# Discrete and Continuous Signals: Analysis, Information and Applications December 11 – 16, 2023

## PLENARY TALKS

### Quantum fingerprinting and hashing for computing, processing and transmitting information

*Farid Ablayev* (*Kazan Federal University*)

Quantum fingerprinting is a family of quantum functions that began to be used in quantum algorithms in the early 2000s. They map classical objects to quantum states in such a way that different arguments can be effectively distinguished. To further highlight the additional cryptographic characteristics of quantum fingerprinting, our group uses the name "quantum hashing". The talk introduces the basic concepts of quantum fingerprinting and hashing, as well as their use in communications protocols, cryptographic signatures, and pattern searching in text.

### Random matrices ensembles and multiple orthogonal polynomials

*Alexander Aptekarev* (*Keldysh Institute of Applied Mathematics*)

Let $\mu(x) := (\mu_1(x), \dots, \mu_d(x))$ be a vector of positive measures. For a given multiindex $n = (n_1, \dots, n_d)$ we consider a polynomial $P_n(x)$ of degree $|n| := n_1 + \cdots + n_p$, which satisfies $n_j$ orthogonality relations to the degrees of the scalar variable $x$ with respect to the measure $\mu_j$, $j = 1, \dots, p$. Such polynomials always exist and they are called multiple orthogonal polynomials. For $p = 1$ we have usual orthogonal polynomials. We discuss several examples of ensembles of random matrices related to the multiple orthogonal polynomials (namely: random matrix model with external source, two matrix model and normal matrix model). An application to the Brownian bridges will be highlighted.

### Approximation by sums of shifts of one function

*Petr Borodin* (*Lomonosov Moscow State University*)

The talk presents a survey of results on the density of the additive semigroup generated by shifts of one function in various function spaces.

## Multiplicative chaos

*Alexander Bufetov* (*Steklov Mathematical Institute, Moscow, Institute for Information Transmission Problems, and Saint Petersburg State University*)

In this survey talk, we will consider different approaches to constructing and studying the exponential of a random field, with a special emphasis on proving the convergence to Gaussian multiplicative chaos of random Euler-like products assigned to realizations of the sine-process.

## Three versions of the nonlinear Fourier transform for real-analytic signals in optical fibers

*Andrei Domrin* (*Lomonosov Moscow State University*)

The evolution of the complex envelopes of signals in fiber-optics communications is described by the focusing nonlinear Schrödinger equation. This equation is integrable by the inverse scattering method, which has three versions corresponding to rapidly decaying, periodic and local holomorphic signals respectively. We mention the remarkable recent advances in the study of the pointwise convergence and time-frequency localization properties of the direct scattering transform (in the first and second versions), which is regarded as the nonlinear Fourier transform. We use the third version to prove the global-in-time real-analytic solvability of the Cauchy problem for any real-analytic initial data and discuss the natural isomorphism between the third version and each of the first two versions in the case of real-analytic signals satisfying the corresponding boundary conditions.

## A curious hypergeometric identity and perfectness of Meixner–Sorokin system of weights

*Alexander Dyachenko* (*Keldysh Institute of Applied Mathematics*)

On two integer lattices with interlacing nodes let us introduce two discrete measures for which a point weight is determined by the product of two classical Meixner weights. It turns out that the resulting measures are positive when the lattices alternate.

This generalisation of the Meixner weight was introduced by Sorokin in 2010. He studied asymptotic behaviour of the corresponding orthogonal polynomials $P_{n,n}$ with diagonal indices. The orthogonal polynomials may be constructed using the (discrete) Rodrigues formula, but the question whether they are uniquely defined remained open.

Indeed, such a system of measures does not form a Nikishin system as the measures have disjoint supports; it does not form an Angelesco systems either since one of these supports lies in the convex hull of the other. Nevertheless, the uniqueness of the orthogonal polynomials (i.e. the normality of diagonal indices for Sorokin's system of measures) would be useful for applying Sorokin's result.

The talk will be aimed at showing that this system of measures is perfect (that is all indices are normal). In the course of our proof we derive an intriguing hypergeometric identity. We also rely on an extension of the aforementioned Rodrigues formula.

This is a joint research with Alexander Aptekarev and Vladimir Lysov.

## Removable sets for Newtonian space $N^{1,p}$

*Yuri Dymchenko (Maritime State University named after G. I. Nevelskoy), Vladimir Shlyk (Far Eastern Federal University)*

Let $X = (X, d, \mu)$ be a complete $p$-Poincaré metric space with distance $d$ and a Borel regular doubling measure $\mu$, $1 < p < \infty$. Following Vodop'yanov and Gol'dstein, we introduce an analogue of $NC_p$-sets in the domain $\Omega$ of $X$ and give the criterion of equality $N^{1,p}(\Omega \setminus E) = N^{1,p}(\Omega)$ in terms of $E$ as an $NC_p$-set in $\Omega$. As a consequence, we obtain that the domains $\Omega_1$ and $\Omega$, $\Omega_1 \subset \Omega$, are $(1, p)$-equivalent if and only if $\Omega \setminus \Omega_1$ is an $NC_p$-set in $\Omega$. Moreover, for a quasisymmetric map $f : X \to Y$ of two complete $p$-regular, $p$-Poincaré metric spaces $X$ and $Y$, we show that $f(E)$ is an $NC_p$-set in the image $f(\Omega)$ if and only if $E$ is an $NC_p$-set in $\Omega \subset X$.

