Probability Techniques in Analysis and Approximation Theory

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ABSTRACTS

Differentiation-invariant subspaces of Ω-ultradifferentiable functions for which weak spectral synthesis fails

Natalia Abuzyarova (Institute of Mathematics with Computing Centre of RAS, Ufa)

We consider a spectral synthesis problem for differentiation-invariant subspaces in a general space $\mathcal{E}_\Omega(a;b)$ of Ω-ultradifferentiable functions, where $(a;b) \subseteq \mathbb{R}$ and $\Omega = \{\omega_n\}$ is a sequence of nonquasianalytic weights subjected some standard restrictions of Ω-ultradifferentiable functions theory. Do there exist differentiation-invariant subspaces $W \subset \mathcal{E}_\Omega(a;b)$ for which weak spectral synthesis fails? Alexandru Aleman, Anton Baranov and Yurii Belov constructed the first example of differentiation-invariant subspace in $C^\infty(a;b)$ which does not admit weak spectral synthesis (2015). We answer the above question using a dual scheme. Namely, we consider a topological module $P = \mathcal{F}(\mathcal{E}'_\Omega(a;b))$, where $\mathcal{F}$ denotes the Fourier-Laplace transform, and find *unlocalisable* primary submodules $J \subset P$. Then, the differentiation-invariant subspaces in $\mathcal{E}_\Omega(a;b)$ which dual submodules are J do not admit the weak spectral synthesis.

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On embedding theorems for function spaces with mixed logarithmic smoothness

Gabdolla Akishev (Kazakhstan Branch of Lomonosov Moscow State University, Astana)

The talk discusses the Lorentz space $L_{p,\tau}(\mathbb{T}^m)$, 2π periodic functions of many variables and $S^{0,\bar{b}}_{p,\theta}\mathbf{B}$, $S^{0,\bar{b}}_{p,\theta}B$ — spaces with mixed logarithmic smoothness, equivalent norms of spaces with mixed logarithmic smoothness, necessary and sufficient conditions for the embedding of spaces $S^{0,\bar{b}}_{p,\theta}\mathbf{B}$, $S^{0,\bar{b}}_{p,\theta}B$ into each other.

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Stability of min- and max-approximation

Alexey Alimov (Lomonosov Moscow State University)

Approximative compactness type properties in min- and max-approximation are studied. Problems of this kind lead in a natural way to "special points" of approximation theory, viz., the spaces characterized in terms of approximative compactness for various problems of approximation. On this way, there appear CLUR-spaces, Day–Oshman spaces, Anderson–Megginson spaces, and CMLUR- and AT-spaces.

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Residue techniques in the study of Euler–Mellin integrals

Irina Antipova (Siberian Federal University, Krasnoyarsk)

The main objects of the theory of multidimensional residues are integrals of rational n-forms over n-dimensional cycles lying in the complement of a polar hypersurface in affine, projective, and toric spaces. Developing ideas of F. Griffiths, V. Batyrev proved (1993) that all periods are A-hypergeometric functions in the sense of I. Gelfand, M. Kapranov and A. Zelevinsky (1990) if the differential form is considered by varying all parameters. The Batyrev class can be significantly expanded by moving from integer parameters to complex ones, and instead of compact homology in integration, we can consider cycles with closed (unbounded) supports. Such a generalization can be achieved by considering Mellin transforms in the class of branching integrals (Euler–Mellin integrals). In the last decade, particular interest has arisen in the study of such integrals in connection with the study of Feynman integrals in quantum field theory and string amplitudes in superstring theory. In Bayesian statistics, such integrals appear as marginal likelihood integrals.

The convergence of the Euler–Mellin integral is ensured by the property of quasi-ellipticity of the integrand denominator, first introduced by T. Ermolaeva and A. Tsikh (1996). In the talk we are going to discuss representations of the Euler–Mellin integrals associated with facets of the Newton polytope of the denominator, and their treatments in the context of the theory of Feynman integrals.

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On integrability of majorants of Fourier sums

Nikolai Antonov (N.N. Krasovskii Institute of Mathematics and Mechanics, Ekaterinburg)

Let $\varphi\colon\ [0,+\infty) \to [0,+\infty)$ be a nondecreasing function, ω be an arbitrary modulus of continuity. Denote by $\varphi(L)$ the set of all 2π-periodic Lebesgue measurable functions f such that $\varphi(|f|)$ is summable on $[0,2\pi)$, and by H_1^ω the set of all $f \in L$ whose L^1-modulus of continuity $\omega(f,\delta)_1$ satisfies the condition $\omega(f,\delta)_1 = O(\omega(\delta))$.

