manifolds with isolated singularities.

Our aim is to extend the $\eta$-invariant of Melrose to parameter-dependent families of boundary value problems. We consider general parameter-dependent boundary value problems elliptic in the sense of Agranovich and Vishik and define $\eta$-invariants for such families. The main analytical result necessary to define the $\eta$-invariant is the asymptotic expansion of the trace of parameter-dependent families for large values of the parameter.

The talk is based on joint work in progress with Konstantin N. Zhuikov. This work was supported by the Ministry of Science and Higher Education of the Russian Federation

(project number FSSF-2023-0016).


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[2] R. B. Melrose, *The eta invariant and families of pseudodifferential operators*, Math. Res. Lett., 2 (1995), No. 5, 541–561.


## Adiabatic evolution generated by a Schrödinger operator: Behaviour of a previously localized quantum particle after the disappearance of the bound state

Fri, Jun 23
15:50–16:15


Vasily Sergeyev

*Saint Petersburg State University, Chebyshev Laboratory*


We consider the Schrödinger equation

$$i\varepsilon \frac{\partial \Psi}{\partial \tau} = -\frac{\partial^2 \Psi}{\partial x^2} + v(x,\tau)\Psi, \qquad \tau \in \mathbb{R},\ x \geq 0,\ \varepsilon \to 0, \tag{1}$$

with the boundary condition $\Psi|_{x=0} = 0$. The potential $v$ is a square potential well that shrinks linearly with time; $v(x,\tau)$ is equal to $-1$ if $0 \leq x \leq 1-\tau$ and to 0 otherwise. The operator $H(\tau) = -\frac{\partial^2}{\partial x^2} + v(x,\tau)$ with the Dirichlet boundary condition at $x = 0$ has (absolutely) continuous spectrum $\sigma_c = [0, +\infty)$, and its point spectrum consists of exactly $n$ negative eigenvalues while $\tau_{n+1} \leq \tau < \tau_n$, where $\tau_n = 1 - \pi(n - 1/2)$, $n \in \mathbb{N}$. When $\tau$ approaches the critical value $\tau_n$, the $n$th eigenvalue $E_n(\tau)$ approaches the edge of $\sigma_c$ and, having reached it, disappears.

A. Fedotov constructed a solution $\Psi_n$ to (1) (see [1]) that has asymptotics of the form

$$e^{-\frac{i}{\varepsilon}\int_{\tau_n}^{\tau} E_n(s)\,ds} \sum_{m=0}^{\infty} \varepsilon^m \psi_{n,m}(x,\tau), \qquad \varepsilon \to 0, \tag{2}$$

while $E_n(\tau)$ exists. Here $\psi_{n,0}(\cdot,\tau)$ is the $n$th eigenfunction of $H(\tau)$. For $\tau \gtrsim \tau_n$ the asymptotic behaviour of $\Psi_n$ changes. Fedotov described the asymptotics of $\Psi_n$ inside the potential well [1], [2]. In this talk we describe the asymptotics outside the well, both near the moment of disappearance of $E_n(\tau)$ and afterwards. In the first case, when $\tau \sim \tau_n$, the probability to find the quantum particle described by $\Psi_n$ is highest in a region of the half-plane $\{(\tau, x) \in \mathbb{R}^2 \colon x \geq 0\}$ resembling a searchlight emanating from the point $(\tau_n, 1-\tau_n)$ at a small angle to the line $\tau = \tau_n$ (see [3]). In the second case, when $\tau > \tau_n$, the particle is most likely to be found outside the potential well at distances of the order of $\varepsilon^{-1/2}$ from the well's boundary, and this probability decays exponentially with $x$. Inside the well and near its boundary, this probability is asymptotically larger for $\tau \sim \tau_k$ with $k < n$ than for other values of $\tau$.

This problem is kindred to the problem of sound propagation in variable-depth shallow water, which has been studied on a physical level of rigour by, for example, A. Pierce [4], and mathematically rigorously by A. Fedotov [5]. The acoustic problem can be studied using similar techniques, and analogous physical effects arise in it.

This talk is based on joint work with Alexander Fedotov. The research is supported by Native Towns, a social investment program of PJSC Gazprom Neft.

