

9-th St. Petersburg Youth Conference in Probability and Mathematical Physics

November 17–20, 2025

ABSTRACTS

Multi-point penalization for symmetric Lévy processes

Temirlan Abildaev

PDMI RAS, Saint Petersburg,
Saint Petersburg State University, Saint Petersburg
t.abildaev23@gmail.com

We consider a one-dimensional symmetric Lévy process $\xi(t)$, $t \geq 0$, that has local time, which we denote by $L(t, x)$, and construct the operator $\mathcal{A} + \sum_{k=1}^n \mu_k \delta(x - a_k)$, $\mu_k > 0$, where $\mathcal{A}$ is the generator of $\xi(t)$, and $\delta(x - a)$ is the Dirac delta function at $a \in \mathbb{R}$. We show that the constructed operator is the generator of $\{U_t\}_{t \geq 0}$ - C_0 -semigroup on $L_2(\mathbb{R})$, which is given by

$$(U_t f)(x) = \mathbf{E} f(x - \xi(t)) e^{\sum_{k=1}^n \mu_k L(t, x - a_k)}, \quad f \in L_2(\mathbb{R}) \cap C_b(\mathbb{R}),$$

and prove the Feynman-Kac formula for the potentials of form of linear combinations of delta functions with positive coefficients.

Furthermore, we construct a family of penalized distributions $\{\mathbf{Q}_{T,x}^\mu\}_{T \geq 0}$ of form

$$\mathbf{Q}_{T,x}^\mu = \frac{e^{\sum_{k=1}^n \mu_k L(T, x - a_k)}}{\mathbf{E} e^{\sum_{k=1}^n \mu_k L(T, x - a_k)}} \mathbf{P}_{T,x},$$

where $\mathbf{P}_{T,x}$ is the measure of the process $\xi(t)$, $t \leq T$. We show that this family weakly converges to a Feller process as $T \rightarrow \infty$, study the Feynman-Kac semigroup generated by this Feller process and prove a limit theorem for the distribution of $\xi(T)$ under $\mathbf{Q}_{T,x}^\mu$.

Schrödinger operator with self-similar properties

Nikolai Andronov

Saint Petersburg State University, Saint Petersburg
nickandronick@gmail.com

We study the one-dimensional Schrödinger equation with a potential given by an infinite sum of delta functions located at points whose positions are described by a quadratic polynomial in the index. The problem is reduced to analyzing a family of difference equations whose parameters depend on the spectral parameter of the original equation. For this family, we establish a renormalization formula that relates solutions with large indices to those with indices of order one. This formula provides a promising tool for further investigation of the spectral properties of the original differential operator.

Non-Extinction Probability of Critical Branching Process in a Random Environment Conditioned on the Value of Minimum of the Associated Random

Walk

Mariia Anokhina

Lomonosov Moscow State University, Moscow

anokhina.mary1@gmail.com

Let $\eta = (\eta_1, \dots, \eta_n, \dots)$ be a sequence of independent identically distributed nondegenerate random variables and $\{f_y(s), y \in \mathbb{R}\}$ — a family of probability generating functions. Let $S_n = X_1 + \dots + X_n$, $n \in \mathbb{N}$, $S_0 = 0$, where $X_i = \ln f'_{\eta_i}(1)$. By a branching process in a random environment (BPRE), we mean the random sequence $Z_0 = 1$, $Z_{n+1} = X_{n+1,1} + \dots + X_{n+1,Z_n}$. In this paper we consider arithmetic random variables X_i with $\mathbf{E}X_1 = 0$, $\mathbf{E}X_1^2 = \sigma^2 \in (0, \infty)$. In this talk we obtain

$$e^{k_n} \mathbf{P} \left(Z_n > 0 \middle| \min_{i \leq n} S_i = -k_n \right) \rightarrow C,$$

where $\{k_n\}$ is an integer-valued sequence, such that $k_n/\sqrt{n} \rightarrow y > 0$, $k_n \rightarrow \infty$, $n \rightarrow \infty$, C is some constant.

Non-extinction of a pair of branching processes in a common random environment

Dmitrii Arapov

Lomonosov Moscow State University, Moscow

dmitrii.arapov@math.msu.ru

We introduce a model of a pair of branching processes $\{\mathbf{Z}_n = (Z_{n,1}, Z_{n,2}), n \in \mathbb{N}_0\}$ in a common random environment (PBPRES) in our report. We assume that for the fixed environment the sequences $\{Z_{n,1}, n \in \mathbb{N}_0\}$ and $\{Z_{n,2}, n \in \mathbb{N}_0\}$ are independent branching process in a varying environment.

This model has a natural biological interpretation. Consider two populations isolated from each other. For instance, we can consider machaon butterflies and Hercules moths. The habitats of these populations do not overlap. At the first sight it seems like reproductions of them are independent. On the other hand, machaons and Hercules moths live on the same planet. Therefore, planet-scale processes (for instance, global temperature changing) affect both populations.

PBPRES is a particular case of multitype branching process in a random environment, MBPRE. However, in MBPRE the particles of one type produce the particles of other types. In our case it's forbidden. This simplification orders to study the process under mild assumptions.

We consider the critical branching process $\{\mathbf{Z}_n\}$. It means that both processes $\{Z_{n,1}, n \in \mathbb{N}_0\}$ and $\{Z_{n,2}, n \in \mathbb{N}_0\}$ are critical. We say that the extinction of $\{\mathbf{Z}_n\}$ is the extinction of one of populations. We show that the non-extinction probability of a pair of branching processes in a common random environment to the moment n is equivalent to the probability of the event that the associated random walk stays “positive” as $n \rightarrow \infty$.

Note that D. Denisov and V. Watchel in paper [1] shed the light on aspects related to the “positivity” of multidimensional random walks.

- [1] Denisov D., Wachtel V., *Random walks in cones revisited*, Ann. Inst. Henri Poincaré, Probab. Stat. – 2024. – Vol. 60, No. 1. – P. 126–166.

Probabilistic representation of the Cauchy problem solution for the discrete nonstationary Schrödinger equation

Ruslan Baiteev

EIMI, Saint Petersburg

altermapper@gmail.com

We study the one-dimensional free Schrödinger equation on $\mathbb{Z}$, which describes the quantum evolution of a discrete wave function $u(n, t)$ with continuous time. The initial state $\varphi(n)$ is prescribed, and the wave function admits the standard interpretation: namely, $|u(n, t)|^2$ represents the probability of observing a free particle at site n at time t . A new approach to solving such an evolution equation is developed, based on the use of discrete analytic functions and symmetric random walks.

The Energy-Casimir Method and Nonlinear Stability in Plasma Physics Problems

Julia Belyaeva

RUDN University, Moscow,

Institute of Applied Mathematics and Mechanics, Donetsk

yilia-b@yandex.ru

The talk is devoted to the study of nonlinear stability of solutions to the Vlasov-Poisson system. This system is a well-known model of high-temperature plasma kinetics. The Energy-Casimir method is applied to the case of a stationary solution in a half-space including the influence of a uniform external magnetic field.

On a conjecture of de Branges

Igor Bereza

Saint Petersburg State University, Saint Petersburg

ibereza@disroot.org

The talk is devoted to an axiomatic description of de Branges spaces. First, a procedure of “twisting” a de Branges space from a special class will be presented, after which, using this construction, L. de Branges’ 1963 conjecture concerning the superfluity of the axiom of continuity, stated in [1], will be refuted. The talk is based on [2].

[1] L. de Branges, *Some Hilbert spaces of analytic functions*, Trans. Amer. Math. Soc. – 1963. – Vol. 106. – P. 445–468.

[2] Bereza I., *On a conjecture of de Branges*, arXiv:2507.12576.

Mixed Volumes of Convex Hulls of Random Processes

Artyom Bolotin

Saint Petersburg State University, Saint Petersburg

bolotin2003@yandex.ru

Let $K_1, K_2, \dots, K_s$ be convex bodies in $\mathbb{R}^d$. Minkowski showed that d -dimensional volume $\text{Vol}_d(\lambda_1 K_1 + \lambda_2 K_2 + \dots + \lambda_s K_s)$ with $\lambda_1, \lambda_2, \dots, \lambda_s \geq 0$ is homogeneous polynomial of degree

d :

$$\text{Vol}_d(\lambda_1 K_1 + \lambda_2 K_2 + \dots + \lambda_s K_s) = \sum_{i_1=1}^s \dots \sum_{i_d=1}^s \lambda_{i_1} \dots \lambda_{i_d} V_d(K_{i_1}, \dots, K_{i_d}),$$

where functions $V_d(K_{i_1}, \dots, K_{i_d})$ are symmetric and called mixed volumes.