## On Dirichlet problem for bianalytic functions and for solutions of general second-order non-strongly elliptic systems with constant coefficients

*Konstantin Fedorovskiy (Lomonosov Moscow State University and Saint Petersburg State University)*

In the talk we plan to discuss Dirichlet problem for non-strongly elliptic second-order PDE with constant complex coefficients in bounded simply connected domains in the complex plane. Starting with the most known case of bianalytic functions (that corresponds to the Bitsadze equation), we will proceed to discuss the case of solutions to general non-strongly elliptic second order PDEs with constant complex coefficients and, moreover, solutions to general non-strongly elliptic systems of second-order PDEs with constant coefficients. We will show that any Jordan domain in the complex plane with sufficiently regular (smooth) boundary is not regular with respect to the Dirichlet problem for any non-strongly elliptic system under consideration, which means that there always exists a continuous complex-valued function on the boundary of the domain under consideration that can not be continuously extended to this domain to a function satisfying the corresponding system therein. Since there exists a Jordan domain with Lipschitz boundary, which is regular with respect to the Dirichlet problem for bianalytic functions, the result obtained is near to be sharp. The discovered phenomena that domains with sufficiently smooth boundaries are not regular with respect to the Dirichlet problem for systems under consideration, while domains having worse boundaries may be regular is rather unexpected and essentially new.

## Free topological groups

*Leonid Genze (Tomsk State University)*

The talk will be devoted to free $n$-periodic and free Abelian $n$-periodic topological groups of the Tikhonoff space $X$. A classification of these groups will be given for the case when $X$ is a compact segment of ordinals. The second part of the report will compare some properties of $n$-periodic (Abelian) groups with the properties of free (Abelian) topological groups, which were introduced and studied in the classical works of A.A. Markov and M.I. Graev in 1945 and 1948, respectively.

## Soft Riemann–Hilbert problems and planar orthogonal polynomials

*Haakan Hedenmalm (Royal Institute of Technology (KTH), Stockholm, and Saint Petersburg State University)*

Recent advances on planar orthogonal polynomials with respect to exponentially varying weights become much easier to work with if we introduce the concept of "soft" Riemann–Hilbert problems. These are actually $\bar{\partial}$-problems but the underlying thinking in terms of jump problems helps a lot. The method applies also in the hard edge case where our work with A. Wennman gives a far-reaching generalization of Carleman's theorem from 1921.

## TBA

*Alexander Holevo (Steklov Mathematical Institute, Moscow)*

## Left-invariant diffusions on compact groups through Dirichlet form perturbation

*Qi Hou (Beijing Insitute of Mathematical Sciences and Applications)*

Gaussian convolution semigroups (also called heat semigroups) on infinite dimensional compact groups exhibit a rich variety of analytic properties. The current knowledge mostly restricts to the product or central types, corresponding to bi-invariant diffusions. In this talk we explain how to obtain heat kernel information for certain (non-product-type, noncentral) Gaussian semigroups through Dirichlet form perturbations. They correspond to certain left-invariant elliptic or sub-elliptic diffusions on compact groups, with generators left-invariant (sub-)Laplacians. This is based on joint work with Laurent Saloff-Coste.

## Integral formulas and inequalities for meromorphic functions and differences of subharmonic functions with applications

*Bulat Khabibullin, Enzhe Menshikova (Institute of Mathematics with Computing Centre, Ufa)*

We consider various formulas relating integrals of delta-subharmonic functions to their Riesz distributions of charges. These new formulas represent far-reaching developments and generalizations of the classical Poisson–Jensen, Shimizu–Ahlfors, I. I. Privalov, T. Carleman, B. Ya. Levin and other integral formulas. A distinctive feature of the new formulas is the absence in them of any derivatives or function values at individual points. The inequalities obtained from these formulas will be applied to uniqueness theorems for entire functions, to approximation in spaces of functions on subsets of the complex plane, etc. We will also indicate possible multidimensional complex generalizations of these results.

## On numerical differentiation of analytic functions by differences of $h$-sums

*Mikhail Komarov* (*Vladimir State University*)

In 2006, V. I. Danchenko suggested the method of $h$-sums for approximation problems and numerical analysis (by *$h$-sum of order $n$* we mean an expression of the form

$$H_n(z) = \sum_{k=1}^{n} \lambda_k h(\lambda_k z), \quad \lambda_k \in \mathbb{C},$$

where $h(z)$ is a fixed function, analytic in the disk $D = \{z : |z| < 1\}$, and $\lambda_1, \ldots, \lambda_n$ are independent numeric parameters). In particular, it was proved that for every $n = 1, 2, \ldots$ there are numbers $\lambda_{n,1}, \ldots, \lambda_{n,n}$ such that the corresponding $h$-sum $H_n$ interpolates the differentiation operator $(zh(z))'$ at the node $z_0 = 0$ with the multiplicity $n$:

$$(zh(z))' = \sum_{k=1}^{n} \lambda_{n,k} h(\lambda_{n,k} z) + O(z^n),$$

and $H_n(z) \rightrightarrows (zh(z))'$ as $n \to \infty$ on compact subsets $K \subset D$ with exponential rate.

We consider an analogous problem of numerical differentiation *by the difference of two $h$-sums of order $n$*. It can be shown that under this modified approach the rate of approximation is much higher.