References


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[2] A. B. Smirnov, A. A. Fedotov, *Adiabatic Evolution Generated by a Schrödinger Operator with discrete and continuous spectra*, Funct. Anal. Its Appl., 50 (2016), 76–79.

[3] V. A. Sergeev, A. A. Fedotov, *On the delocalization of a quantum particle under the adiabatic evolution generated by a one-dimensional Schrödinger operator*, Math. Notes, 112 (2022), 726–740.

[4] Allan D. Pierce, *Guided mode disappearance during upslope propagation in variable depth shallow water overlying a fluid bottom*, J. Acoust. Soc. Am., 72 (1982), 523–531.

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Wed, Jun 21
10:50–11:35

## Short-wave asymptotic solutions for hyperbolic systems with rapidly varying coefficients

Andrei Shafarevich

*Moscow State University*

Short-wave asymptotic solutions for equations with smooth coefficients are deeply connected with geometric objects - Lagrangian surfaces or complex vector bundles over isotropic manifolds. If coefficients of equations are not regular (for example, varying rapidly near certain surface), the general theory can not be applied directly. In particular, geometric objects, mentioned above, have to be modified. We study short-wave asymptotics for strictly hyperbolic systems, assuming that coefficients vary rapidly near a hypersurface. We discuss modifications of the corresponding geometric objects and construct asymptotic series for the solution of the Cauchy problem.

Mon, Jun 19
10:00–10:45

## Dissipative pencils and operator differential equations in Hilbert space

Andrei A. Shkalikov

*Moscow State University*

We shall work with polynomial operator pencils of the form

$$A(\lambda) = A_0 + \lambda A_1 + \cdots + \lambda^n A_n,$$

where $A_j,\ j = 1, \dots, n$ are generally unbounded operators in Hilbert space $H$. The main attention will be paid to the case $n = 2m$. The pencil $A(\lambda)$ is said to be dissipative if $(A(\lambda)x, x) \geq 0$ for all $\lambda \in \mathbb{R}$ and all $x$ belonging to the domain of $A$. We shall write some useful identities which imply apriori estimates for the solutions of the equation $A(id/dt)u(t) = 0$ vanishing as $t \to +\infty$, and prove the factorization theorems for the pencil $A(\lambda)$. Some appllications will be given.

Support from the Russian Science Foundation grant No. 20-11-20261 is acknowledged.

## Homogenization of a non-self-adjoint non-local convolution type operator

Tue, Jun 20
14:30–15:15

Vladimir A. Sloushch

*Saint Petersburg State University*

In $L_2(\mathbb{R}^d)$, we consider a bounded operator $\mathbb{A}_\varepsilon$, $\varepsilon > 0$, given by

$$(\mathbb{A}_\varepsilon u)(x) \coloneqq \varepsilon^{-d-2} \int_{\mathbb{R}^d} a((x-y)/\varepsilon)\, \mu(x/\varepsilon, y/\varepsilon)\, (u(x) - u(y))\, dy, \quad x \in \mathbb{R}^d,\ u \in L_2(\mathbb{R}^d).$$

Operators of this type are used to describe the behavior of random systems of a large (infinite) number of particles. It is assumed that $a(x)$ is a non-negative function of class $L_1(\mathbb{R}^d)$ such that $\|a\|_{L_1} = 1$; $\mu(x, y)$ is a non-negative, bounded and separated from zero function. Suppose that

$$\mu(x+m, y+n) = \mu(x, y), \qquad x, y \in \mathbb{R}^d,\ m, n \in \mathbb{Z}^d;$$
$$M_k = \int_{\mathbb{R}^d} |x|^k a(x)\, dx < +\infty, \qquad k = 1, 2, 3;\ \sum_{n \in \mathbb{Z}^d} a(\cdot + n) \in L_{2,\mathrm{loc}}(\mathbb{R}^d).$$

Under these assumptions, the operator $\mathbb{A}_\varepsilon$ is bounded, the spectrum of $\mathbb{A}_\varepsilon$ is contained in the right half-plane $\{\lambda \in \mathbb{C} \colon\ \mathrm{Re}\, \lambda \geq 0\}$; the operator $\mathbb{A}_\varepsilon$ is not assumed to be self-adjoint.