We consider convex hulls of independent random walks with exchangeable increments and calculate the mathematical expectation of their mixed volumes. As a corollary, we obtain a similar result for independent symmetric stable Lévy processes.

Sine-process and Gaussian multiplicative chaos

Alexander Bufetov

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow

bufetov@mi-ras.ru

Multiplicative chaos was born in the works of A.N. Kolmogorov: on December 17, 1940, Kolmogorov submitted a brief note “On the logarithmically normal distribution law of particle sizes under fragmentation” to the Reports of the Academy of Sciences of the USSR. A few days later, on December 28, 1940, Kolmogorov submitted his famous note “The local structure of turbulence in an incompressible viscous fluid for very large Reynolds numbers” to the same journal. Kolmogorov’s theory of homogeneous isotropic turbulence was criticized by Landau, who pointed out the need to account for the highly chaotic behavior of energy dissipation in a turbulent flow. In 1961, in Luminy, Kolmogorov presented a lecture “A refinement of the concepts of the local structure of turbulence in an incompressible viscous fluid for large Reynolds numbers”, where he addressed Landau’s objection, stating (in Kolmogorov’s own formulation) that “the variability of energy dissipation must increase without bound”. Kolmogorov and Obukhov formulated a new hypothesis—that of the “normality of the distribution of the logarithm” of energy dissipation in a turbulent flow.

The log-normal hypothesis of Kolmogorov and Obukhov—formulated at a physical level of rigor—inspired Mandelbrot’s work on multiplicative cascades, and subsequently, the work of Peyrière and Jean-Pierre Kahane, who provided the existence theorem for Gaussian multiplicative chaos.

It turned out that the Gaussian multiplicative chaos constructed by Kahane arises in a wide variety of problems, including, as noted by the St. Petersburg physicist Yan Valeryevich Fedorov, in problems of random matrix theory. The convergence to Gaussian multiplicative chaos in various matrix models has been studied by many mathematicians, in particular (the list is far from complete) — Berestycki, Chhaibi, Lambert, Nikeghbali, Ostrovsky, Simm, Webb.

In these lectures, we will begin with a brief review of Kolmogorov’s theory and the Kolmogorov-Obukhov log-normal hypothesis, continue with a discussion of the Mandelbrot-Peyrière-Kahane theory, and then move on to consider the convergence to Gaussian multiplicative chaos for random holomorphic functions—specifically, stochastic Euler products corresponding to the sine-process.

Recent Results on the Mittag-Leffler Distribution

Ekaterina Chernyshenko

Lomonosov Moscow State University, Moscow

`ekaterina.chernyshenko@math.msu.ru`

In stochastic models with heavy tails, the Mittag-Leffler distribution often appears as a limiting law. The example of three different random processes demonstrates almost sure convergence to this distribution. In particular, in the model known as the Chinese restaurant process (see [1]), random partitions of a finite set $1, 2, \dots, n$ are analyzed. As the size of the set n tends to infinity, the number of resulting partition blocks converges almost surely to a random variable with the Mittag-Leffler distribution, as established in [4]. Subsequently, a similar model for continuous time is considered—the Yule pure birth process (see [2]). In this model, particle coloring follows this algorithm: initially, the system contains a single particle with a unique color. Each newly appearing particle, with a given probability, either inherits the color of its parent particle or acquires a completely new color that did not previously exist in the population. In [2], it is established that as t tends to infinity, the number of different colors that have appeared converges almost surely to a random variable with the Mittag-Leffler distribution. We will demonstrate how the Mittag-Leffler distribution describes the limiting distribution of the time spent at the origin for a symmetric, homogeneous random walk on a one-dimensional lattice under the condition of infinite jump variance (see [3]). We will then strengthen this result by proving almost sure convergence.

- [1] Pitman J., *Exchangeable and partially exchangeable random partitions*, Probab. Theory Relat. Fields – 1995. – Vol. 102. – P. 145–158.
- [2] Pitman J., *Combinatorial stochastic processes*, Lecture Notes in Math. 1875. Springer, Berlin – 2006.
- [3] Aparin A.A., Popov G.A., Yarovaya E.B., *On the sojourn time distribution of a random walk at a multidimensional lattice point*, Theory Probab. Appl. – 2021. – Vol. 66, No. 4. – P. 657–675.
- [4] Bercu B., Favaro S., *A martingale approach to Gaussian fluctuations and laws of iterated logarithm for Ewens-Pitman model*, Stoch. Proc. Appl. – 2024. – Vol. 178. – P. 1–19.

Smoothness of minimal locally concave functions

Egor Dobronravov

Saint Petersburg State University, Saint Petersburg

`yegordobronravov@mail.ru`

In the theory of partial differential equations, the study of solutions to the degenerate Monge-Ampère equation is of significant interest. Concave solutions of the degenerate Monge-Ampère equation are minimal locally concave functions. Minimal locally concave functions also arise as optimizers of integral functionals in Bellman function theory. In this regard, the question of the structure and smoothness of minimal locally concave functions is of particular interest. A significant limitation of existing works on this subject is the absence of a free boundary – a part of the boundary where the boundary value is not prescribed – and they also assumed the existence of a smooth, strictly concave majorant. We will show that

even in the absence of these restrictions, a minimal locally concave function is $C^{1,1}$ smooth both inside the domain and up to the fixed boundary. We will also discuss the degree of smoothness that a minimal locally concave function can maintain up to the free boundary

Dimension of measure with small Fourier transform

Nikita Dobronravov

Saint Petersburg State University, Saint Petersburg
dobronravov1999@mail.ru

The Uncertainty Principle (UP) in mathematical analysis is a family of facts that state: both function and its Fourier transform cannot be simultaneously small. One version of Uncertainty Principle is the Theorem that there does not exist non zero function in $L_p(\mathbb{R}^d)$ if its Fourier transform is supported by a set of finite α -Hausdorff measure with $\alpha < 2d/p$. We proved that this UP does not hold at the endpoint. We proved that for any $2 < p < \infty$ and $d \in \mathbb{N}$ there exists a probability measure with compact support μ in $\mathbb{R}^d$ such that $\mathcal{H}_{\frac{2d}{p}}(\text{supp}(\mu)) = 0$ and $\hat{\mu} \in L_p(\mathbb{R}^d)$.

Here $\mathcal{H}_\alpha$ is Hausdorff measure of dimension α .

Stochastic Dynamics Near Critical Points in Stochastic Gradient Descent

Dmitry Dudukalov

Sobolev Institute of Mathematics, Novosibirsk
d.dudukalov@g.nsu.ru

The talk is devoted to limit theorems for additive stochastic gradient descent as the step size tends to zero. We identify the conditions under which convergence (almost surely or in probability) to a local minimum occurs, provided the process is initialized within its basin of attraction, as well as the conditions under which such convergence fails to occur. In addition, we discuss the stochastic dynamics arising when the gradient descent is initialized in a neighborhood of a non-smooth maximum.

Asymptotic analysis of some integrals with highly degenerate denominators

Alexey Elokhin

National Research University "Higher School of Economics", Moscow,
Steklov Mathematical Institute of Russian Academy of Sciences, Moscow
aelokhin@hse.ru

In the problem of rigorous justification of Peierls's theory of thermal conductivity, the asymptotic analysis of integrals of the form $\int \frac{F dx}{\Omega^2 + \nu^2}$, as $\nu \rightarrow 0$, plays a crucial role. The existence of the asymptotics is determined by the properties of the function Ω , in particular by its behavior in the vicinity of its critical points. In my talk I am going to describe the approach to obtain the asymptotics when critical points of the function Ω are degenerate, and to discuss some difficulties related to it.

degenerate denominators. *arXiv preprint arXiv:2509.21604*.

The Kolmogorov-Chapman equation and its connection with the Markov property of the process

Elena Filichkina

Lomonosov Moscow State University, Moscow
elena.filichkina1999@yandex.ru

The report considers examples of processes that satisfy the Kolmogorov-Chapman equation, but are not Markov processes. One of the first such examples was proposed by Feller in [1], where a process with three or more states is considered. Based on Bernstein's example of pairwise independent variables which are not mutually independent, examples of two-state processes can be constructed. It is shown that the Kolmogorov-Chapman equation do not uniquely determine a non-Markov process. It is also established that for a non-degenerate Gaussian process with a continuous covariance function, the satisfying of the Kolmogorov-Chapman equation for its transition densities is equivalent to the Markov property of the process.