## Correct use of Padé approximants and Hermite–Padé polynomials in practice

*Aleksandr Komlov* (*Steklov Mathematical Institute, Moscow*)

Let $f$ be an algebraic function and $f_0$ be its germ at some point $z_0 \in \mathbb{C}$. Padé approximants are the best rational approximations of a given degree for the germ $f_0$. The convergence and the asymptotic behaviour of Padé approximants is described by Stahl theory. Nevertheless, sometimes this theory is used incorrectly in practice even in industrial applications. We give one example, where such mistake led to collapses in global electrical networks. We explain this mistake and explain how it was fixed by Trias in 2012 due to correct understanding of Stahl theory. Further we consider such generalizations of Padé polynomials as Hermite–Padé polynomials. Unfortunatelly, for these polynomials there is no analogue of Stahl theory for general $f$. But, sometimes Hermite–Padé polynomials are also used in practical applications without theoretical justification. Moreover, in some problems they are much more effective than usual Padé polynomials. We give such example from the molecular chemistry and justify it in the model case, when the function $f$ is 3-valued.

## Time-frequency localization operator

*Aleksei Kulikov (Tel Aviv University)*

Given a measurable set $U \subset \mathbb{R}$, we define the projection onto $U$ as $P_U : L^2(\mathbb{R}) \to L^2(\mathbb{R})$ given by $(P_U f)(x) = f(x)\chi_U(x)$. Similarly, for the set $V \subset \mathbb{R}$ we define the Fourier projection onto $V$ as $Q_V = \mathcal{F}^{-1} P_V \mathcal{F}$, where $\mathcal{F}$ is a Fourier transform. The operator $S_{U,V} = P_U Q_V P_U$ is called a time-frequency localization operator, associated with $U$ and $V$.

It is easy to see that $S_{U,V}$ is a non-negative definite operator of norm at most 1. If both $U$ and $V$ have finite measure it turns out that $S_{U,V}$ is a Hilbert–Schmidt operator with $||S_{U,V}||_{HS}^2 = |U||V|$. In particular, it is a compact operator and as such it has a sequence of eigenvalues $1 \geq \lambda_1(U,V) \geq \lambda_2(U,V) \geq \ldots > 0$.

In this talk, we will focus on the case when both $U$ and $V$ are intervals. In this case the eigenvalues depend only on the product of length of the intervals $c = |U||V|$, so we have a sequence $1 > \lambda_1(c) > \lambda_2(c) > \ldots > 0$. It turns out that these eigenvalues exhibit a phase transition: first $\approx c$ of them are very close to 1, then there are $\approx \log c$ intermediate ones and the remaining eigenvalues decay to zero extremely fast. We will discuss the behaviour of eigenvalues in these regimes, with focus on the most interesting, intermediate region. If time permits we will also mention a new exponential lower bound for the eigenvalues $\lambda_n(c)$ when $n < (1-\varepsilon)c, \varepsilon > 0$.

## Coefficient approach to the problem of describing homogeneous manifolds

*Alexander Loboda (Voronezh State Technical University and Voronezh State Technical University)*

The problem of describing holomorphically homogeneous real hypersurfaces of spaces $\mathbb{C}^n$, $n = 2, 3, 4$, is discussed as well as some close questions about the homogeneity of embedded submanifolds. We consider an approach related to the normalization of the surfaces themselves and the Lie algebras of vector fields associated with them, as well as to the determination of these objects through the lower Taylor coefficients of the functions representing them.

The use of canonical (Moser normal) equations of real hypersurfaces in the space $\mathbb{C}^2$ made it possible to obtain a complete description of holomorphically homogeneous hypersurfaces of this space. By using a similar technique in the space $\mathbb{C}^3$ a local description was given of fairly representative families of homogeneous Levi-nondegenerate hypersurfaces with "rich" symmetry algebras.

Reduction to canonical form of the basic holomorphic vector fields of 5-dimensional Lie algebras in the space $\mathbb{C}^3$ made it possible to obtain a complete description of all holomorphically homogeneous Levi-nondegenerate hypersurfaces with trivial symmetry algebras. A similar prospect emerges for 7-dimensional Lie algebras in the space $\mathbb{C}^4$. At the same time, questions about the dimension of the symmetry algebras of the resulting homogeneous manifolds can also be studied using normal equations, the coefficient approach and computer programs of symbolic calculations.

## The spaces of delta-subharmonic functions of finite order with respect to the model function of growth

*Konstantin Malyutin* (*Kursk State University*)

The classes of standard functions introduced by B. N. Khabibullin are considered. The concept of a model function of growth covers a large class of functions. Functions $f$ of finite order with respect to the model function can have an order of growth in the classical sense equal to infinity or zero. For example, the model functions of growth include functions of $r > 0$ of the form $\exp^{\circ n} r$, where $\exp^{\circ n}$ is the $n$-th superposition with $n = 1, 2, \dots$ of the exponential function exp, degree of the logarithmic function $\ln^p(e+r)$ for any $p \geq 1$, and in general any differentiable function $M(r) > 0$ such that the function $rM'(r) > 0$ increases for $r > 0$. We prove that for any function $f$ defined on $\mathbb{R}^+$ whose growth is determined by a model function of growth $M$, there exist its own proximate growth functions with respect to the model growth function $M$ that estimate $f$ above and below. Thus, we solve the extended Hadamard problem for a fairly wide class of entire and subharmonic functions. The proof is constructive.