Suppose that $f \in L(\mathbb{T})$, denote by $S_n(f,x)$ the nth partial sum of the trigonometric Fourier series (nth Fourier sum) of f, and by

$$M(f,x) = \sup_{n\geq 1} |S_n(f,x)|$$

the majorant of the Fourier sums of f. We consider the problems of conditions for the almost everywhere convergence of the Fourier series and the integrability of the majorant of the Fourier sums of f in terms of the belonging of this function to classes $\varphi(L)$ and H_1^ω. We propose to discuss multidimensional analogs of these problems for the case of rectangular partial sums of multiple trigonometric Fourier series.

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Normal random matrices and recurrence relations for multiple orthogonal polynomials

Alexander Aptekarev (Keldysh Institute of Applied Mathematics of RAS)

Our main attention will be devoted to the normal matrices ensembles, which have many interesting applications (Laplacian growth, Diffusion limited aggregation). An important feature of the orthogonal polynomials ensembles of random matrices is that the joint probability density of their eigenvalues is represented by means of the determinants composed by Christoffel–Darboux (CD) kernels of orthogonal polynomials or their generalizations. For the normal matrices ensembles the corresponded CD kernel is taken for polynomials orthogonal with respect to an area measure. We show that for some special cases of the normal random matrices (related with discrete Painlevé equation) these polynomials are the multiple orthogonal polynomials. This fact makes their asymptotical analysis much easier.

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Random unconditional convergence of Rademacher chaos in L_∞ and its applications to graph theory

Sergei Astashkin (Samara National Research University),
Konstantin Lykov (Institute of Mathematics of the National Academy of Sciences of Belarus)

According a recent result due to the authors of this talk, both multiple Rademacher system and Rademacher chaos possess the property of random unconditional convergence in L_∞. This fact combined with some novel connections between L_∞-norms of linear combinations of elements of these systems and some special norms of matrices of their coefficients allows us to establish sharp two-sided estimates for the discrepancy of weighted graphs and hypergraphs. Some of these results extend the classical theorems proved by Erdös and Spencer for the unweighted case.

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On distributed optimization problems under similarity: optimal algorithms and fast extensions

Aleksandr Beznosikov (Moscow Institute of Physics and Technology),
Alexander Gasnikov (Innopolis University and Steklov Mathematical Institute, Moscow)

In this talk, we consider distributed methods for solving optimization problems. In the distributed formulation, the target function is divided into parts, and each of these parts can be accessed only by a local agent/worker. We deal with the case where the local functions are "similar" to each other in some sense. Due to the "similarity" it is possible to achieve a significant acceleration of the theoretical guarantees of convergence of methods in terms of estimates on communication complexity. Besides the issue of convergence of algorithms and obtaining upper bounds, we touch upon lower complexity bounds and verify the optimality of the proposed methods. In the remaining time, we try to discuss the question of how we can "break through" the lower estimates and construct an even faster method, in particular, we additionally introduce the possibility of compressing the transmitted information, modify the proposed algorithms and obtain upper and lower bounds in a new formulation.

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Random sampling discretization of integral norms in finite-dimensional spaces

Feng Dai (University of Alberta, Edmonton, Canada)

In this talk, I will present recent advancements in the Marcinkiewicz discretization problem using random sampling in finite-dimensional spaces. The goal is to establish two-sided estimates for the integral norm of functions in the space via a finite sum of function values evaluated at randomly selected points that are independent of the individual functions in the space. The main challenge is to determine the "nearly" optimal number of random points required for the Marcinkiewicz discretization inequalities to hold with high probability.

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Coefficientwise total positivity of some matrices defined by linear recurrences

Aleksandr Dyachenko (Keldysh Institute of Applied Mathematics of RAS, Moscow)

A matrix of polynomials is called coefficientwise totally positive (CTP) if all its minors are polynomials with positive coefficients. We verify this property for a few families of infinite lower-triangular matrices. During the talk we will, in particular, touch upon CTP triangular matrices stemming from orthogonal and multiple orthogonal polynomials.

It is also intriguing to consider triangles generated by other types of recurrence relations. Almost 30 years ago Brenti conjectured that the Eulerian triangle (the lower-triangular matrix of Eulerian numbers, A008292 in OEIS) is totally positive. The Eulerian numbers appear in polylogarithms of negative integer orders and count the number of permutations of $1, 2, ..., n+1$ with k excedances. We introduce a more general family of matrices that experimentally appear to be CTP. Then we prove that its special subfamily including the reversed Stirling subset triangle (A008278) is indeed CTP. This result is new and required a more delicate approach, than total positivity in the non-reversed case (cf. A008277 in OEIS).