- [1] W. Feller, *Non-Markovian processes with the semigroup property*, Ann. Math. Statist. 30 (1959).

Central limit theorem for the coulumb gas at high temperatures

Sergei Gorbunov

MIPT, Moscow

The circular β -ensemble is a measure on n -point configurations of the unit circle

$$d\mathbb{P}_\beta^n(\theta_1, \dots, \theta_n) = Z^{-1} \prod_{1 \leq m < l \leq n} |e^{i\theta_m} - e^{i\theta_l}|^\beta \prod_{k=1}^n d\theta_k, \quad \theta_j \in (-\pi, \pi).$$

From the physical viewpoint it is a Boltzmann distribution of a system of particles, interacting with the pair potential $U(\theta_1, \theta_2) = -\ln|e^{i\theta_1} - e^{i\theta_2}|$ at the inverse temperature β . Killip and Stoiciu [5] established the convergence of the measure under the scaling $\theta_j \rightarrow n\theta_j$ as $n \rightarrow \infty$ to a limit point process — the sine- β process $\mathbb{P}_\beta$, which is a measure on infinite locally finite subsets of the real line.

Theorem Assume that $\beta \leq 2$. Assume a function f on the real line and its derivative are square-integrable. Then the sum $\sum_{x \in X} f(x/R)$ of the values of the function in the points of a random configuration $X \sim \mathbb{P}_\beta$ without the expectation converges to the gaussian distribution as $R \rightarrow \infty$. Given the function is real-valued, the convergence holds under the Kolmogorov-Smirnov metric at the rate of $1/\sqrt{\ln R}$.

For $\beta = 2$ the circular ensemble coincides with the radial part of the Haar measure on the unitary group. The respective limit process $\mathbb{P}_2$ is the sine-process. At $\beta = 1, 4$ the point

processes $\mathbb{P}_\beta^n$, $\mathbb{P}_\beta$ are pfaffian. However, in general there are no explicit formulae for the correlation functions of the sine- β process.

In 1970 Henry Jack [3], motivated by the problem of calculation of matrix integrals, introduced a family of symmetric functions, parametrized by partitions and a parameter β . 17 years later Macdonald [7] gave their algebraic-combinatorial description. In particular, he proved that they have an analogue of the Cauchy identity and are orthogonal with respect to the circular- β ensemble. Jiang and Matsumoto [4] used these results to derive precise estimates on moments of sums of powers of coordinates under the circular β -ensemble. We pursue their approach. Following Borodin and Okounkov [1], we derive the Gessel-type expansion [2] for the expectations of multiplicative functionals. Using the formula for the expectation of the size of a random partition under the Jack measure, we deduce an estimate, holding under the Killip-Stoiciu limit.

Surprisingly, the restriction on the temperature $\beta \leq 2$ is due to exponential growth of the Jack polynomials norms at $\beta > 2$. This is a combinatorial expression of the phase transition of the considered measures at $\beta = 2$, which was first noted by Valkó and Virág [8]. Another expression, conjectured by Lambert [6], is sufficiency of $1/2$ -Sobolev regularity for the counterpart of the Szegő Theorem to hold. In the construction of Valkó and Virág of the sine- β process by a random Dirac operator the value $\beta = 2$ separates the limit point and the limit-circle cases.

- [1] Borodin A., Okounkov A., *A Fredholm determinant formula for Toeplitz determinants*, Integral Equations Operator Theory – 2000. – Vol. 37. – P. 386–396.
- [2] Gessel I. M., *Symmetric functions and P-recursiveness*, J. Comb. Theory, Ser. A – 1990. – Vol. 53. – P. 257–285.
- [3] Jack H., *A class of symmetric polynomials with a parameter*, Proc. Roy. Soc. Edinburgh, Sect. A – 1970. – Vol. 69. – P. 1–18.
- [4] Jiang T., Matsumoto S., *Moments of traces of circular beta-ensembles*, Ann. Probab. – 2015. – Vol. 43. – P. 3279–3336.
- [5] Killip R., Stoiciu M., *Eigenvalue statistics for CMV matrices: From Poisson to clock via random matrix ensembles*, Duke Math. J. – 2009. – Vol. 146. – P. 361–399.
- [6] Lambert G., *Mesoscopic central limit theorem for the circular β -ensembles and applications*, Electron. J. Probab. – 2021. – Vol. 26. – P. 1–33.
- [7] Macdonald I. G., *Symmetric functions and Hall polynomials*, The Clarendon Press, 2nd ed., (1995).
- [8] Valkó B., Virág B., *Continuum limits of random matrices and the Brownian carousel*, Invent. Math. – 2009. – Vol. 177. – P. 463–508.

The spectral problem for the Kronig-Penney model in terms of Schur's algorithm

Pavel Gubkin

PDMI RAS, Saint Petersburg,
Saint Petersburg state university, Saint Petersburg

`gubkinpavel@pdmi.ras.ru`

The relativistic Kronig-Penney model describes the one-dimensional Dirac operator $\mathcal{D}_Q$ on the half-line $\mathbb{R}_+$ of the form $\mathcal{D}_Q : X \mapsto JX' + QX$, where constant matrix J is a square root of the minus identity matrix and $Q = \sum_{k \geq 0} Q_k \delta_{hk}$ is a measure-valued potential supported on the half-lattice $h\mathbb{Z}_+$ for some $h > 0$. As well as in more classical cases, e.g., $Q \in L^2(\mathbb{R}_+)$, for such a Dirac operator one can define the Weyl function m_Q and the corresponding Schur function $f_Q = \frac{m_Q - i}{m_Q + i}$. The following theorems solve the direct and inverse spectral problems for $\mathcal{D}_Q$.

Theorem. *The mass Q_0 of Q at the point 0 can be explicitly calculated in terms of $f_Q(\infty) = \lim_{y \rightarrow \infty} f_Q(iy)$.*

Theorem. *The Schur functions f_Q and f_{Q_h} of $Q = \sum_{k \geq 0} Q_k \delta_{hk}$ and $Q_h = \sum_{k \geq 0} Q_{k+1} \delta_{hk}$ are related by the step of the classical Schur's algorithm*

$$e^{2ihz} f_{Q_h}(z) = \frac{f_Q(z) - f_Q(\infty)}{1 - \overline{f_Q(\infty)} f_Q(z)}, \quad z \in \mathbb{C}_+.$$

The two theorems above reduce the spectral theory of the Kronig-Penney model to the theory of orthogonal polynomials on the unit circle. In the talk we will show how this reduction can be used to obtain the explicit two-sided uniform stability estimate for the mapping $Q \mapsto m_Q$. The talk is based on the joint work with Roman Bessonov.

- [1] R. Bessonov and P. Gubkin, *Direct and inverse spectral continuity for Dirac operators*, arXiv:2505.00485.

On spatiotemporal structures of a simple symmetric branching random walk on $\mathbb{Z}$

Aleksandr Gusarov

Lomonosov Moscow State University, Moscow

`aleksandr.gusarov@math.msu.ru`

We consider a simple symmetric continuous-time branching random walk on a one-dimensional lattice $\mathbb{Z}$. In this case, the random walk is described by a difference Laplacian with diffusion coefficient $\varkappa > 0$. It is also assumed that the only branching source (i.e., the point at which particles can reproduce and die) is at zero, and its intensity (i.e., the first derivative of the generating function of the offspring) is assumed to be $0 < \beta < \infty$. At the initial time $t = 0$, there is a single particle at the point x . The results for the particle numbers at some fixed point $y \in \mathbb{Z}$ as $t \rightarrow \infty$ are well known, see, for example, [1] and [2]. We show how to explicitly establish the change in the behavior of the particle numbers as a function of the power-law relation between the time and spatial coordinates of a one-dimensional lattice.

- [1] Yarovaya E. B., *Branching random walks in a heterogeneous environment*, MCCME, (2025).
- [2] Smorodina N. V., Yarovaya E. B., *On one limit theorem for branching random walks*, Theory Probab. Appl. – 2024. – Vol. 68, No. 4. – P. 630–642.

