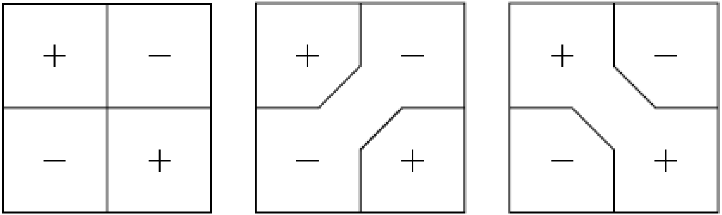
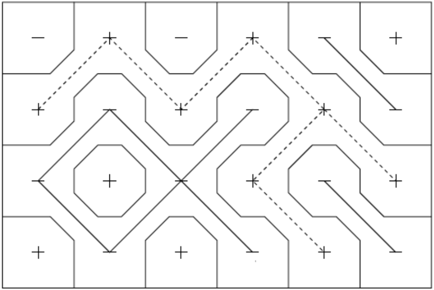
Bogomolny-Schmit Percolation Model



Picture from Bogomolny-Schmit paper.

Bogomolny-Schmit Percolation Model

Using this analogy we can think of the nodal domains as percolation clusters on the square lattice.

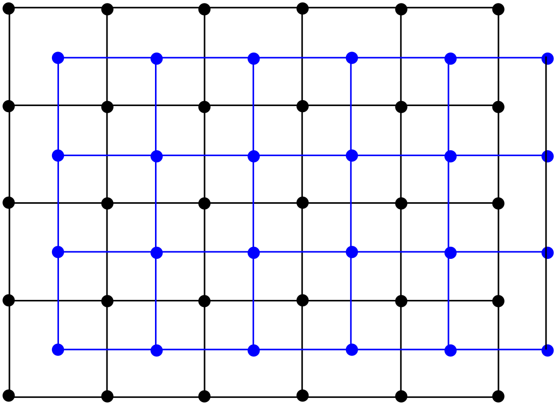


This leads to the conjecture that

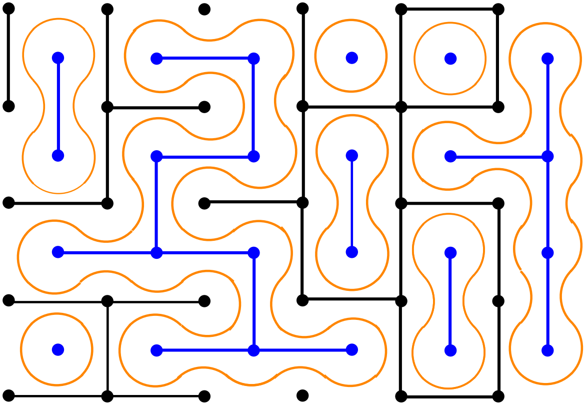
$$\mathbb{E}(N(f, R\Omega)) = R^2 \mathrm{Area}(\Omega) \frac{3\sqrt{3} - 5}{4\pi^2}$$

$$\mathrm{Var}(N(f, \Omega)) \approx \mathrm{const}\, R^2.$$

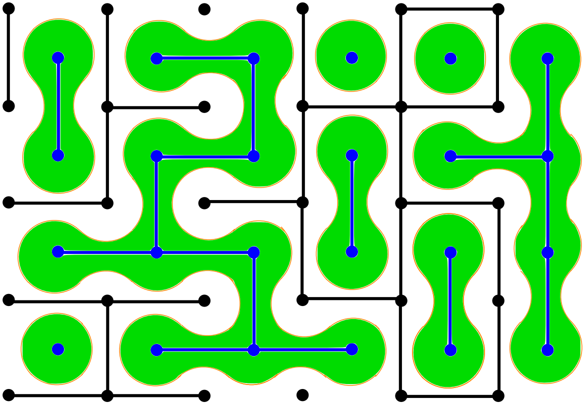
Critical Square Lattice Bond Percolation