The research is supported by Russian Science Foundation (project No. 22-21-00012).

## Bipolar Green function and Nuttall decomposition of a three-sheeted torus

*Semen Nasyrov* (*Kazan Federal University*)

We study the Nuttall decomposition of a three-sheeted torus $T$ which is a Riemann surface $S$ of the algebraic function $w = \sqrt[3]{(z-a_1)(z-a_2)(z-a_3)}$. This decomposition is induced by a harmonic function $U$ with two logarithmic singularities, it is called bipolar Green function. It is important to describe this function and investigate its properties. With the help of Weierstrass elliptic functions, we construct the universal covering of $T$ by the complex plane $\mathbb{C}$ and study the harmonic function $\widetilde{U}$ on $\mathbb{C}$ corresponding to $U$. The sets $\widetilde{U}(z) = 0$, $\widetilde{U}(e^{2\pi i/3}z) = 0$ and $\widetilde{U}(e^{-2\pi i/3}z) = 0$ fully define decomposition of the complex plane into three parts which induces the Nuttall decomposition of $T$. According to the Nuttall conjecture, this decomposition defines the convergence domains for the Hermite–Padé approximants.

The study was supported by the grant of the Russian Science Foundation No. 23-11-00066.

## Mellin–Barnes integrals and equivalences of triangulated categories

*Romo Jorquera* (*Yau Mathematical Science Center, Tsinghua University*)

In this talk we will present a pedagogical survey on how convergence of certain Mellin–Barnes integrals, that arise from open quantum field theory, can be used to predict equivalences between different triangulated categories. We will focus on the case of how monodromies give rise to autoequivalences of derived categories of coherent sheaves and present several examples, as time allows.

## Seiberg–Witten equations and pseudoholomorphic curves

*Armen Sergeev (Steklov Mathematical Institute, Moscow)*

Seiberg–Witten equations (SW-equations for short) were proposed in order to produce a new kind of invariant for smooth 4-dimensional manifolds. These equations, opposite to the conformally invariant Yang–Mills equations, are not invariant under scale transformations. So to draw a useful information from these equations one should plug the scale parameter $\lambda$ into them and take the limit $\lambda \to \infty$.

If we consider such limit in the case of 4-dimensional symplectic manifolds solutions of SW-equations will concentrate in a neighborhood of some pseudoholomorphic curve (more precisely, pseudoholomorphic divisor) while SW-equations reduce to some vortex equations in normal planes of the curve. The vortex equations are in fact static Ginzburg–Landau equations known in the superconductivity theory. So solutions of the limiting adiabatic SW-equations are given by families of vortices in the complex plane parameterized by the point z running along the limiting pseudoholomorphic curve. This parameter plays the role of complex time while the adiabatic SW-equations coincide with a nonlinear $\bar{\partial}$-equation with respect to this parameter.

## Domains of univalence for classes of bounded holomorphic functions

*Aleksei Solodov (Lomonosov Moscow State University)*

The problem of finding domains of univalence for classes of holomorphic self-mappings of the disc is concidered. In 1926, E. Landau found sharp radius of the disc of univalence for the class of such mappings with a given value of derivative at the inner fixed point. In 2017, V. V. Goryainov discovered the existence of domains of univalence for the classes of holomorphic self-mappings of the disc with two fixed points and conditions on the values of angular derivatives at the boundary fixed points. The report is devoted to the development of these results. The sharp domains of univalence for classes of holomorphic self-mappings of the disc with repulsive boundary fixed point are found depending on the localization of the attracting fixed point and the value of the angular derivative at the repulsive fixed point.

## Information disclosure in networks

*Nikolay Vereshchagin (Lomonosov Moscow State University)*

We consider the network consisting of three nodes 1, 2, 3 connected by two open channels $1 \to 2$ and $1 \to 3$. The information present in the node 1 consists of four strings $x$, $y$, $z$, $w$. The nodes 2, 3 know $x$, $w$ and need to know $y$, $z$, respectively. We want to arrange transmission of information over the channels so that both nodes 2 and 3 learn what they need and the disclosure of information is as small as possible. By information disclosure we mean the amount of information in the strings transmitted through channels about $x, y, z, w$ (or about $x, w$). We are also interested in whether it is possible to minimize the disclosure of information and simultaneously minimize the length of words transferred through the channels.

## On the construction of the Green’s function for strongly elliptic systems in domains on the plane

*Astamur Bagapsh* (*Bauman Moscow State Technical University and Federal Research Center “Computer Science and Control” of RAS*)

We consider strongly elliptic second-order systems with constant coefficients in domains on the plane reduced to the canonical form with only two real parameters $\tau$, $\sigma \in [0, 1)$ and written in complex form. When both parameters are equal to zero, we obtain the complex Laplace equation. When only $\sigma = 0$, we have the skew-symmetric system which is a perturbation of the Laplace one by the parameter $\tau$. Its solutions are represented as a sum of two holomorphic functions of variables $z_\tau = z - \tau\overline{z}$ and $\overline{z}$. In some regions, it is possible to construct the Green’s function for a skew-symmetric system using the analogue of Schwarz function which holomorphically expresses $z_\tau$ on the boundary in terms of $\overline{z}$. Although different Schwartz functions may arise for different parts of the boundary, for such domains as, for example, a strip or an angle, it is possible to select functions that are invariant under the replacement of one Schwarz function by another. Using such functions, the Green’s function is constructed. The general case of a two-parameter system is studied by considering it as a perturbation of a skew-symmetric system with respect to the parameter $\sigma$.

