PLAN OF TALK 2 ON B-THEORY

1. Reminder of HTQM and its 3 examples. Local observables in HTQM

$$d\, I(t,dt) + [Q, I(t,dt)] = 0 \quad \Rightarrow \quad I(t,dt) = e^{-\{Q,G\}t - G\,dt}$$

1. $V = \Omega^{\cdot}(X) \qquad Q = d \qquad G = d^{*} \qquad H = \Delta$

2. $V = \Omega^{\cdot}(X) \qquad Q = d \qquad G = \iota_v \qquad H = \mathcal{L}_v$

3. $V = \tilde{V} \otimes \mathbb{C}^2 \qquad Q = \begin{pmatrix} 0 & \tilde{H} \\ 0 & 0 \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad H = \begin{pmatrix} \tilde{H} & 0 \\ 0 & \tilde{H} \end{pmatrix}$

2. Lagrangian interpretation of examples of HTQM, discussion about Plank constant

1. $\mathcal{L} = p_i \dot{X}^i + \pi_i \dot{\psi}^i + G^{ij} p_i p_j$

2. $\mathcal{L} = p_i(\dot{X}^i - v^i(x))dt + \pi_i\left(\dot{\psi}^i - \frac{\partial v^i}{\partial x^j} \psi^j\right)$

3. $\mathcal{L} = p_i \dot{X}^i + (G^{ij} p_i p_j - m^2) + b\dot{c}$

3. Cohomology of example 3 and equations of motion for free fields

$$Q = \begin{pmatrix} 0 & \tilde{H} \\ 0 & 0 \end{pmatrix} \qquad H_Q \rightsquigarrow \tilde{H} = 0$$

$$\tilde{H} = G^{ij} p_i p_j - m^2 = \Box - m^2$$

$\tilde{H} = 0 \Rightarrow$ solutions of Klein-Gordon equations

$$H_Q = \bigoplus_i \left(\begin{pmatrix} \text{solution}_\alpha \\ 0 \end{pmatrix} \oplus \begin{pmatrix} 0 \\ \text{solution}_\alpha \end{pmatrix} \right) = \bigoplus_\alpha c \otimes \text{solution}_\alpha \oplus 1 \otimes \text{solution}_\alpha$$

4. HTQM and contraction of acyclic complex

$$V = V_{IR} \oplus V_{UV} \qquad h: V_{IR} \to 0$$

h-homotopy $\qquad h: V_{UV} \to V_{UV}\ , \quad h^2 = 0$

$$\{h, Q\} = \mathrm{Proj}_{V_{IR}} \qquad Q = Q \qquad G = h \qquad H = \mathrm{Proj}_{V_{IR}}$$

Let $\{\tilde{h}, Q\} = \tilde{H}$ – positive invertable $\qquad G = \tilde{h} \qquad H = \tilde{H}$

$$h = \frac{\tilde{h}}{\tilde{H}} = \int_0^{+\infty} dt\, \tilde{h}\, e^{-t\tilde{H}}$$

5.Deformations of QM and HTQM, Schwinger times and Witten's descent in dimension 1

$$e^{-t(H+\Delta H)} = e^{-tH} + \int_0^t e^{-(t-\tilde{t})H} \Delta H\, e^{-\tilde{t}H} d\tilde{t} + \dots$$

$$\Delta H = \int_0^t e^{-(t-\tilde{t})H} G\, d\tilde{t}\ \ c\Delta H\, e^{-\tilde{t}H} d\tilde{t} +$$

$$+ \int_0^t e^{-(t-\tilde{t})H}\ c\underset{\Delta Q}{\underset{\uparrow}{\Delta H}}\ G\, d\tilde{t}\ e^{-\tilde{t}H}$$

$$e^{-\{Q+\Delta Q, G\}t + G\,dt} = I_0 + \int_0^t e^{-(t-\tilde{t})H} \{G, \Delta Q\} d\tilde{t}\, e^{-\tilde{t}H} +$$

Witten's descend $[Q, \{G, \Delta Q\}] = \langle \{G,\Delta Q\}(\tilde{t}) \rangle d\tilde{t} +$

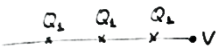
$= d\Delta Q$

6. Deformations of cohomology, HTQM and IR divergences

$$(Q_0 - \epsilon Q_1)(V_0 + \epsilon V_1 + \epsilon^2 V_2 + \dots) = 0$$

$Q_0 V_0 = 0 \qquad Q_0 V_1 = Q_1 V_0$ if $Q_1 V_0$ is zero in cohomology

$$V_1 = h Q_1 V_0 = \int_0^{+\infty} e^{-tH} G Q_1 V_0$$



$$V_K = \int_0^{+\infty} dt_1 \dots \int_0^{+\infty} dt_K \; e^{-t_1 H} G Q_1 e^{-t_2 H} G Q_1 \dots G Q_1 V_0$$

obstructions $\Rightarrow$ IR divergences

This formula works if $\{Q_0, Q_1\} = 0 \qquad Q_1^2 = 0$

Bicomplex

7. Contact terms, obstructions and IR divergences

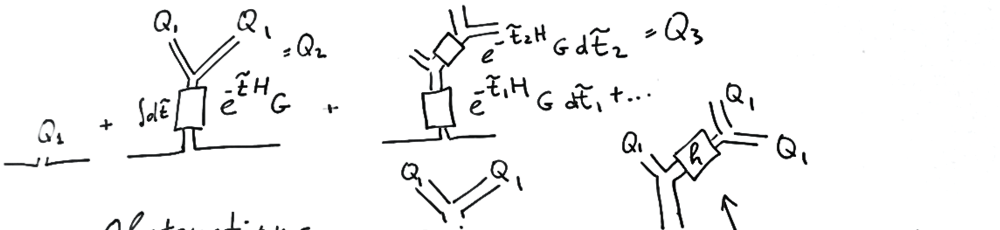
solve $(Q + \epsilon Q_1 + \epsilon^2 Q_2 + \dots)^2 = 0$

$\{Q, Q_1\} = 0 \qquad \{Q, Q_2\} = Q_1^2$

$Q_1^2 \neq 0 \Rightarrow$ UV problem

$Q_2 = [h, Q_1^2], \dots$

Q_2 - contact term



Obstructions $\qquad \mathrm{Proj}_{H_Q} \qquad \mathrm{Proj}_{H_Q}$ Massey operations