New condition for a probability distribution of an absolutely continuous random variable to be uniquely determined by its moments

Maksim Iakovenko

Lomonosov Moscow State University, Moscow

`maksim.iakovenko@math.msu.ru`

We consider two random variables: $X \sim F$ with values in $\mathbf{R}$ and $Y \sim G$ with values in $\mathbf{R}_+$. The distributions F and G are assumed to be absolutely continuous with densities f and g , respectively. All moments of the random variables X and Y are assumed to be finite. In paper [1], conditions on the densities f and g are introduced such that Carleman's condition is satisfied, from which the M -determinacy of the random variables X and Y immediately follows. These conditions on the densities can be weakened, leading to a generalization of the result. Explicit examples of densities for the boundary cases are provided.

- [1] Wei Y., Zhang R., *A new moment determinacy condition for probability distributions*, Theory Probab. Appl. – 2025. – Vol. 70, No. 1. – P. 129–139.

The adaptive goodness-of-fit chi-square test for scale-location parametric families

Ruslan Iakupov

Lomonosov Moscow State University, Moscow

`ruslan.iakupov@math.msu.ru`

The classical chi-square test is used to check the goodness-of-fit between empirical and theoretical distributions in the discrete case. To apply this test to continuous distributions on $\mathbb{R}$, one usually performs discretization: the data are divided into several intervals, and the number of observations in each interval is counted. However, in the standard setting, the intervals are fixed before the experiment. As a result, the test actually checks the agreement of probabilities somehow in the chosen blocks, rather than the original hypothesis. This can make the test less powerful in detecting local deviations in the data.

In this work we consider goodness-of-fit hypothesis for some scale-location parametric family. We propose a modification of the chi-square test, where the block boundaries are adapted to the data using sample quantiles. This approach takes into account the actual structure of the data and keeps the test sensitive to the shape of the empirical distribution.

We use the following method. The sample is divided into N blocks according to the empirical quantiles i/N . Then, for this discrete parametric family we estimate the scale and location parameters.

For a given number of blocks N , all possible combinations of these blocks into groups of k are considered. For each combination, the chi-square statistic is computed, and the final statistic is the sum of these values over all possible groupings.

The main result of the work is the limit theorem for the test statistic. It is shown that the statistic has an asymptotic weighted chi-square distribution. This result provides the asymptotic test for for scale-location goodness-of-fit problem.

Rate of convergence in limit theorems for the local time of random walk at a

point in $\mathbb{Z}^d$

Olga Iushkova

Lomonosov Moscow State University, Moscow

olga.ushkova@math.msu.ru

In [1], limit theorems on the distribution of the sojourn time of a random walk at a point were proved depending on the lattice dimension under the assumption of finite variance and under a condition leading to an infinite variance of jumps. In this paper, theorems on the estimation of the rate of convergence to the obtained limit distributions were proved by the Stein method in the Wasserstein metric. Using a discrete approximation of the sojourn time, asymptotic properties of the transition probabilities of a random walk, and the generalized hyper-Airy functions considered in [2-3], new results were obtained under different assumptions about the variance of the jumps of the random walk.

In the case of finite jump variance and dimension $d = 1$, the rate of convergence estimate to the half-normal distribution is $O(t^{-1/2})$. In the case of infinite jump variance in dimension $d = 1$ with parameter value $\alpha \in (1, 2)$, convergence to the Mittag-Leffler distribution takes place, and under the condition $\frac{\alpha}{\alpha-1} \in \mathbb{N}$ the estimate $O(t^{1/\alpha-1})$ is valid.

- [1] Aparin A. A., Popov G. A., Yarovaya E. B., *On the sojourn time distribution of a random walk at a multidimensional lattice point*, Theory Probab. Appl. (2022).
- [2] Mainardi F., *On the initial value problem for the fractional diffusion-wave equation*, World Sci. Publ., River Edge, NJ (1994).
- [3] Cinque F., Orsingher E., *General Airy-type equations, heat-type equations and pseudo-processes*, J. Evol. Equ. (2025).

On branching random walks in homogeneous and nonhomogeneous random environments

Oleg Ivlev

Lomonosov Moscow State University, Moscow

olivlegerr@gmail.com

Two models of continuous-time symmetric branching random walks on a multidimensional integer lattice $\mathbb{Z}^d$ in random environments are considered. The first model represents a homogeneous branching environment, where the intensities of particle death and splitting at each $x \in \mathbb{Z}^d$ are determined by a pair of non-negative random variables $(\xi^-(x) = \xi^-(x, \omega), \xi^+(x) = \xi^+(x, \omega))$, defined on a certain probability space $(\Omega, \mathcal{F}, \mathcal{P})$. Thus, the environment (i.e., the set of branching characteristics at the sources) in the first model is a collection of pairs of random variables $(\xi^-(x), \xi^+(x))$, where $x \in \mathbb{Z}^d$. Let us assume that the pairs $(\xi^-(x), \xi^+(x))$ are independent and identically distributed. We will denote the mathematical expectation with respect to the measure $\mathcal{P}$ by $\langle \cdot \rangle$. The second model studies a nonhomogeneous branching environment, which is determined by a single pair of non-negative random variables $(\xi^-(0), \xi^+(0))$, which specify the intensities of particle death and splitting at zero, respectively. The aim of this work is to study the so-called “annealed” moments $\langle m_n^p \rangle$, $p \geq 1$ of the local and total particle numbers for both models. The

corresponding asymptotics for the moments have been obtained, confirming the hypothesis about their form put forward in [1].

- [1] Yarovaya E., *Symmetric branching walks in homogeneous and non homogeneous random environment*, Communications in Statistics-Simulation and Computation – 2012. – Vol. 41, No. 7. – P. 1232–1249.
- [2] Albeverio S. et al., *Annealed moment Lyapunov exponents for a branching random walk in a homogeneous random branching environment*, Markov Process. Relat. Fields – 2000. – Vol. 6, No. 4. – P. 473–516.

Entropy analysis of distributions of stable Levy processes

Viktor Khamzin

Saint Petersburg State University, Saint Petersburg,
EIMI, Saint Petersburg
viktokhamzinwork@gmail.com

mm -the entropy of a metric space with a measure is a quantity that shows how many balls of the same radius must be taken to cover the set of the desired measure. It was defined in the classical work of K.Shannon [2], but until recently it was practically not studied. A.M.Vershik and M.A.Lifshits in [1] found the value of the mm -entropy of a Banach space with a Gaussian measure. The talk will focus on the latest results obtained in the non-Gaussian case: we will consider the trajectory space of the α -stable Levy process and find its mm -entropy.

- [1] Vershik A. M., Lifshits M. A., *mm -entropy of a Banach space with a Gaussian measure*, Theory Probab. Appl. – 2023. – Vol. 68, No. 3. – P. 532–543, in Russian; 431–439, in English.
- [2] Shannon C. E., *A mathematical theory of communication*, Bell Syst. Tech. J. – 1948. – Vol. 27, No. 3. – P. 379–423; Vol. 27, No. 4. – P. 623–656.

Determinantal processes and interpolation of functions from values in points of a random configuration

Alexey Klimenko

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow
National Research University “Higher School of Economics”, Moscow
klimenko@mi-ras.ru

The talk is based on the joint work with A. Borichev, A. Bufetov, and Zhaofeng Lin.

Determinantal process is a class of random point fields, that is, of probability measures on the space of *configurations*, discrete subsets of some phase space E , that have some special form of correlation functions. A determinantal process can be constructed from a contraction operator in the space $L^2(E)$. In most examples this operator is the projection to a subspace $H \subset L^2(E)$, and functions in H are sufficiently regular so that their values in any given point

of E are well-defined. Then one can connect the determinantal process (that is, a random configuration X) and the space H by the following question: is a function $f \in H$ uniquely defined by its values in the points of (almost any) configuration X ? This question will be the main topic of the talk.

Balance of forces as a mechanism for community formation in random graphs

Ivan Kobzev

Lomonosov Moscow State University, Moscow

kobzev.cmc@yandex.ru

Exponential random graph models (ERGMs) often suffer from degeneracy when modeling networks with communities, leading to the formation of either a sparse graph or a single giant dense component.

This talk proposes a mechanism that explains the spontaneous emergence of communities as a result of a competition between two forces: local attraction, which forms dense groups, and stabilizing repulsion, which prevents them from merging. This principle is studied using a minimal ERGM where attraction is modeled by promoting triangles, and repulsion by penalizing paths of length three.

The main theorem on the phase transition to a state with several stable communities is formulated. Key ideas of the proof are discussed, and results of numerical simulations confirming the theoretical conclusions are presented.