We find approximation of the resolvent $(\mathbb{A}_\varepsilon + I)^{-1}$ in the operator norm on $L_2(\mathbb{R}^d)$ for small $\varepsilon$. To prove the results, we modify the operator-theoretic approach for the case of a non-self-adjoint operator.

The talk is based on joint work with A. L. Piatnitski, T. A. Suslina and E. A. Zhizhina.
Support from the Russian Science Foundation grant No. 22-11-00092 is acknowledged.

## Geometry of surfaces and a formation of singularities of 2D soliton equations represented by $L$, $A$, $B$-triples

Wed, Jun 21
10:00–10:45

Iskander A. Taimanov

*Sobolev Institute of Mathematics*

We expose recent results on a certain geometrical mechanism of a formation of singularities of the modified Novikov–Veselov and Davey–Stewartson II equations. The singular solutions are constructed by means of surface theory. These equations are represented by $L$, $A$, $B$-triples and the formation of singularities is related to a degeneration of the zero level discrete spectra of the corresponding $L$-operators.

Wed, Jun 21
17:00–17:25

## Asymptotic of solution of the Helmholtz equation with a linear refractive index, in which the conical Arnold's singularity of rays family appears

Anton A. Tolchennikov

*Ishlinsky Institute for Problems in Mechanics RAS*

The talk will be devoted to the 2-dimensional model Helmholtz equation with a linear refractive index and a localized right-hand side. In similar problems, according to the work of Dobrokhotov, Nazaikinskii, Anikin, Rouleau, the construction of asymptotic solution is based on the Lagrangian surface composed of the semi-trajectories of the Hamilton system, released from the circle (the intersection of the vertical plane with the zero surface Hamiltonian level). However, the result of this article cannot be applied directly, since there is one trajectory that comes to a singular point in infinite time. This leads to the fact that the asymptotic solution will be localized not only in the neighborhood projection of a 2-dimensional Lagrangian manifold into physical space, but also in the vicinity of the projection of a special ray that "breaks" off the Lagrangian surface. In addition, the Lagrangian surface is not smooth and its normal form (the Legendrian version) was described by V. I. Arnold in the book "Singularities of Caustics and Wave Fronts".

This joint work with S. Yu. Dobrokhotov and I. A. Bogaevsky was supported by a grant from the Russian Science Foundation No. 21-11-00341.

Wed, Jun 21
12:50–13:15

## Asymptotics of multiple orthogonal Hermite polynomials. Approach based on recurrence relations

Anna Tsvetkova

*Ishlinsky Institute for Problems in Mechanics RAS*

The multiple orthogonal Hermite polynomials $H_{n_1,n_2}(z,a)$ are defined by the following recurrence relations

$$H_{n_1+1,n_2}(z,a) = (z+a)H_{n_1,n_2}(z,a) - \frac{1}{2}\Big(n_1 H_{n_1-1,n_2}(z,a) + n_2 H_{n_1,n_2-1}(z,a)\Big),$$
$$H_{n_1,n_2+1}(z,a) = (z-a)H_{n_1,n_2}(z,a) - \frac{1}{2}\Big(n_1 H_{n_1-1,n_2}(z,a) + n_2 H_{n_1,n_2-1}(z,a)\Big).$$

We construct a uniform asymptotics in terms of the Airy function Ai of diagonal polynomials $H_{n,n}(z,a)$ as $n \to \infty$.

The idea of the method we are using is the following: we introduce a small parameter $h \sim \frac{1}{n}$ and a continuous function $\varphi(x,z,a)$ such that $H_{n,n}(z,a) = \varphi(nh,z,a)$. This allows us to reduce the system that defines the polynomials to a pseudo-differential equation for $\varphi$, where $x$ is a variable and $(z,a)$ are parameters. Obtained equation is related to the complex-valued Hamiltonian connected with the third-order algebraic curve. In general case, to obtain the result for such problems, a transition from real variable $x$ to a complex one is made. In this problem, we propose a different approach based on a reduction of the original problem to three equations, two of which have asymptotics with purely imaginary phases, and the symbol of the third one has the form $\cos p + V_0(x) + hV_1(x) + O(h^2)$. The asymptotic solution of the last equation can be expressed in terms of Airy function of the real-valued argument, what allows us to obtain a uniform asymptotics for $H_{n,n}(z,a)$.