## Weissler type inequalities in Bergman spaces

*Diana Khammatova* (*Kazan Federal University*)

For $r \in (0, 1)$ let $f_r(z) = f(rz)$. A well-known Weissler inequality states that for the Hardy spaces $H^p$ and $H^q$, $p \le q$,

$$\|f_r\|_{H^q} \le \|f\|_{H^p} \qquad \Longleftrightarrow \qquad r \le \sqrt{\frac{p}{q}}.$$

We consider Bergman spaces $A^p(w)$ with the weight $w(r) \ge 0$, consisting of functions $f$ analytic in the unit disk $\mathbb{D}$ and such that

$$\|f\|_{A^p(w)} := \left( \frac{1}{2\pi} \int\limits_{\mathbb{D}} |f(z)|^p w(|z|)\, dz \right)^{1/p} < \infty.$$

Obtaining a counterpart of the Weissler inequality for Bergman spaces is a problem that is currently being actively researched. For classical Bergman weights $w_\alpha(r) = 2(\alpha - 1)(1-r^2)^{\alpha-2}$, $\alpha > 1$, a significant progress was achieved by K. Seip, F. Bayart, O. F. Brevig, J. Ortega-Cerdà, K.-M. Perfect and, recently, A. Kulikov, and P. Melentijević.

In our work, we considered a counterpart of the Weissler inequality for Bergman spaces $A^{2n}(w)$ and $A^2(w)$ with general radial weights. We found the conditions on the weight $w$ in terms of its moments ensuring that the counterpart is true for $n \in \mathbb{N}$ and $0 < r \le \frac{1}{\sqrt{n}}$.

We also examined the case of noninteger exponents for the particular function $f(z) = e^z$. Then the corresponding inequality can be considered as a certain analog of the Bernoulli inequality. An example of a monotonic weight was constructed for which these inequalities are no longer true.

The talk is based on a joint work with A. D. Baranov, I. R. Kayumov and R. Sh. Khasyanov.

## V.V. Chernikov's method for investigating functionals on the class of conformal univalent mappings

*Ivan Kolesnikov (Tomsk State University)*

The variational method is one of the main methods for finding a region of a functional and boundary mappings. The greatest difficulty of this method is the study of the functional differential equation for the boundary mapping. V. V. Chernikov proposed a way for finding a solution to such a functional differential equation using the area method. We use the idea of V. V. Chernikov to study some functionals defined on the class $S$, and consider questions about improving the method.

The work was supported by RSF Grant No. 23-21-00080.

## On the scalar approach to the weak asymptotic problem for the model $\mathcal{GN}$-system

*Elijah Lopatin (Steklov Mathematical Institute, Moscow)*

Traditionally, weak asymptotics of Hermite–Padé polynomials for various systems of Markov functions (such as the Angelesco and Nikishin systems) is described in terms of the vector potential problem on the complex plane. This approach has a number of limitations in context of generalisation of Stahl's theory on Hermite–Padé polynomials. Development of the alternative language was initiated by Sergey Suetin (Steklov Institute, Moscow) in 2018. He considered a very special generalised Nikishin system ($\mathcal{GN}$ system) of two functions; corresponding weak asymptotic problem was solved via scalar potential problem with a harmonic exterior field on a Riemann surface of genus zero. This technique was generalised on the slightly wider class of $\mathcal{GN}$ systems by me in 2021; the corresponding potential problem lives on a Riemann surface on genus $g > 0$. In this talk I will summarise recent results on the development of this scalar approach.

## Analytic continuation of power series by means of coefficients interpolation

*Aleksandr Mkrtchyan (Siberian Federal University and Institute of Mathematics NAS RA)*

We consider the problem of analytic continuation of power series in the sectoral domain by means of interpolation of coefficients by holomorphic functions. We explore the relationship between the growth (indicator) of the interpolating function and the set, to which the series can be extended. To estimate the growth of a holomorphic function of several complex variables, we use a multidimensional indicator after Ivanov, which is one of generalizations of the indicator introduced by Phragmén and Lindelöf for analytic functions of one complex variable. Also, a piecewise affine majorant for some subclass of functions will be used.

We obtain a multivariate version of the Le Roy and Lindelöf theorem, i.e., establish a connection between the growth of the interpolating function of the coefficients on the imaginary subspace and the multivariate sectoral domain to which the multiple series extends analytically.

## Avkhadiev–Wirths' conjecture for Hardy inequalities on balls

*Ramil Nasibullin (Kazan Federal University)*

The conjecture states that among all $n$-dimensional domains with given inradius $\delta_0$ the maximum of the best Brezis–Marcus constants $\lambda(\Omega)$ is given by $\lambda(B_n)$, where $B_n$ is the $n$-dimensional ball of radius $\delta_0$. In the talk the known results in Hardy type inequalities with additional terms are presented and we will give review of our own results in this subject. We improve estimates of $\lambda(\Omega)$.

## Conformal spectral estimates of the Dirichlet-Laplacian

*Valerii Pchelintsev (Tomsk State University)*

In this talk we give applications of the conformal geometry to the spectral estimates of the Dirichlet-Laplace operator in bounded simply connected domains with highly non-rectifiable boundaries. Our method is based on the geometrical theory of composition operators on Sobolev spaces. These composition operators are generated by conformal mappings. The talk is based on a joint work with Ivan Kolesnikov.