Probabilistic and statistical properties of a random graph with independent vertices' weights

Anna Kotova

Saint Petersburg State University, Saint Petersburg

kotann2710@mail.ru

We study a random graph model constructed according to the following principle. Each vertex is assigned a random weight (fitness) w_i , where the weights w_i are independent identically distributed random variables. Then, when the weights of the vertices are fixed, an edge is drawn independently between each pair of vertices i, j with probability $f(w_i, w_j)$, where f is a pre-selected function ([1], [2]).

For this model, we obtain limit theorems for the distribution of vertex degrees, and, in case of bilinear edge function f , construct statistical estimators for the vertex weight distribution function.

- [1] Caldarelli G., Capocci A., Rios P., Muñoz M., *Scale-free networks from varying vertex intrinsic fitness*, Phys. Rev. Lett. – 2002. – Vol. 89, No. 25, 258702.
- [2] Stegehuis C., Zwart B., *Scale-free graphs with many edges*, Electron. Commun. Probab. – 2023. – Vol. 28. – P. 1–11.

Steady state Partial Asymmetric Simple Exclusion Process from the prism of 1D Magnetic Random Walks: phase transitions and beyond

Elizaveta Kovalenko

MIPT, Moscow

kovalenko.elizavebeth@gmail.com

The Partially Asymmetric Simple Exclusion Process (PASEP) model is considered. Starting from the matrix ansatz for the steady state, one obtains an algebra that represents a q -deformation of the matrix algebra for the Totally Asymmetric Simple Exclusion Process (TASEP). The steady-state solution can be expressed in terms of q -Hermite polynomials, which admit an interpretation in terms of magnetic random walks. The phase transition in the PASEP model is expected to correspond to the localization of magnetic random walks at the boundary.

- [1] Derrida B., Evans M. R., Hakim V., Pasquier V., *Exact solution of a 1D asymmetric exclusion model using a matrix formulation*, J. Phys. A: Math. Gen. – 1993. – P. 1493–1517.
- [2] Valov A., Gorsky A., Nechaev S., *Equilibrium mean-field-like statistical models with KPZ scaling*, Phys. Part. Nucl. – 2021. – Vol. 52, No. 2. – P. 185–201.
- [3] Koekoek R., Lesky P. A., Swarttouw R. F., *Hypergeometric orthogonal polynomials and their q -analogues*, Springer (2010).

On L. V. Firsov’s Method for Determining the Length of the Attic Stadion

Mikhail Krakovskiy

Lomonosov Moscow State University, Moscow

krakovskiyma@my.msu.ru

This work is devoted to the statistical analysis of L. V. Firsov’s method for determining the length of the stadion — the basic unit of distance used by ancient geographers. Firsov’s method was criticized by Engels, who pointed out the presence of “large outliers” in the data. Our goal is to conduct a statistical test of L. V. Firsov’s hypothesis concerning the existence of a stable measure of length, approximately equal to 157–158 m, which may have been adopted in the Hellenistic scientific tradition, using the Student’s t -test and correlation analysis. The obtained results confirm the accuracy of Firsov’s method.

The relationship between the spectrum structure of the evolution operator for a branching random walk on $\mathbb{Z}$ and the configuration of branching sources

Mikhail Krotov

Lomonosov Moscow State University, Moscow

mikhail.krotov@math.msu.ru

We consider a continuous-time branching random walk (BRW) on the one-dimensional lattice $\mathbb{Z}$. At the initial time $t = 0$, there is a single particle at an arbitrary point of $\mathbb{Z}$. For a detailed approach to describing BRW models, see, for example, [1]. We examine configurations of a

finite number of branching sources, i.e., points on $\mathbb{Z}$ where particles can produce a finite number of offspring or die. The branching intensity of particles at the branching sources is assumed to be equal. The study focuses on a supercritical BRW, characterized by exponential growth of particle numbers at each lattice point. For this case, limit theorems on the almost sure convergence of normalized particle numbers hold true, see [2]. Phase transitions in the supercritical BRW are determined by the structure of the discrete positive spectrum of the evolution operator, i.e., the operator appearing on the right-hand side of the equation describing the evolution of the first moments of particle numbers, see [3]. This work investigates the conditions for the existence of positive isolated eigenvalues, as well as their behavior depending on the configuration of branching sources and their intensities. In contrast to similar BRW models on multidimensional lattices, for the BRW on $\mathbb{Z}$, the computations are presented in an explicit form, which significantly facilitates the application of numerical simulation methods

- [1] Yarovaya E. B., *Branching random walks in an inhomogeneous medium*, Center Appl. Stud. Mech. Math. Moscow State Univ., M. (2007).
- [2] Smorodina N. V., Yarovaya E. B., *One limit theorem for branching random walks*, Theory Probab. Appl. – 2023. – Vol. 68, No. 4. – P. 779–795.
- [3] Yarovaya E. B., *Spectral asymptotics of supercritical branching random walk*, Theory Probab. Appl. – 2017. – Vol. 62, No. 3. – P. 518–541.

Distances between random points inside and on the boundary of a convex body

Alexey Lotnikov

EIMI, Saint Petersburg,
PDMI RAS, Saint Petersburg
alex.lotnikov@gmail.com

In 2019, A. S. Tarasov and D. N. Zaporozhets formulated a conjecture stating that the mean distance between two random points on the boundary of a convex body is not less than the mean distance between two random points inside it. This talk will focus on a special case of this conjecture for centrally symmetric planar bodies. We will present results obtained in this direction, including precise relationships between the mean distances for circumscribed figures, which serve as analogues of Kingman’s formula connecting the mean distances between interior points with the length of a random chord. In conclusion, we will discuss potential avenues for weakening the original hypothesis for arbitrary planar bodies.

- [1] Kingman J., *Random secants of a convex body*, J. Appl. Probab. – 1969. – Vol. 6, No. 3. – P. 660–672.
- [2] Moseeva T., *Random sections of convex bodies*, Zap. Nauchn. Semin. POMI – 2019. – Vol. 486. – P. 190–199.
- [3] Bonnet G., Gusakova A., Thäle C., Zaporozhets D., *Sharp inequalities for the mean distance of random points in convex bodies*, Adv. Math., 326 (2021).
- [4] Gorshkov A., Nikitin I., *Mean distance between random points on the boundary of a convex figure*, J. Math. Sci., NY. – 2024. – Vol. 286, No. 5. – P. 798–806.

Mathematical foundations of the Kalman filter

Vladislav Maksimov

Lomonosov Moscow State University, Moscow
vladislav.maksimov@math.msu.ru

The Kalman filter is undoubtedly one of the most important discoveries in applied mathematics of the last century. While numerous engineering books have been written about it, there are relatively few that focus specifically on its mathematical foundations. We will examine a rigorous mathematical formulation of the problem and prove a theorem on the existence and uniqueness of the solution to the optimal estimation problem using the apparatus of matrix-valued scalar products. If time permits, we will also discuss issues related to the efficient numerical implementation of the filtering algorithm.

- [1] Kailath T., Sayed A. H. and Hassibi B., *Linear Estimation*, Prentice Hall, Englewood Cliffs, (2000).
- [2] <https://github.com/rllabbe/Kalman-and-Bayesian-Filters-in-Python>
- [3] Matasov A.I., *Fundamentals of Kalman Filter Theory*.

On the minimal integral energy of majorants of the Wiener process

Sergey Nikitin

PDMI RAS, Saint Petersburg
nikitin97156@mail.ru

We consider the asymptotic behavior (over long time intervals) of the minimal integral energy

$$|h|_T^\psi := \int_0^T \psi(h'(t)) dt$$

of majorants of the Wiener process $W(\cdot)$ satisfying the constraints $h(0) = r$, $h(t) \geq W(t)$ for $0 \leq t \leq T$.

The results significantly generalize previous asymptotic estimates obtained for the case of kinetic energy $\psi(u) = u^2$, revealing that this case, where the minimal energy grows logarithmically, is a critical one, lying between two different asymptotic regimes.

On the law of iterated logarithm for random walks on a multidimensional lattice

Elina Nizamova

Lomonosov Moscow State University, Moscow
elina.nizamova@math.msu.ru

This work establishes the functional law of the iterated logarithm for a symmetric continuous-time random walk on a multidimensional lattice. For a description of the random walk model with finite jump variance, see, for example, [2]. We have proved this law under a slightly stronger assumption than the existence of a finite variance for the random walk jumps,

namely, under the existence of a moment of order $2 + \epsilon$, $\epsilon > 0$. The proof of the main result uses the functional laws of the iterated logarithm for discrete random walks and the Wiener process, combined with the strong approximation theorem by a Wiener process from [1] and [3]. The proposed approach made it possible to overcome the difficulties arising when passing from discrete to continuous time. The main result is a generalization of the classical law of the iterated logarithm for continuous-time random walks.