The talk is based on the joint work with A. I. Aptekarev, S. Yu. Dobrokhotov and D. N. Tulyakov.

## Asymptotics of block Toeplitz determinants with piecewise continuous symbols

Fri, Jun 23
10:50–11:35

Jani Virtanen

*University of Reading*

I discuss recent results on the asymptotics of the block Toeplitz determinants generated by matrix-valued piecewise continuous functions with finitely many jumps under mild additional conditions. The approach is based on a new localization theorem for Toeplitz determinants, a new method of computing the Fredholm index of Toeplitz operators with piecewise continuous matrix-valued symbols, and other operator theoretic methods. As an application of our results, we consider piecewise continuous symbols that arise in the study of entanglement entropy in quantum spin chain models. Joint work with Estelle Basor and Torsten Ehrhardt.

## Spectral theory of Jacobi operators with increasing coefficients

Mon, Jun 19
17:00–17:45

Dmitri R. Yafaev

*IRMAR-UMR CNRS 6625 & Saint Petersburg State University*

Spectral properties of Jacobi operators $J$ are intimately related to an asymptotic behavior of the corresponding orthogonal polynomials $P_n(z)$ as $n \to \infty$. We study the case where the off-diagonal coefficients $a_n$ and, eventually, diagonal coefficients $b_n$ of $J$ tend to infinity as $n \to \infty$.

We suppose that $a_n \sim n^\sigma$ for some $\sigma > 0$ and that the ratio $\gamma_n := 2^{-1} b_n (a_n a_{n-1})^{-1/2}$ has a finite limit $\gamma$ as $n \to \infty$. In the case $|\gamma| < 1$ (and $\sigma \le 1$) asymptotic formulas for $P_n(z)$ generalize those for the Hermite polynomials and the corresponding Jacobi operators $J$ have absolutely continuous spectra covering the whole real line. If $|\gamma| > 1$, then spectra of the operators $J$ are discrete.

Our main concern in this talk is to investigate the critical case $|\gamma| = 1$ that occurs, for example, for the Laguerre polynomials. Our results show that there are two “phase transitions”: for $\sigma = 1$ and for $\sigma = 3/2$. If $\sigma \le 1$, then the absolutely continuous spectrum of the Jacobi operator $J$ coincides with some half-axis. In the case $\sigma \in (1, 3/2]$, the spectrum of $J$ is either absolutely continuous and covers the whole real-axis for relatively small diagonal coefficients $b_n$ or it is discrete for large $b_n$. If $\sigma > 3/2$, then the minimal Jacobi operator has deficiency indices $(1, 1)$ and the spectra of all its self-adjoint extensions are discrete.

Tue, Jun 20
11:40–12:25

## Homogenization of non-local parabolic problems with non-symmetric kernels in stationary ergodic media

Elena Zhizhina

*Institute for Information Transmission Problems RAS & The Arctic University of Norway, Campus Narvik*

In my talk I present results of our joint work with A. Piatnitski. We study homogenization of parabolic problems for integral convolution type operators with a non-symmetric jump kernel that is periodic in spatial variables and stationary random in time:

$$\frac{\partial u^\varepsilon}{\partial t} = \frac{1}{\varepsilon^{d+2}} \int\limits_{\mathbb{R}^d} a\Big(\frac{x-y}{\varepsilon}\Big)\ \mu_\omega\Big(\frac{x}{\varepsilon}, \frac{y}{\varepsilon}; \frac{t}{\varepsilon^2}\Big)\Big(u^\varepsilon(y,t) - u^\varepsilon(x,t)\Big)\ dy,$$

where $\varepsilon > 0$ is a small parameter, $\mu_\omega(\xi, \eta; \tau)$ is periodic in spatial variables and stationary random in time. The function $\mu_\omega(\cdot)$ as well as the function $a(\cdot)$ are not assumed to be symmetric in spatial variables. Under good mixing properties on $\mu_\omega$, we show that the limit homogenization problem takes the form of a SDE. It is worth noting that the observed significant change in the behavior of the limit equation occurs due to a lack of symmetry of the kernel.