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Approximation by simplest fractions and simplest bianalytic sums

Konstantin Fedorovskiy (Lomonosov Moscow State University and Saint Petersburg State University)

We will discuss the question on approximation by simplest fractions (i.e., sums of Cauchy kernels with unit coefficients) and by simplest bianalytic sums (i.e., sums of fundamental solutions to the Bitsadze equation with unit coefficients). We will start with Chui's conjecture and its version for weighted (Hilbert) Bergman spaces. For a wide class of weights, it will be shown that for every N, the simplest fractions with N poles on the unit circle have minimal norm if and only if the poles are equidistributed on the circle. Next, we describe the closure of the simplest fractions in weighted Bergman spaces under consideration. These results were obtained at 2021 in the joint work by the speaker with E. Abakumov (Univ. Gustave Eiffel, Paris, France) and A. Borichev (Aix–Marseille University, France). Finally, we discuss the problem on approximation of functions by simplest bianalytic fractions,and several new effects and phenomena that appeared in this connection. This part is based on the joint work in progress by the speaker with P. Borodin (Lomonosov Moscow State University).

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The method for solving the Delsarte problem for designs on homogeneous spaces

Dmitry Gorbachev (Tula State University)

We study the problem of finding lower bounds for the cardinality of weighted designs on compact rank-1 spaces. To solve this problem, P. Delsarte, J. Goethals, and J. Seidel introduced what is known as the linear programming bound, based on a two-point distribution of the design. This bound is based on solving an extremal problem known as the Delsarte problem for Jacobi–Fourier series. Earlier, V.V. Arestov, A.G. Babenko, and their students proposed a solution scheme for a similar problem in the case of spherical codes, based on the primal-dual problem. We adapt this scheme to the case of designs. The scheme is based on convex analysis and consists of several steps, including: formulating the dual problem for the Stieltjes measure, proving the existence of an extremal function and measure, deriving duality relations, characterizing extremal functions and measures based on these relations, reducing the problem to a polynomial system of equations in specific cases, proving the existence of an analytical solution to the system through its certification or by using a special Gröbner basis, and applying the uniform Stieltjes–Bernstein estimate. The described method has been used to solve several new Delsarte problems. These results are useful in the problem of integral norm discretization when estimating the number of nodes in discrete norms.

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Hyperbolic Fourier series and the Klein–Gordon equation

Haakan Hedenmalm (Royal Institute of Technology (KTH), Stockholm, and Saint Petersburg State University)

The Klein–Gordon equation in 1+1 dimensions is one of the truly basic second order PDEs with constant coefficients. It models the time evolution of a one-dimensional relativistic boson with spin 0. Since it is relativisitic, the temporal relation between points is felt, and a given pair of points is either time-like or space-like. If the pair of points is space-like, we cannot say that one or the other event happens before or after the other. If we study a space-like cone, and place equidistributed points on the edges, do we get a uniqueness set for Klein–Gordon solutions? The answer turns out to depend on the density of points, and the shape of the solution.

As a consequence, we are led to study hyperbolic Fourier series, a topic which is natural but is a recent discovery only. The first installment is a paper with A. Montes-Rodriguez (Annals of Mathematics, 2011). The second only exists as a 2024 preprint, but it builds on insights in the work of Radchenko and Viazovska (2019).

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Maximal operators on spaces BMO and BLO

Grigor Karagulyan (Yerevan State University and Institute of Mathematics of National Academy of Sciences of RA)

We consider maximal kernel-operators on measure spaces (X, μ) equipped with a ball-basis. We prove that under certain asymptotic condition on the kernels those operators maps boundedly BMO(X) into BLO(X), generalizing the well-known results of Bennett–DeVore–Sharpley and Bennett for the Hardy–Littlewood maximal function. As a particular case of such an operator one can consider the maximal function

$$\mathcal{M}_\varphi f(x) = \sup_{r>0} \frac{1}{r^d} \int_{\mathbb{R}^d} |f(t)| \varphi\left(\frac{x-t}{r}\right) dt, \tag{1}$$

and its non-tangential version, where $\varphi(x) \geq 0$ is a bounded, integrable spherical function on $\mathbb{R}^d$, decreasing with respect to $|x|$. We prove that $\mathcal{M}_\varphi$ is bounded from BMO$(\mathbb{R}^d)$ to BLO$(\mathbb{R}^d)$ if and only if

$$\int_{\mathbb{R}^d} \varphi(x) \log(2 + |x|) dx < \infty.$$

Our main result is an inequality, providing an estimation of certain local oscillation of the maximal function $\mathcal{M}(f)$ by a local sharp function of f.

