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# Chow-weight homology: a "mixed motivic decomposition of the diagonal"

Basic properties of Chow motives imply that if a Chow motif $M$ is $r$-effective ($r > 0$) then its Chow groups $\mathrm{CH}_i(M, \mathbb{Q}) = \{0\}$ for $i < r$. 40 years ago S. Bloch noted that the converse is true if the base field is a universal domain. This easily yielded the corresponding effectivity properties of the cohomology of a smooth projective $P$ whose lower Chow groups are trivial; moreover, the diagonal cycle on $P \times P$ decomposes into a certain sum.

Together with V. Sosnilo and D. Kumallagov, I extended these results (in several ways) to Voevodsky motives, motives with compact support of arbitrary varieties, and cohomology with compact support.

**Theorem** (B.+V. Sosnilo)**.** *Let $r > 0$, $X$ is a $k$-variety (that is, a reduced separated scheme of finite type over $k$), $e$ is the exponential characteristic of $k$, $K_0$ is a universal domain containing $k$.*

*Assume that $\mathrm{CH}_j(X_{K_0}, \mathbb{Q}) = \{0\}$ for $0 \le j < r$. Then the following statements are valid.*

*1. There exists $E > 0$ such that $E \cdot \mathrm{CH}_j(X_{k'}, \mathbb{Z}[1/e]) = \{0\}$ for all $0 \le j < r$ and all field extensions $k'/k$.*

*2. If $k$ is a subfield of $\mathbb{C}$ and $q > 0$ then the (highest) $q$-th weight factor of the mixed Hodge structure $H^q_c(X_{\mathbb{C}})$ (the singular cohomology of $X_{\mathbb{C}}$ with compact support) is $r$-effective (as a pure Hodge structure).*

*Moreover, the same property of the Deligne weight factors of $H^q_c(X_{k^{alg}})$ is fulfilled for étale cohomology with values in the category of $\mathbb{Q}_\ell[\mathrm{Gal}(k)]$-modules if $k$ is the perfect closure of a finitely generated field, $\ell \in \mathbb{P} \setminus \{e\}$.*

*In particular, these factors are zero if $q < 2r$.*

*3. The motif $\mathcal{M}^c_{\mathbb{Q}}(X)$ (of $X$ with compact support) is an extension of an element of $DM^{\mathrm{eff}}_{gm}(k, \mathbb{Q})_{w_{\mathrm{Chow}} \ge 1}$ by an object of $\mathrm{Chow}^{\mathrm{eff}}(k, \mathbb{Q})\langle r \rangle$; here $DM^{\mathrm{eff}}_{gm}(k, \mathbb{Q})_{w_{\mathrm{Chow}} \ge 1}$ is the extension-closure of $\cup_{i>0} \mathrm{Chow}^{\mathrm{eff}}(k, \mathbb{Q})[i]$.*

Moreover, condition 3 implies condition 1; well-known motivic conjectures yield that condition 2 implies condition 1 as well.

The most general of our motivic formulations involve the new Chow-weight homology theories. Those are defined in terms of the (exact and conservative) weight complex functor $DM^{\mathrm{eff}}_{gm} \to K^b(Chow^{\mathrm{eff}})$. The latter is defined and studied by means of the theory of weight structures on triangulated categories.