- [1] Bulinski A. V., *A new version of the functional law of the iterated logarithm*, Theory Probab. Appl. – 1980. – Vol. 25, No. 3. – P. 502–512, in Russian.
- [2] Yarovaya E. B., *Branching random walks in an inhomogeneous medium*, Publishing House of the Center for Applied Research, Faculty of Mechanics and Mathematics, Moscow State University. Moscow (2007), in Russian.
- [3] Bashtova E., Shashkin A., *Strong Gaussian approximation for cumulative processes*, Stoch. Proc. Appl. – 2022. – Vol. 150. – P. 1–18.

Martingale transforms and minimal biconcave functions

Mikhail Novikov

PDMI RAS, Saint Petersburg,

The work is supported by the RSF grant №24-71-10011.

Novikov.3.14@yandex.ru

This talk is devoted to sharp estimates for $\mathbb{E}[f(\psi_\infty)]$, where f is arbitrary and ψ_∞ may be described, informally, as the limit value of a martingale transform of an indicator function. We show that this problem can be completely reduced to the computation of a minimal biconcave function, explain the connection with the BMO space, and illustrate the approach with several inequalities. Our main result is a precise criterion for the minimality of a biconcave function on the strip $\{(x, y) \in \mathbb{R}^2: |x - y| \leq 1\}$.

Final product of a random recurrence sequence

Fedor Obergan

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow

oberganfedor@mail.ru

Consider the model of a random recurrence sequence $(Y_n, n \geq 0)$ defined by the recurrence relation $Y_{n+1} = A_n Y_n + B_n$, where A_n are positive independent and identically distributed random variables and B_n may depend on $(A_k, B_k, 0 \leq k < n)$ for any $n > 0$. The model was introduced and studied by A.V. Shklyaev ([1]). It is quite interesting to study such sequences because a lot of models of branching processes can be represented as random recurrence sequences. For instance, branching process in a random environment with and without immigration, bisexual branching process in a random environment and many others. Large deviations probabilities for such branching processes in a random environment were investigated using random recurrence sequences in [2] and [3]. In the report author will present a theorem about large deviations probabilities for the n -th partial sum U_n of the series whose terms are the elements of the random recurrence sequence.

In the report models of final products of the random recurrence sequence will also be considered. Note that for branching processes the final product was studied in [4]. Let $(Y_n, n \geq 0)$ be the random recurrence sequence with positive integer values described above and let $\zeta = (\zeta_1, \zeta_2, \dots)$ be a sequence of independent and identically distributed random variables, which is called the random environment. Given the environment, consider a sequence of independent random variables $C_{i,j}$ distributed according to g_{ζ_i} , where $\{g_y(s), y \in \mathbb{R}\}$ are probability generating functions. It is also required that for any fixed $k \geq 0$ the random variables $(C_{k+1,i}, i > 0)$ and $(Y_j, j \leq k)$ are independent. The local final product of the random recurrence sequence $(Y_k, k \geq 0)$ for a fixed $n \geq 0$ is defined as the random variable $L_n = \sum_{i=1}^{Y_n} C_{n+1,i}$, and the random variable $F_n = \sum_{m=0}^n L_m$ is called the overall final product for a fixed $n \geq 0$.

In the report author will show theorems about large deviations probabilities for L_n and F_n . The applications of these results to branching processes in a random environment also will be presented in the report.

The study was supported by a grant from the Russian Science Foundation (project 24-11-00037) at the Steklov Mathematical Institute of Russian Academy of Sciences.

- [1] Shklyaev A. V., *Large deviations of branching process in a random environment*, Discrete Math. Appl. – 2021. – Vol. 31, No. 4. – P. 281–291.
- [2] Shklyaev A. V., *Large deviations of branching process in a random environment. II*, Discrete Math. Appl. – 2021. – Vol. 31, No. 6. – P. 431–447.
- [3] Shklyaev A. V., *Large deviations of bisexual branching process in random environment*, Discrete Math. Appl. – 2025. – Vol. 35, No. 3. – P. 173–186.
- [4] Vatutin V. A., *Polling systems and multitype branching processes in a random environment with final product*, Theory Probab. Appl. – 2011. – Vol. 55, No. 4. – P. 631–660.

Dynamical construction of the GFF on a graph

Daniil Panov, Pavel Mozolyako

Saint Petersburg State University, Saint Petersburg

panovdan2003@gmail.com

About 13 years ago H. Hedenmalm and P. Nieminen published a dynamical construction of the Gaussian free field on planar domains via the white noise field and the Hadamard's variational formula ([1]). We show that an analogous result holds in the discrete context, on graphs. Also, a discrete analogue of the Hadamard's formula is obtained. As in the continuous setting, this construction provides a convenient representation of the Gaussian free field and some of its well-known properties follow.

- [1] Hedenmalm H., Nieminen P. J., *The Gaussian free field and Hadamard's variational formula*, Probab. Theory Relat. Fields – 2014. – Vol. 159. – P. 61–73.

Universal local-linear kernel estimation for derivative of the regression function

Sergey Petrenko

Novosibirsk State University, Novosibirsk, Russia

s.petrenko@g.nsu.ru

The problem of nonparametric regression is considered, specifically that of estimating the derivative of a regression function, when the values of the regression function, observed with random errors, are available at a known set of deterministic or random points (the set of regressors). A vast literature is devoted to this problem, including approaches based on kernel smoothing methods. The talk will present consistency and asymptotic normality conditions for a new class of locally linear kernel estimators, which rely on a more general condition on the regressors than those previously known for this problem and offer several advantages.

It should be noted that in previous studies, models with deterministic and random regressors were typically treated separately. In the former case, a regular design condition on the regressors was usually imposed to ensure adequate coverage of the domain of the regression function, while in the latter case, some form of weak dependence among the regressors was assumed. In this work, the set of regressors is treated as a sequence of random variables within the series scheme, with the series length serving as the sample size parameter. This framework allows deterministic-design models to be included as a special case.

When studying the asymptotic properties of the new kernel estimator with respect to the regressor design, it is required only that, with high probability, the regressors form a refining partition of the domain of the regression function. In the context of dense data, this condition is essentially necessary for the recovery of the regression function and its derivatives. It is universal with respect to the stochastic nature of the regressors and allows models with deterministic and random regressors to be treated within a unified framework—without imposing regularity or weak dependence conditions.

Finally, this simple and intuitive condition makes it possible to estimate the regression function of interest without any information about the dependence structure of the regressors, which is especially important for practical applications.

Power Series Truncation Method for SIS-model with different velocities of migration of susceptible and infectious

Dmitry Podolin

National Research University “Higher School of Economics”, Moscow

d-podolin@mail.ru

Let us consider the following modification of the SIS model of a disease:

$$\frac{\partial S}{\partial t} + V \frac{\partial S}{\partial x} = -\beta SI + \gamma I, \quad \frac{\partial I}{\partial t} + U \frac{\partial I}{\partial x} = \beta SI - \gamma I. \quad (1)$$

In system (1) $S(x, t)$ is the linear density of susceptible to disease and $I(x, t)$ is the linear density of infectious ones. At $t = 0$, susceptible begin to migrate along the x -axis at a constant velocity V , and infectious begin to migrate in the same direction at a constant velocity U . It is natural to assume that susceptibles move faster than infectious, so there should be $V > U$.

Using dimensionless variables we obtain the normalized system:

$$\begin{cases} \frac{\partial \bar{S}}{\partial t} + \frac{\partial \bar{S}}{\partial \bar{x}} = -\bar{S}\bar{I} + \bar{I} \\ \frac{\partial \bar{I}}{\partial t} + \bar{U} \frac{\partial \bar{I}}{\partial \bar{x}} = \bar{S}\bar{I} - \bar{I} \end{cases}, \quad \bar{U} = \frac{U}{V} < 1. \quad (2)$$

To solve the Cauchy problem we apply the power series truncation method. Solutions are sought as power series:

$$S(x, t) = \sum_{k=0}^{\infty} S_k(t)x^k, \quad I(x, t) = \sum_{k=0}^{\infty} I_k(t)x^k. \quad (3)$$

Substituting series (3) into system (2) and truncating at order N , we obtain a system of ordinary differential equations for coefficients $S_k(t)$ and $I_k(t)$.