Research was supported by RSF Grant No. 23-21-00080.

## On the range of a convolution operator in spaces of ultradifferentiable functions

*Daria Polyakova (Southern Federal University, Rostov-on-Don)*

We consider a nonsurjective convolution operator in the Beurling space of ultradifferentiable functions of mean type generated by the weight function $\omega$. We establish necessary and (separately) sufficient conditions on the symbol under which the range of the operator contains the space defined by another weight function and of another type. These results are applied to convolution operators in the Roumieu spaces of mean type.

## Quantum Gaudin model and isomonodromic deformations

*Ilya Tolstukhin (HSE University, Moscow)*

The Gaudin model is a quantum integrable system originally introduced to describe the interaction of multiple charged particles on a line. It consists of $n$ commuting Hamiltonian operators, dependent on $n$ pairwise distinct complex parameters and acting on the tensor product of $n$ irreducible representations of the Lie algebra $\mathfrak{sl}_2$. One of the main tasks of the Gaudin model is to diagonalize these operators and understand how their joint spectrum changes as the parameters vary. In joint work with Natalia Amburg, branched coverings of the parameter space with joint spectra of Hamiltonians were studied in the case of $n = 3$. The base of such coverings is the Riemann sphere, and algebraic curves act as total spaces. The remarkable structure of these curves will be described, along with their connection to isomonodromic deformations.

## Continuation of the family of projectors to a positive operator-valued measure and restoration of the quantum state after measurement

*Anton Alekseev (Moscow Institute of Physics and Technology)*

We build POVM, which is absolutely continuous relative to the scalar measure with a projector-valued density, and study the quantum channel generated by it. Also, we consider the possibility of restoring states when measured by the Davis–Lewis instrument. An experiment with two outcomes is considered, and we compare two cases: with post-selection and without post-selection. We study how states are transformed in these cases and whether it is possible to restore them.

## On some properties of classical and quantum random walks

*Stanislav Grishin (Moscow Institute of Physics and Technology)*

This talk is devoted to homogeneous random walks on a line. In the classical case, the generating function for the positive half-line first passage time is discussed. It often turns out to admit an algebraic equation and is useful in statistical computations. Besides, the corresponding algebraic curve sometimes is rational and has a positive genus which can be accounted via analyzing singularities. An open quantum system $\rho \mapsto \sum_i V_i \rho V_i^*$ where $V_i$ are operators of shift on $a_i$ with coefficients $\sqrt{p_i}$ is similar to the classical random walk with increases $a_i$ and their probabilities $p_i$. Some characteristics of it are computed.

## On generating quantum channels

*Renat Gumerov, Ruslan Khazhin (Kazan Federal University)*

For composite quantum systems, we consider quantum channels that uniquely determine the channels of subsystems. Such channels of composite systems are called generating quantum channels. The talk is devoted to properties of those quantum channels. We give a criterion for a quantum channel to be generating. Algebraic and topological properties of sets of generating quantum channels are discussed. For composite systems consisting of two qubits, we construct generating phase-damping channels which generate phase-damping and depolarizing quantum channels of subsystems.

## Entanglement-based approach to quantum coherence, path information, and uncertainty

*Aleksei Kodukhov (Terra Quantum AG, Switzerland),*
*Dmitry Kronberg (Steklov Mathematical Institute, Moscow)*

We propose a framework in which quantum measurement is applied to a subsystem of an entangled state, thus producing a quantum ensemble in the other part. This approach generalizes the seminal multibeam Young's interference experiment which established the wave-particle duality principle, connecting quantum properties (interference) and path information. Our framework provides a connection between such fundamental quantum information concepts as entanglement, coherence, and uncertainty.

## Example of a non-covariant channel constructed using the unitary irreducible representation of a finite group, for which classical capacity is calculated

*Lev Ryskin (Moscow Institute of Physics and Technology)*

Recently, classical capacity of quantum channels generated by projective unitary irreducible representations of finite group was calculated for some strong limitations of the group structure, on the representation $U_g$, on the probability distribution $\pi_g$, on the finite group $G$ and finally that the obtained channel $\Phi_G$ has to be covariant with respect to representation $U_g$.

The channel $\Phi_G$ is not always covariant regarding to representation $U_g$ when the group $G$ is not abelian – it depends on $\pi_g$. But, fortunately, the limitation of covariance of the quantum channel $\Phi_G$ can be omitted if other conditions are preserved. In addition, the real example of such non covariant quantum channel will be given on the group $S_4 \times S_3$.

## Symmetries of $C^*$-algebras

*Ekaterina Turilova (Kazan Federal University)*

There are many faces of $C^*$-algebras whose symmetries encode important aspects of their structures. We show that in surprisingly different situations these symmetries are implemented by Jordan $*$-isomorphisms and lead to full Jordan invariants. In this respect we study the following structures: one dimensional projections in a Hilbert space with transition probability and orthogonality relation (Wigner type theorems); projection lattices of von Neumann algebras and $AW^*$-algebras (Dye type theorems); abelian $C^*$-subalgebras with set theoretic inclusion (Bohrification program in quantum theory).