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Uniqueness theorems for holomorphic functions in the unit disk and completeness of various systems of functions

Bulat Khabibullin (Institute of Mathematics with Computing Centre of RAS, Ufa)

We present new uniqueness theorems for holomorphic functions in the unit disk with given subharmonic majorants for the logarithms of the modules of these holomorphic functions. The results are formulated in terms of zero distributions of these holomorphic functions and Riesz mass distributions of these subharmonic majorants. They take into account the distributions of zeros and masses both by radius and by argument. We also present applications of these uniqueness theorems to questions of completeness of various systems of holomorphic functions in weight spaces of holomorphic functions. The research was supported by a grant from the Russian Science Foundation No. 24-21-00002.

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On reverse Markov–Nikol'skii inequalities for polynomials with zeros on a segment

Mikhail Komarov (Vladimir State University)

Let Π_n be the class of algebraic polynomials P of degree n, all of whose zeros lie on the segment $[-1,1]$. In 1995, S. P. Zhou has proved the following Turán type reverse Markov–Nikol'skii inequality: $\|P'\|_{L_p[-1,1]} > c\,(\sqrt{n})^{1-1/p+1/q}\,\|P\|_{L_q[-1,1]}$, $P \in \Pi_n$, where $0 < p \le q \le \infty$, $1 - 1/p + 1/q \ge 0$ ($c > 0$ is a constant independent of P and n). We show that Zhou's estimate remains true in the case $p = \infty$, $q > 1$. Some of related Turán type inequalities are also discussed.

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Constructive recovery of values of an algebraic function via Hermite–Padé polynomials

Aleksandr Komlov (Steklov Mathematical Institute of RAS, Moscow)

Let f be an algebraic function of degree $m+1$ and f_∞ be its holomorphic germ at the point ∞. Hermite–Padé polynomials of type I for the tuple $[1, f_\infty, f_\infty^2, \dots, f_\infty^m]$ of order n at ∞ are $m+1$ polynomials $Q_{n,j}$, $j = 0, \dots, m$, such that $\deg Q_{n,j} \le n$ and

$$Q_{n,0}(z) + Q_{n,1}(z) f_\infty(z) + Q_{n,2}(z) f_\infty^2(z) + \dots + Q_{n,m}(z) f_\infty^m(z) = O(z^{-m(n+1)})$$

as $z \to \infty$.

In 1984 J. Nuttall (not in general case and not with full proofs) and in 2017 E. Chirka, R. Palvelev, S. Suetin and A. Komlov (in general case and with full proofs) showed that $Q_{n,m-1}/Q_{n,m}$ asymptotically recover the sum of the values of f on first m sheets of Nuttall partition of the Riemann surface of f. So, this ratio recovers sum of m values of $(m+1)$-valued function f.

In 2021 the polynomial Hermite–Padé m-system was introduced. With the help of this system we show that for generic function f the ratio of some minors of size $m+1-k$ of the $(m+1)\times(m+1)$ matrix consisting of Hermite–Padé polynomials of order $n, n-1, \dots, n-m$ asymptotically recover the sum of the values of f on first k sheets of Nuttall partition of the Riemann surface of f for each $k = 1, \dots, m$. Hence we constructivelly recover m values of $(m+1)$-valued algebraic function f.

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Density of homogeneous polynomials

András Kroó (Alfréd Rényi Institute of Mathematics, Budapest)

In this talk we will consider the following central problem on the uniform approximation by **homogeneous polynomials**:

For which 0-symmetric star like domains $K \subset \mathbb{R}^d$ and which $f \in C(\partial K)$ there exist homogeneous polynomials h_n, h_{n+1} of degree n and $n+1$, respectively, so that uniformly on ∂K

$$f = \lim_{n\to\infty}(h_n + h_{n+1})?$$

This is the analogue of the classical Weierstrass approximation problem with polynomials of total degree being replaced by homogeneous polynomials. The answer to the above problem has an intrinsic connection to the geometry of the underlying domain. We will give a survey of various results related to the above question and will also list some important open problems.