For validation we use the exact Bateman-type solution:

$$I(x, t) = \frac{1 - U}{1 + e^{-2x+(1+U)t}}, \quad S(x, t) = U + (1 - U) \frac{e^{-2x+(1+U)t}}{1 + e^{-2x+(1+U)t}}. \quad (4)$$

Quantitative analysis shows significant accuracy improvement with increasing N . The method demonstrates convergence and applicability for solving SIS-model with migration.

- [1] Brauer F., Driessche P., Wu J., eds. *Mathematical epidemiology*, Springer Berlin, Heidelberg, (2008).

Probabilities of the Late Extinction of Branching Processes

Alexander Shklyaeu

Steklov Mathematical Institute of Russian Academy of Sciences, Moscow

Lomonosov Moscow State University, Moscow

ashklyaeu@gmail.com

Usually, a supercritical Galton-Watson branching process in a random environment has two alternative: early extinction or exponential growth. However, what can we say about the trajectories with the late extinction?

The answer is quite simple: the probability of the extinction after a late moment n is of order ca^n for some constants c and $a \in (0, 1)$. The corresponding trajectory is close to this of some subcritical branching process.

What about the branching processes in a random environment (BPRE)? In this case the question is quite complicated. First of all, we have three different types of supercritical BPRE and only for one type (strongly supercritical) the behavior is close to the above. If consider only the strongly supercritical case, the result is quite complicated too – the only one paper in this area is [1], published by V.I. Afanasyev in 2024 (to this moment the English version of this article is not published). He considered a particular case of geometric distribution of the number of descendants of one particle.

In my opinion, the problem is that the supercritical BPRE, conditioned on the late extinction, is not a subcritical BPRE like in the Galton-Watson case. However, it's a positive recurrent Markov chain. Thus, we need to use general Markov chain theory, not p.g.f. techniques.

Using general R -positivity theory (see [2]), we generalize the results of Afanasyev to a general distribution of the descendants number, assuming only some moment conditions.

More accurately, we show that the probability of the late extinction has the order cR^n , where c, R are some positive constants. For the case of geometric distribution of number of descendants of one particle, R is known:

$$R = \mathbf{E} \frac{1}{\mu},$$

where μ is the conditional expectation of the number of descendants of one particle, conditioned on the environment.

In the report we will discuss the result above and will motivate the interest in the late extinction problem.

- [1] Afanasyev, V. I. *Strongly Supercritical Branching Process in a Random Environment Conditioned on Dying at a Distant Moment*. Diskretnaya matematika, (2024), in Russian; Discrete Mathematics & Applications, in print.
- [2] Ferrari P. A., Kesten H., Martínez S., *R-positivity, quasi-stationary distributions and ratio limit theorems for a class of probabilistic automata*, Ann. Appl. Probab. – 1996. – Vol. 6, No. 2. – P. 577–616.

Limit theorem for a number of II-type particles in a branching process with mutations in one gene

Mikhail Shvaikov

Lomonosov Moscow State University, Moscow
mikhail.shvaikov@math.msu.ru

We consider a triangular array of branching processes with infinitely many types. At each step, a number n is fixed, representing the number of generations observed. This number n defines the probabilities p_n and q_n . After the birth each particle may change its type to the next (higher) type with the probability p_n or to the previous (lower) type with the probability q_n . The process starts with a single particle of the first type, which has a mean number of offspring $\mu_1 > 1$. Numbers of particles of each type are independent and identically distributed, whereas particles of a higher type has higher mean number of offspring: $\mu_{i+1} > \mu_i$, $i \in \mathbb{Z}$. We assume that $q_n = o(1)$, $n \rightarrow \infty$ and analyze the limit distribution of the number of second-type particles.

Similar process (in a random environment) is described in the V.A. Vatutin's paper [1], however, his model is different. In particular, he considers a finite number of types and the particles of his processes can only change their type to a higher one. Related problems are known in literature. More specifically, Marek Kimmel and David E. Axelrod examine (in [2]) general properties of branching processes with infinitely many types and genome (probabilities p_n and q_n should be interpreted as mutations of a gene). In [3] Gonzalez et al.

consider bisexual branching processes of the similar nature. This talk, on the other hand, addresses the specific problem: investigating the first moments the initial particle of its type is born and analyzing the limiting distribution of the number of particles of each type.

Current work shows that as $p_n \sim C\mu_1^{-n}$, $n \rightarrow \infty$, where C is arbitrary constant, the problem can be reduced to the process, where particle types only increase. It is proved that particles of the second type emerge a finite time prior to termination of the process. Furthermore, the explicit form of the limiting distribution of the number of such particles is derived.

- [1] Vatutin V. A., *The structure of the decomposable reduced branching processes. I. Finite-dimensional distributions*, Theory Probab. Appl. – 2015. – Vol. 59, No. 4. – P. 641–662.
- [2] Kimmel M., Axelrod D. E., *Branching processes in biology*, Springer New York, NY (2015).
- [3] González M., Hull D. M., Martínez R., Mota M., *Bisexual branching processes in a genetic context: the extinction problem for Y-linked genes*, Math Biosci (2006).

Scattering and radiation of acoustic waves in discrete waveguides with several cylindrical outlets to infinity

Danil Smorchkov

Saint Petersburg State University, Saint Petersburg
st076101@student.spbu.ru

A discrete waveguide is a graph G that consists of several discrete semi-cylinders connected by a finite number of edges and nodes. By a discrete cylinder we mean a graph that is periodic when shifted by a given vector and has a finite periodicity cell. An equation of the form $-\operatorname{div}_a \nabla u - \mu u = f$ is considered on the graph G , where the given function f and the unknown function u are functions on the set V of nodes of the graph, and div and ∇ are difference analogs of the corresponding differential operators. The spectral parameter μ is assumed to be real and fixed. The weight function a is defined on the set of edges, is assumed to be positive and stabilizing at infinity with an exponential rate.

The *continuous spectrum eigenfunction* (CSE) is by definition a solution to the homogeneous problem that is bounded and doesn't belong to $\ell_2(V)$. We construct a basis of CSEs subject to the asymptotics at infinity: $Y_j^+ = u_j^+ + \sum_{k=1}^{\Upsilon} S_{j,k} u_k^- + o(1)$. Here $u_1^+, \dots, u_{\Upsilon}^+$ stand for *incoming waves*, while $u_1^-, \dots, u_{\Upsilon}^-$ stand for *outgoing waves*. The matrix $S = \|S_{j,k}\|$ is called the *scattering matrix*.

We establish a well-posed statement of problem with the *intrinsic radiation conditions*: $u = c_1 u_1^- + \dots + c_{\Upsilon} u_{\Upsilon}^- + o(1)$. The coefficients c_j are computed by the formulas $c_j = i(f, Y_j^-)_V$, where $(\cdot, \cdot)_V$ is the expansion of the inner product in $\ell_2(V)$ and $Y_1^-, \dots, Y_{\Upsilon}^-$ is another basis of CSEs given by $Y_j^- = \sum_{k=1}^{\Upsilon} (S^{-1})_{j,k} Y_k^+$.

The talk is based on a joint research with A.S. Poretskii.

Conditional Measures of Perfect Measures are Perfect

Igor Sokolov

MIPT, Moscow

sokolov.igor506@yandex.ru

The main result of the talk is that the conditional measure of a determinantal point process, which is perfect in the sense of G. I. Olshanski, is also perfect. The talk is based on joint work with A. I. Bufetov.

- [1] Olshanski G., *Determinantal point processes and fermion quasifree states*, arXiv:2002.10723.
- [2] Bufetov A., *Quasi-symmetries of determinantal point processes*, arXiv:1409.2068.
- [3] Bufetov A. I., Qiu Y., Shamov A., *Kernels of conditional determinantal measures and the proof of the Lyons-Peres conjecture*, arXiv:1612.06751 – 2018.
- [4] Bufetov A. I., *Conditional measures of determinantal point processes*, arXiv:1411.4951 – 2016.