## A semigroup approach to describing the stochastic dynamics of a quantum system

*Andrey Utkin (Moscow Institute of Physics and Technology and Steklov International Mathematical Center)*

The talk is devoted to one method to derive the equation of stochastic dynamics of a quantum system for a wave function and its applications in quantum information theory. The main example of stochastic dynamics is the process of continuous-time weak measurements of an observable $L = L^*$ described by the stochastic differential Belavkin equation

$$d|\psi_t\rangle = -iH|\psi_t\rangle dt - \frac{1}{2}L^2|\psi_t\rangle dt + L|\psi_t\rangle dW_t.$$

Here $|\psi_t\rangle = |\psi_t(\omega)\rangle \in \mathcal{H}$ is a wave function, as a random vector process on the probability space $\Omega$, $H$ is a Hamiltonian, $W_t$ is a real Wiener process.

In my talk we obtain a generalization of the Belavkin equation by writing the equation not for a random vector $|\psi_t\rangle$, but for its distribution. Thus, we get rid of stochasticity and deal with Markov semigroups and their generators. The conditions of approximation of semigroups of diffusion processes corresponding to a continuous random process in Hilbert space by discrete ones are investigated. Formally, the semigroup operators are considered on the space of bounded weakly continuous functions $C_{BW}(\mathcal{H})$ on the Hilbert space, which carry out the evolution of the characteristic functions $\varphi_t(v) = \mathbb{E}e^{i\langle v|\psi_t\rangle_{\mathbb{R}}}$ of the process. The proof uses the developed powerful theory of continuous and bi-continuous semigroups on a Banach space.

In quantum physics, stochastic processes continuous in time find interesting applications: from models of collisions of particles in a gas to random circuits.

## Characterization of non-adaptive Clifford channels

*Vsevolod Yashin (Steklov Mathematical Institute, Moscow)*

We show that multiqubit quantum channels which may be realised via stabilizer circuits without classical control (Clifford channels) have a particularly simple structure. They can be equivalently defined as channels that preserve mixed stabilizer states, or the channels with stabilizer Choi state. Up to unitary encoding and decoding maps any Clifford channel is a product of stabilizer state preparations, qubit discardings, identity channels and dephasing channels. This simple structure allows to characterise information-theoretic properties of such channels.

## SECTION "THEORETICAL COMPUTER SCIENCE"

### Search of structures computable in polynomial time

*Pavel Alaev (Sobolev Institute RAS)*

We consider some questions related to the problem of searching a structure $A$ computable in polynomial time (in short, $P$-computable), which is isomorphic to a given abstract structure $B$. In particular, a general criterion for the existence of such a $P$-computable structure $B$ is formulated. As an application, we discuss some questions about the existence of $P$-computable presentations for a series of classical objects, including fields and Abelian groups.

### On primitive recursive and automatic structures

*Nikolay Bazhenov (Sobolev Institute RAS)*

In the talk we consider subrecursive algebraic structures. One of the key notions in this direction is the notion of a punctual structure introduced by Kalimullin, Melnikov, and Ng (2017). An infinite algebraic structure $S$ is punctual if the domain of $S$ is equal to the set of natural numbers, and the signature functions and relations of the structure $S$ are primitive recursive. The methods of punctual structure theory found their applications not only in the classical constructive model theory, but also in other areas of mathematical logic and theoretical computer science. In particular, the paper "Automatic and polynomial-time algebraic structures" (2019) proves that the class of automatic structures (in the sense of Khoussainov and Nerode) does not admit a simple syntactic characterization even within the framework of infinitary logic). In the talk, we discuss recent results on punctual structure theory, and its applications to the theory of numberings and equivalence relations. The talk is based on joint works with Harrison–Trainor, Kalimullin, Melnikov, and Ng.

### Lower bounds for regular resolution over parities

*Dmitry Itsykson (Ben-Gurion University)*

The proof system resolution over parities (Res($\oplus$)) operates with disjunctions of linear equations (linear clauses) over $GF(2)$; it extends the resolution proof system by incorporating linear algebra over $GF(2)$. Over the years, several exponential lower bounds on the size of tree-like Res($\oplus$) refutations have been established. However, proving a superpolynomial lower bound on the size of dag-like Res($\oplus$) refutations remains a highly challenging open question. We prove an exponential lower bound for regular Res($\oplus$). Regular Res($\oplus$) is a subsystem of dag-like Res($\oplus$) that naturally extends regular resolution. This is the first known superpolynomial lower bound for a fragment of dag-like Res($\oplus$) which is exponentially stronger than tree-like Res($\oplus$). The talk is based on joint work with Klim Efremenko and Michal Garlik.

## Lower bounds for monotone minimal perfect hashing revisited

*Dmitry Kosolobov (Ural Federal University)*

The monotone minimal perfect hash (MMPH) is a data structure that, for a set of $n$ integers $k1 < \ldots < kn$ from an interval $[1, u]$, supports the following queries: given $x$, the query either returns $i$ such that $ki = x$ if such $i$ exists, or returns any number otherwise. The MMPH is a well-known algorithmic workhorse with many applications, both theoretical and practical. The most space-efficient known MMPHs utilize only $O(n \log\log\log u)$ bits of space. Recently, Assadi, Farach-Colton, and Kuszmaul proved that, surprisingly, this weird space bound is tight in the worst case, for a very broad range of parameters $n$ and $u$ (essentially including all interesting cases). This result, presented in their paper "Tight bounds for monotone minimal perfect hashing" at SODA'23, was obtained using a series of non-trivial combinatorial techniques like fractional chromatic numbers, LP duality, and non-standard graph products. In this talk, a simplified (albeit still non-trivial) approach for the proof is presented using only relatively elementary combinatorial methods.