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Holomorphic self-maps of a disc with fixed points

Olga Kudryavtseva (Lomonosov Moscow State University),
Aleksei Solodov (Lomonosov Moscow State University)

Fixed points of a holomorphic self-map of the unit disc have a decisive influence on its geometric and analytic properties. In the talk, we give an overview of known and new results on classes of such functions. The presentation focuses on approaches to solving extremal problems on classes of functions with several fixed points.

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Fekete lemma in Banach spaces

Aleksei Kulikov (University of Copenhagen, Denmark)

The classical Fekete lemma says that if the sequence of real numbers a_n satisfies the inequality $a_{n+m} \leq a_n + a_m$ for all $n, m \in \mathbb{N}$ then the limit $\lim_{n\to\infty} \frac{a_n}{n}$ exists. In this talk we will discuss what happens when a_n are the elements of some Banach space. The main result that we will discuss is the following theorem.

Theorem. *Let X be a uniformly convex Banach space and let a_n be a sequence of vectors in X such that $||a_{n+m}|| \leq ||a_n + a_m||$ for all $n, m \in \mathbb{N}$. Then the limit $\lim_{n\to\infty} \frac{a_n}{n}$ exists.*

Interestingly, the condition of uniform convexity is essential – if X is not convex (that is, if the unit sphere of X contains an interval) then it is not hard to see that the Fekete lemma fails, but even for convex, but not uniformly convex spaces there might be a counterexample.

The talk is based on a joint work with Feng Shao.

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Singularity of random Bernoulli matrices

Alexander Litvak (University of Alberta, Edmonton, Canada)

We discuss recent progress on singularity of random matrices with i.i.d. 0/1 Bernoulli entries. This talk is partially based on a joint work with K. Tikhomirov.

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Automorphisms of Hopf manifolds of dimension $n \geq 2$

Elijah Lopatin (Steklov Mathematical Institute of RAS, Moscow)

Describing the group of automorphisms $\mathrm{Aut}(X)$ of a compact complex manifold X is among the classical issues of complex geometry. According to the Bochner–Montgomery [1] theorem, such groups are complex Lie groups and it is almost everything we can a priori say about them: for a majority of X, it is extremely complicated (or nearly impossible) to find the generating set of $\mathrm{Aut}(X)$ or some other explicit characterisation.

Therefore it is natural to investigate classification properties, i.e. such properties that the group $\mathrm{Aut}(X)$ possesses one when X is a complex manifold, and does not for other X. It seems the Jordan property [8] to be the most promising.

Let G be a group. We say that G is *Jordan* (or has the *Jordan property*) if there is a constant $J = J(G) \in \mathbb{N}$ such that for any finite subgroup $H \subset G$ there is a normal abelian subgroup $A \trianglelefteq H$ of index at most $J(G)$.

It is known that automorphism groups of complex projective varieties [5] and, more generally, compact Kähler manifolds [7] are Jordan. For non-Kähler compact complex manifolds there are only a few known results on the Jordan property for automorphism groups: for compact complex manifolds in Fujiki's class $\mathcal{C}$ [6], for compact complex surfaces [9] and for some examples [3,4] of non-Kähler holomorphically symplectic manifolds [2].

Hopf manifold H_n, i. e. a compact complex manifold of dimension $n \geq 2$ such that its universal cover is isomorphic to $\mathbb{C}^n \setminus 0$, is a natural example of non-Kähler complex manifold for studying structural properties of its automorphism group. H_n is realized as a quotient of $\mathbb{C}^n \setminus 0$ by a free action of a group isomorphic to $\mathbb{Z}$, which acts on $\mathbb{C}^n \setminus 0$ via biholomorphic contractions $\mathbb{C}^n \setminus 0 \to \mathbb{C}^n \setminus 0$. Recently it was shown [10] that $\mathrm{Aut}(\mathsf{H}_n)$ is Jordan. We expand on the results of [10] proving that the group $\mathrm{Aut}(\mathsf{H}_n)/\mathrm{Aut}^0(\mathsf{H}_n)$ is finite; here $\mathrm{Aut}^0(\mathsf{H}_n)$ is the connected component of unity in $\mathrm{Aut}(\mathsf{H}_n)$. We also provide the explicit structure of mentioned biholomorphic contractions.

This is a joint research with Constantin Shramov, Steklov Mathematical Institute of RAS, Moscow.