The Characteristic Polynomial of a Random Unitary Matrix: A Probabilistic Approach

Ksenia Sologubova

MIPT, Moscow

shushasl@mail.ru

This work reviews the paper by Bourgade, Hughes, Nikeghbali, and Yor, which proposes a different approach to studying the characteristic polynomial of a random unitary matrix. Previously, its distribution was analyzed using complex analytical methods, such as the Selberg integral and the Weyl density. The authors propose a simpler and more intuitive probabilistic approach, based on a recursive construction of the Haar measure.

The main result of the work is the construction of two equivalent representations of the characteristic polynomial: as a product of independent random variables and as a sum of independent random variables when considering its logarithm.

We examine how these representations enable more straightforward proofs of known facts. In particular, we focus on a new proof of the Keating-Snaith central limit theorem, which relies solely on classical probability theory results, such as the multivariate central limit theorem, and avoids heavy calculations.

On Modeling Catalytic Branching Random Walks

Marina Susorova

Lomonosov Moscow State University, Moscow

susorovama@gmail.com

This presentation focuses on the computer modeling of continuous-time catalytic branching random walks on one-dimensional and two-dimensional lattices.

While classical studies of such systems are typically concerned with their asymptotic behavior at large times [1], this work is centered on the numerical analysis of process evolution over finite time intervals. The key objective is to analyze the spatiotemporal dynamics of particle density and its distribution across the lattice nodes, which requires the application of specialized algorithms.

The main result of this work is the development and comparative analysis of two alternative computational approaches—recursive and non-recursive algorithms—that enable detailed modeling. The presentation will cover the implementation principles of these methods, discuss their computational features, and demonstrate modeling results, including a series of visualizations and animations that clearly illustrate system dynamics.

- [1] Bulinskaya E.Vl., *Probabilistic-geometric properties of spatial branching random walk*, Dissertation for the degree of Doctor of Physical and Mathematical Sciences, Moscow (2024), in Russian.
- [2] Yermishkina E.M., Yarovaya E. B., *Simulation of branching random walks on a multi-dimensional lattice*, J. Math. Sci. – 2020. – Vol. 250, No. 3. – P. 443–459.

Limit Theorems for a Class of Maximal Branching Processes

Gregory Talpa

Lomonosov Moscow State University, Moscow

`grigorii.talpa@math.msu.ru`

Let $\{X_{i,j}\}$ be a sequence of i.i.d. random variables with a c.d.f. F . We define the random sequence $\{M_n\}$ by:

$$M_0 = 1, \quad M_n = \max(X_{n,1}, \dots, X_{n,M_{n-1}}), \quad n \geq 1.$$

This process is called a maximal branching process (MBP) and was introduced in [1]. One can say that in every generation of maximal branching process, descendants of only one particle survive – namely, the particle that has the largest number of descendants.

Now let us add a random environment to the model. Fix an i.i.d. sequence of random variables $\{\eta_i\}$ and consider random variables $\{X_{i,j}\}$ with a c.d.f. F_{η_i} , where $\{F_y\}$ is a family of distribution functions. We define the random sequence $\{M_n\}$ by:

$$M_0 = 1, \quad M_n = \max(X_{n,1}, \dots, X_{n,M_{n-1}}), \quad n \geq 1.$$

This process is called a maximal branching process in a random environment (MBPRE) and was introduced in [2].

In this work, we consider the case where the distribution function has the form

$$F_y(x) = 1 - \frac{c(y)}{x} + o\left(\frac{1}{x}\right), \quad x \rightarrow +\infty.$$

Depending on the value of $\mathbf{E} \ln c(\eta)$, MBPREs are classified as supercritical, critical, or subcritical. For this family of distribution functions F_y (under some additional assumptions),

we obtain the asymptotic behaviour of the extinction probability $\mathbf{P}(M_n > 0) \sim c/\sqrt{n}$ in the critical case, prove the central limit theorem, and find the asymptotics of large deviation probabilities in the supercritical case.

- [1] Lamperti J., *Maximal branching processes and long range percolation*, J. Appl. Probab. – 1970. – Vol. 7, No. 1. – P. 89–96.
- [2] Lebedev A. V., *Maximal branching processes in random environment*, arXiv:2104.08849.

On multipliers that make martingale square functions bounded

Anton Tselishchev

PDMI RAS, Saint Petersburg
celis_anton@pdmi.ras.ru

Let f be a function on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a fixed discrete filtration $(\mathcal{F}_n)$, $0 \leq f \leq 1$ a.e. It is well known that in this case, the martingale square function $S(f)$ lies in all L^p spaces for $1 < p < \infty$, but it is not necessarily bounded.

Nevertheless, we will show how to *explicitly construct* another function (a "multiplier") m , such that $0 \leq m \leq 1$, for which the product $m \cdot f$ has a bounded martingale square function: $\|S(m \cdot f)\|_\infty \leq C$, while at the same time this product is "not too small": $\mathbb{E}(f) \geq c\mathbb{E}(m \cdot f)$.

The case of a continuous-time filtration, as well as the case of a discrete dyadic filtration, were previously considered by Peter Jones and Paul Müller. We will briefly discuss the history of this problem and two approaches to its solution for the case of an arbitrary discrete filtration: a constructive one, which allows writing a more or less explicit formula for the function m , and a completely non-constructive one, which allows to impose more conditions on the function m but has its drawbacks.

Meeting, Coalescence and Consensus Times of Random Walks on Random Graphs

Roman Vasiliev

Lomonosov Moscow State University, Moscow
r.a.vasiliev1998@gmail.com

The study concerns meeting, coalescence, and consensus times of random walks on undirected random graphs. Three model classes are considered: random d -regular graphs, Erdős–Rényi graphs $G(n, p)$, and configuration models with power-law degree distributions. For regular and $G(n, p)$ graphs, we prove an exponential limit for the normalized meeting time of two independent walks started from stationarity:

$$\frac{T_{\text{meet}}}{E[T_{\text{meet}}]} \xrightarrow{d} \text{Exp}(1), \quad E[T_{\text{meet}}] \sim \frac{n}{d}.$$

In heavy-tailed settings, the expected time becomes sublinear, $E[T_{\text{meet}}] = o(n)$. These results imply asymptotics for coalescence and consensus times: $E[T_{\text{coal}}] \sim \frac{n}{d} \log m$ and $E[T_{\text{cons}}] \sim \frac{n}{d} \log n$. Comparison with previous work (Avena et al., Benjamini–Kozma–Wormald, Oliveira) shows consistency with known orders and refines asymptotic behavior for undirected models.

- [1] Avena L., Capannoli F., Hazra R. S., Quattropiani M., *Meeting, coalescence and consensus time on random directed graphs*, 2024.
- [2] Benjamini I., Kozma G., Wormald N., *The mixing time of the giant component of a random graph*, 2016.
- [3] Oliveira R. I., *Mean field conditions for coalescing random walks*, 2013.

Characterization geometry distribution with k-th records

Bogdan Yakovlev

Saint Petersburg State University, Saint Petersburg
 bogdanrnd1@gmail.com

Let $X_1, X_2, \dots$ – iid values (distributed like X), with $P(X_1 < n) < 1$.

Let

$$L(0) = 1,$$

$$L(n+1) = \min\{j > L(n) | X_j > X_{L(n)}\}$$

Define $R_n(X) = X_{L(n)}$ as n -th strong record and $geom(\beta)$ is a geometry distribution with support natural numbers with 0.

Let $A_k(\beta)$ – distribution generated by k -th record of $geom(\beta)$. Then there are results:

- 1) Let $k \geq 2$, X – random value with natural numbers with 0, the result approve that if

$$R_k(X) \sim A_k(\beta_1), R_{k-1}(X) \sim A_{k-1}(\beta_2),$$

then

$$\beta_1 = \beta_2, X \sim geom(\beta_1).$$

- 2) We can describe class of distributions X such that $R_1(X) = A_1(\beta)$ for fixed $\beta \in (0, 1)$.

LOCAL INSTITUTIONS PARTICIPATING IN THE ORGANIZATION OF THE EVENT

- St. Petersburg Department of Steklov Mathematical Institute of the Russian Academy of Sciences
- Leonhard Euler International Mathematical Institute in St. Petersburg
- St. Petersburg University
- Steklov International Mathematical Center
- Steklov Mathematical Institute of Russian Academy of Sciences



Steklov International Mathematical Center



The conference is financially supported by a grant of the Ministry of Education and Science of the Russian Federation for the establishment and development of the Leonhard Euler International Mathematical Institute in Saint Petersburg, agreements 075-15-2025-343, 075-15-2025-344 and the grant to the Steklov International Mathematical Center, agreement no. 075-15-2025-303.