Critical Square Lattice Bond Percolation



Critical Square Lattice Bond Percolation



Off-critical Percolation

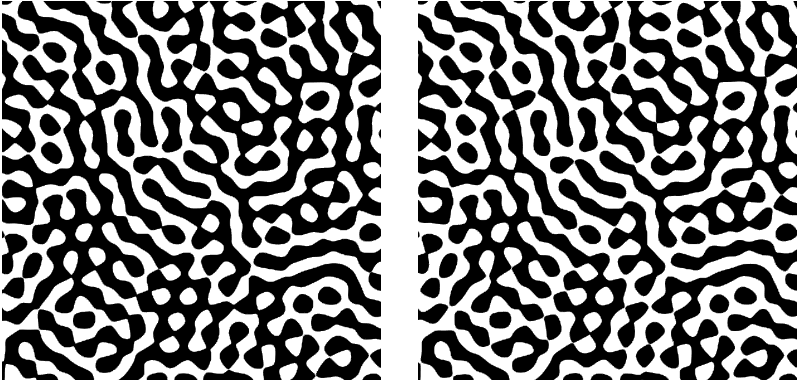


Figure: Excursion sets for levels 0 (nodal domains) and level 0.1

Off-critical Percolation

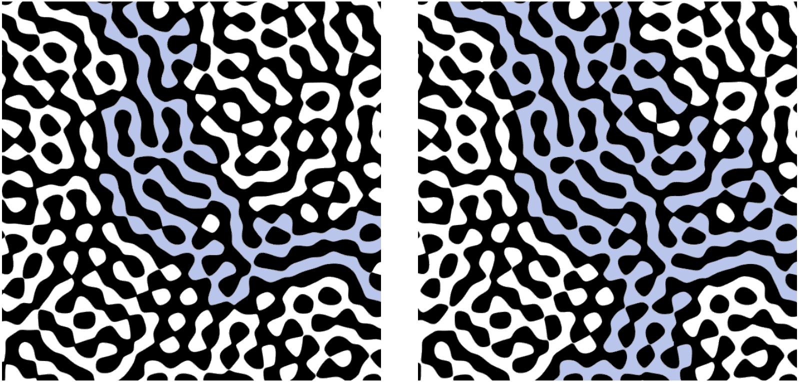
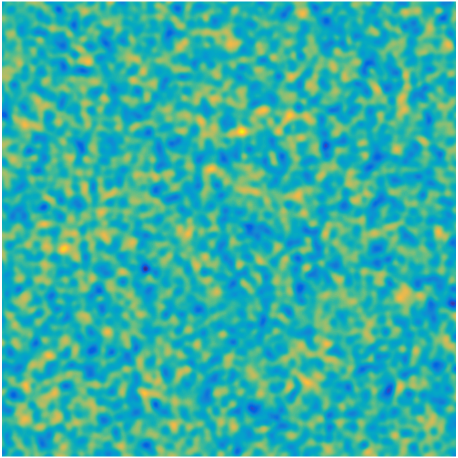


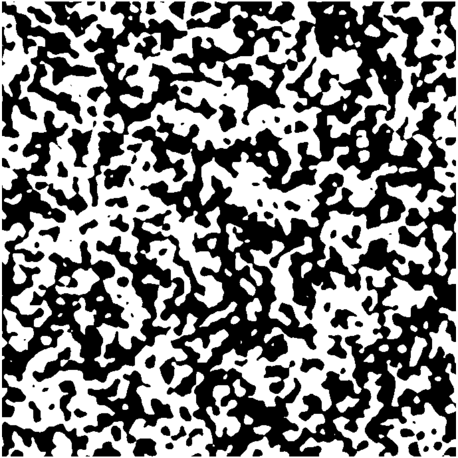
Figure: Excursion sets for levels 0 (nodal domains) and level 0.1

A Good Example: Bargmann-Fock field



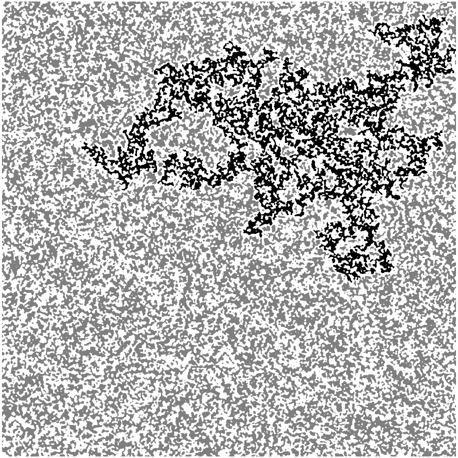
Bargmann-Fock field heat-map

A Good Example: Bargmann-Fock field



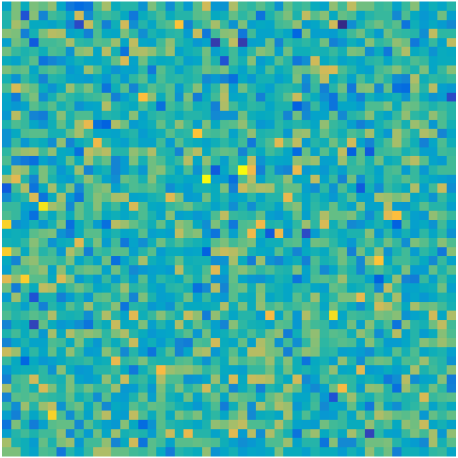
Nodal domains

A Good Example: Bargmann-Fock field



Nodal domains with highlighted largest domain

A Bad Example: discrete white noise



Nodal domains are exactly Bernoulli site percolation clusters with $p = 1/2$ which is not critical.

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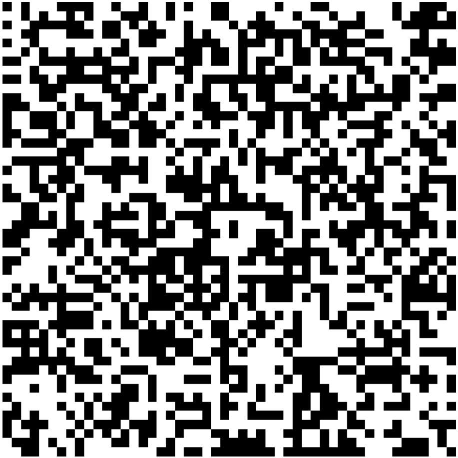

Nodal domains are exactly Bernoulli site percolation clusters with $p = 1/2$ which is not critical.