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Generalizations of Bernstein and Videnskii operators

Alexey Lukashov (Moscow Institute of Physics and Technology)

Bernstein operators are associated with Bernoulli scheme as follows:

$$B_n(f,x) = \mathbb{E}\left(f \circ Z(n,x)\right),$$

where $Z(n,x) = \frac{1}{n}\sum_{i=1}^{n} Y(i,x)$, $Y(i,x)$ is the sequence of independent Bernoulli random variables with parameters $P\{Y(i,x)=1\}=x$ and $P\{Y(i,x)=0\}=1-x$. V.S. Videnskii in a series of papers studied generalizations of Bernstein operators to the case of rational functions. They can be written in the form

$$V_n(f,x) = \mathbb{E}\left(f \circ (\mathbb{E}Z(n,x))^{-1} \circ Z(n,x)\right),$$

where $Y(i,n)$ has now parameters $p_{in}(x) = \frac{\rho_{in}x}{1+\rho_{in}-x}$, $\rho_{i,n} > 0$, instead of x.

We give a survey of results to compare approximation properties of Bernstein and Videnskii type generalizations for one or several intervals, and for the semi-axis.

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Extremal measures and asymptotics of orthogonal polynomials of a discrete variable

Vladimir Lysov (Keldysh Institute of Applied Mathematics of RAS, Moscow)

In the series of works of the 1980s, A.A. Gonchar and E.A. Rakhmanov developed a method for studying the asymptotic behavior of polynomials orthogonal with respect to varying (i.e., depending on the degree of the polynomial) weight. Orthogonality was considered both on real intervals and on curves with special symmetry (S-property). We extend these results in two directions. First, we consider multiple orthogonality, and second, discrete orthogonality measures. During the talk we formulate some general results and provide specific examples of their use.

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To Birman–Krein–Vishik theory of nonnegative symmetric operators with compact preresolvent

Mark Malamud (Saint Petersburg State University)

Let $A \geq 0$ be a closed densely defined non-negative symmetric operator in a Hilbert space $\mathfrak{H}$, let $\mathfrak{H}_1 := \operatorname{ran}(A + I)$, and let P_1 be the orthoprojection in $\mathfrak{H}$ onto $\mathfrak{H}_1$. Let also A_F and A_K be, respectively, the maximal (Friedrichs') and minimal (Krein's) non-negative selfadjoint extensions of A.

Next, assuming A to be positive definite, $A \geq m_A > 0$, Krein characterized the extension A_K as follows: $\operatorname{dom} A_K = \operatorname{dom} A \dotplus \mathfrak{N}_0$ where $\mathfrak{N}_0 := \ker A^*$. Therefore Krein's extension A_K admits the following representation: $A_K = A'_K \oplus (\mathbb{O} \upharpoonright \mathfrak{N}_0)$ where $A'_K := A_K \upharpoonright \mathfrak{M}_0$, and $\mathfrak{M}_0 := \mathfrak{N}_0^{\perp} = \operatorname{ran} A$.

The operator A'_K is called the reduced Krein extension.

We will discuss relations between certain spectral properties of the operators A_F, A'_K, and A **assuming the operator** $(A + I)^{-1} : \mathfrak{H}_1 \to \mathfrak{H}$ **to be compact.**

First we discuss the validity of the following equivalence:

$$P_1(I_{\mathfrak{H}} + A)^{-1} \in \mathfrak{S}(\mathfrak{H}_1) \quad \Longleftrightarrow \quad (I_{\mathfrak{M}_0} + A'_K)^{-1} \in \mathfrak{S}(\mathfrak{M}_0), \tag{2}$$

which improves and complements the known Krein's result. Here $\mathfrak{S}$ is arbitrary symmetrically normed ideal $\mathfrak{S}$ including Neumann-Schatten ideals $\mathfrak{S} = \mathfrak{S}_p$, $p \in (0, \infty]$, as well as ideals Σ_p (a compact operator T is put in the class $\Sigma_p(\mathfrak{H})$, if $s_n(T)) = O(n^{-1/p})$, $p \in (0, \infty)$).

Secondary we will discuss the improvement of equivalence (2) for $\mathfrak{S} = \Sigma_p$. It happens that the inclusion $P_1(I_{\mathfrak{H}} + A)^{-1} \in \Sigma_p(\mathfrak{H}_1)$ for some $p \in (0, \infty)$, does not ensure coincidence of the eigenvalues asymptotics of operators in (2), i.e. the following equivalence with some $a \geq 0$:

$$\lambda_n\big(P_1(I_{\mathfrak{H}} + A)^{-1}\big) = an^{-1/p}\,(1 + o(1)) \quad \Longleftrightarrow \quad \lambda_n\big((I_{\mathfrak{M}_0} + \widehat{A}'_K)^{-1}\big) = an^{-1/p}\,(1 + o(1))\,. \tag{3}$$

In fact, it will be explained that the validity of (3) as $n \to \infty$ depends on A_F.