## Complexity for theories with Kleene star

*Stepan Kuznetsov (Steklov Mathematical Institute RAS, Moscow)*

We give a survey of algorithmic complexity results for theories of algebraic structures including Kleene star. Those include Kleene algebras, Kleene lattices, and their extensions with division operations, also called action algebras/lattices. We focus on equational and Horn theories, since already on this level of expressivity we get high levels of undecidability. The talk includes both well-known old results (following Kozen, 2002 and Buszkowski, 2007), as well as new ones recently obtained by the speaker. The latter include undecidability and complexity of the logic of action algebras, both non-commutative and commutative, and that of the logic of Kleene algebras with commutativity conditions.

## Phase shift and multi-controlled $Z$-type gates

*Andrey Novikov (Sobolev Institute RAS)*

One of the most famous algorithms in quantum computations is the Grover search algorithm. Under certain assumptions, this algorithm provides quantum speedup for the search problem. We always assume that we can build it efficiently, but assume for a moment that you are limited in the gates you are allowed to use for the implementation of the Grover diffusion operator. For example, if you need to use at most two-channel gates. What would be the complexity of the decomposition circuit for the diffusion operator itself? Here we show that it is sufficient to use just only the $Z$-base operators (such $2n\sqrt{Z}$ and controlled $CZ$) and Hadamard operators, but also we show that in this case, the number of used gates can grow exponentially. At least, the number of the multi-controlled $Z$-gates needed to build diffusion operator decomposition circuit only of the following gates: $Z$, controlled $Z$, multi-controlled $Z$-gates, or is summed into $2n - 1$ for the $n$-dimensional phase shift and cannot be decreased if we will not allow other gates.

## Exact real computation for differential equations

*Svetlana Selivanova* (*A.P. Ershov Institute of Informatics Systems RAS*)

In this talk we discuss the exact real computation paradigm, which turns into practice the rigorous but theoretical computable analysis approach (see, e.g., [Weihrauch 2000]). We also highlight some of its recent applications in computing solutions of differential equations with guaranteed prescribed precision. Exact real computation packages allow to conveniently implement imperative algorithms involving real numbers, converging sequences, and smooth functions without using the Turing machines formalism (which computability analysis heavily relies on). This approach differs from traditional reliable numerics in considering real numbers as exact entities (as opposed to intervals) while guaranteeing output approximations up to error $1/2^n$ (as opposed to intermediate precision propagation). Here $n$ is the output error parameter, i.e., the number of computed digits of the output, which can be arbitrary and is given by the user. One of the crucial theoretical problems to address within this approach is estimating the bit-complexity of computation w.r.t. the parameter $n$ (see, e.g., [Ko 2003] on complexity theory for real functions), which reflects the speed of implementations. Thus, the talk also contains a brief discussion on algorithmic complexity classification of differential equations, compared to their other known classifications.

## Recursion elimination: from olympiad problems to program optimization

*Nikolay Shilov* (*Innopolis University*)

The problem of recursion elimination is a broad topic, including purely mathematical as well as practical formulations. Mathematical formulation may be presented as follows: What are decidable sufficient conditions to find a primitive recursive function that is equivalent to a given recursive function. A practical formulation, for instance, may be presented as follows: what are syntactic conditions for functional programs that are sufficient for implementation of recursion via iteration with static memory. In the talk I will provide an overview of known results in this direction and present some new results.

## On the decision problem for quantified probability logic

*Stanislav Speranski* (*Steklov Mathematical Institute RAS, Moscow*))

We shall be concerned with a natural one-sorted language $L$ for talking about probability spaces, where quantifiers are intended to range over events in a given space. In this language one can express statements like 'for every event $E$, if $P(E) > 0$, then there exists an event $F$ such that $0 < P(F) < P(E)$' – here $P$ denotes the measure in question. It is known that the validity problem for arbitrary $L$-sentences is highly undecidable. So one may wish to know more about prefix fragments of $L$. It turns out that the validity problem for $AE$-sentences n $L$ is decidable, while that for $EA$-sentences is not. This solves the decision problem for prefix fragments of $L$. Moreover, similar results can be obtained for some natural one-sorted fragments of Halpern's first-order logics of probability.

# Monomial combinatorics of polynomial ideals and related algorithms

*Nikolay Vasilyev (St. Petersburg Department of Steklov Mathematical Institute)*

Monomial combinatorics describes the geometric and combinatorial properties of supports of polynomials belonging to polynomial ideals. A classic example is the theory of Gröbner bases, which allows us to find sets of polynomials in an ideal with minimal leading terms, with respect to any admissible orderings. Other tasks, the simplest in terms of formulation, but by no means simple algorithmically, is the search for monomials or binomials in ideals. The first task is equivalent to determining the intersection of an ideal with a lattice of monomials. The second problem is closely related to the torical geometry. I will describe various methods and algorithms of monomial combinatorics that allow, for example, to build monomial bases in factoralgebras, as well as alternative methods to Buchberger's algorithm for constructing Gröbner bases of polynomial ideals based on interesting combinatorial structures such as involutive divisions.

We will also discuss some applications of monomial combinatorics to the tropical varieties and give tropical interpretation of the universal Gröbner basis.