We will also discuss the abstract Alonso-Simon problem [1] on the eigenvalues asymptotics of A_F and A'_K, and the explicit solution to the Birman problem.

Besides, we discuss improvement of Birman's and Grubb's results (see [2], [3]) regarding equivalence of semiboundedness properties of an extension $\widetilde{A} = \widetilde{A}^*$ of A and the corresponding boundary operator.

A part of results of the talk are announced in [4] and published in [5].

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Average Kolmogorov width and its applications

Yuri Malykhin (Steklov Mathematical Institute of RAS, Moscow)

The classical notion of Kolmogorov width of a set in a normed space measures the error of approximation of this set by n-dimensional linear subspaces. Here we consider the "worst-case" error of approximation.

If we take the "average-case" error instead, we arrive to the notion of average Kolmogorov width. We will discuss some new bounds for the average widths and some applications for the classical Kolmogorov widths.

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Inequalities for quasinorms of rational functions in a domain and on its boundary

Tatsiana Mardvilko (Belarusian State University, Minsk)

Previously (in 2011), the author together with A.A. Pekarski obtained an inequality connecting the quasinorms of rational functions with respect to the linear measure on $\mathbb{R}$ and the planar measure in the half-plane $\Pi = \{z \in \mathbb{C} : \Im z > 0\}$. In this context, the rational functions belonged to the weighted Lebesgue space in Π, where the quasinorm is defined as follows

$$\|f\|_{L_{p,\mu}(\Pi)} = \left(\int_{\Pi} (\Im z)^{p\mu-1} |f(z)|^p \, dm_2(z)\right)^{1/p}, \quad p > 0, \quad \mu > 0.$$

Here m_2 is the planar Lebesgue measure in $\mathbb{C}$.

The report will discuss some applications of the noted inequality. Furthermore, a generalization of this inequality for a domain whose boundary is a Lavrent'ev curve will be presented.

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Analytic extension of simple and multiple power series by means of coefficients interpolation

Aleksandr Mkrtchyan (Siberian Federal University, Krasnoyarsk, and Institute of Mathematics NAS RA)

One of the methods of studying the problem of analytical continuation of power series is interpolation of the coefficients of the series. With this approach Le Roy and Lindelöf obtained conditions under which a series analytically extends into a sector. Note that the theorem, gave a connection between the sector and the growth of the interpolation function. More precisely, the type of interpolation function must be less than π on the closed half-plane $Rez \geq 0$.

We weakened the condition of less than π on the fact that the sum of the indicator (growth) on directions $\frac{\pi}{2}$ and $-\frac{\pi}{2}$ is less than 2π. Also we obtain the multivariate version of this theorem, i.e. establish a connection between the growth of the interpolating function of the coefficients on the imaginary subspace and the multivariate sectoral domain where the multiple series is analytically extends.

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On the wavelet transform of periodic ultradifferentiable functions

Ildar Musin (Institute of Mathematics with Computing Centre of RAS, Ufa)

The main part of the talk is devoted to wavelet transform on the space of periodic ultradifferentiable functions of Roumieu type. It is based on recent results on Gelfand–Shilov spaces and description of periodic ultradifferentiable functions of Roumieu type in terms of decay of their Fourier coefficients.

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Integral potential type operator for infinitely differentiable and real analytic functions

Simona Myslivets (Siberian Federal University, Krasnoyarsk)

We prove the infinite differentiability of an integral operator of the potential type for an infinitely differentiable function defined on the boundary of the domain in $\mathbb{C}^n$ with the boundary of the class $\mathcal{C}^\infty$, up to the boundary of the domain on both sides.

We also prove the real analyticity of the Bochner–Martinelli integral for a real analytic function given at the boundary of the domain.

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TBA

Natalia Obukhova (Saint Petersburg Electrotechnical University «LETI»)

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Ivan Oseledets (Skolkovo Institute of Science and Technology, Moscow)

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Mauricio Romo (Shanghai Institute for Mathematics and Interdisciplinary Sciences (SIMIS) and Fudan University)

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Approximation of locally-constant functions by algebraic polynomials and some applications

Konstantin Ryutin (Lomonosov Moscow State University and Moscow Center of Fundamental and Applied Mathematics),
Yuri Malykhin (Steklov Mathematical Institute of RAS, Moscow)

We plan to talk about explicit explicit easily implementable polynomial approximations of sufficiently high accuracy for locally constant functions on the union of disjoint segments and some more general disjoint sets.

This problem has important applications in several areas of numerical analysis, complexity theory, quantum algorithms, etc. The one, most relevant for us, is the amplification of approximation method: it allows to construct approximations of higher degree M and better accuracy from the approximations of degree m. Such constructions are used in linear algebra, computer science (communication complexity).

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K-theory of graded C^*-algebras in the tight-binding model of solid state theory

Armen Sergeev (Steklov Mathematical Institute, Moscow)

After the discovery of quantum Hall effect and its topological explanation the mathematical methods based on the theory of C^*-algebras and their K-theory enter firmly into the arsenal of solid state physics.

A key role in the theory of solid states is played by their symmetry groups. It was Kitaev who has pointed out the relation between the symmetries of solid bodies and Clifford algebras.

In this talk we pay main attention to the class of solid bodies called the topological insulators. They are characterized by having a broad energy gap stable under small deformations. The algebras of observables of such solid bodies belong to the class of graded C^*-algebras for which there is a variant of K-theory proposed by Van Daele. It makes possible to define the topological invariants of insulators in K-theory terms.

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Factorization representation and properties of zero sets of some weighted classes of analytic functions in the unit disk

Faizo Shamoyan (Saratov State University)

In the talk we consider the problem the factorization of certain classes of analytic functions in the unit disk, for which the logarithm of modulus belongs to the weighted L^p classes.

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Generalization of the Rossovskii problem on the limit of a special product of sines

Vladimir Sherstyukov (Lomonosov Moscow State University)

Some time ago, in the theory of functional differential equations with affine transformations, the problem of calculating the spectral radius for a certain parametric family of operators arose. The question comes down to finding the limit of the special product of sines with arguments generated by a given geometric progression. The report plans to discuss a more general problem in which an arbitrary infinitely large sequence is taken as the generating sequence.

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Discrete models for differentially constrained spaces

Dmitriy Stolyarov (Saint Petersburg State University and St. Petersburg Department of Steklov Institute of Mathematics)

I will speak about a mysterious correspondence between inequalities for special discrete time martingales and solutions to certain PDEs. Usually these inequalities involve the L_1 norms of functions and martingales and, thus, have applications to geometric measure theory. The said correspondence also works for questions in geometric measure theory (quantifying singularities of martingales and singularities of PDE solutions). It seems that there are much more questions than answers here.

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Multishift in a Hilbert space

Pavel Terekhin (Saratov State University)

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Low-rank approximation analysis

Eugene Tyrtyshnikov (Marchuk Institute of Numerical Mathematics of RAS, Moscow)

Tensor decompositions become a very popular tool for modelling data in many application problems. However, a better understanding of why they are so efficient is still a hot issue with a machinery based on some relevant probability models for data. We discuss some open questions and new developments of cross-approximation approach to optimization problems with the tensor-train model.

References

1. *E. Tyrtyshnikov*, Tensor decompositions and rank increment conjecture, Russian Journal of Numerical Analysis and Mathematical Modelling, 25 (4), 239–246 (2020).

2. *D. Zheltkov, E. Tyrtyshnikov*, Global optimization based on TT-decomposition, Russian Journal of Numerical Analysis and Mathematical Modelling, 25 (4), 247–261 (2020).

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On the power of adaption, randomization and non-linear measurements

Mario Ullrich (Johannes Kepler University, Linz, Austria)

We show that the maximal gain of adaption and randomization is limited when considering approximation of functions from convex sets based on arbitrary linear measurements in a worst-case setting. We also discuss the situation when arbitrary non-linear, Lipschitz-continuous measurements are allowed, where some (surprising) improvements hold.

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TBA

Tatyana Vavilova (Almazov National Medical Research Centre)

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Infinite-dimensional conic Steiner formula

Dmitry Zaporozhets (St. Petersburg Department of Steklov Institute of Mathematics)

The classical Steiner formula expresses the volume of the neighborhood of a convex compact set in $\mathbb{R}^d$ as a polynomial in the radius of the neighborhood. In the work of Tsirelson (1985), this result was extended to the infinite-dimensional case. A spherical analogue of the Steiner formula for convex subsets of $\mathbb{S}^{d-1}$ is also well-known. Using Tsirelson's idea of applying the theory of Gaussian processes, we obtain an infinite-dimensional version of this spherical analogue.

The talk is based on a joint work with Maria Dospolova.