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Book of Abstracts

Fri, Jun 26
12:35–13:00

On the location of eigenvalues in the one-dimensional involutive Friedrichs model

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The involutive Friedrichs model is described by the operator

$$Af(x) = ixf(-x) + \int_{-1}^{1} K(x,y)f(y)\,dy$$

acting in $L_2[-1,1]$.

In [1] it was shown for a more general case that if K is Hölder continuous with exponent $\alpha > 1/2$ and vanishes on $\partial[-1,1]^2$, then the operator A can only have finite number of discrete eigenvalues. In the one-dimensional case $K(x,y) = k(x)k(y)$, however, this holds for just $k \in C[-1,1]$. We will independently show this for our particular case and discuss the problem of eigenvalues lying on the essential spectrum $[-1,1]$.

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Tue, Jun 23
12:00–12:30

Short-wave asymptotic solution of the Cauchy problem for the one-dimensional wave equation with a smoothed velocity jump

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In this work, we study the wave equation whose velocity has a localized perturbation at some point x_0. The initial condition is a rapidly oscillating wave packet whose wavelength is not comparable to the scale of the inhomogeneity.

We will analyze the general situation for the one-dimensional wave equation, where the characteristic wavelength of the initial perturbation is described by a small parameter ε, and the characteristic width of inhomogeneity is described by the parameter ε^α, where α is any positive number.

Thus, we consider the problem

$$\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{x-x_0}{\varepsilon^\alpha}, x\right)\frac{\partial^2 u}{\partial x^2}, \ \ x \in \mathbb{R}, \quad u|_{t=0} = \varphi^0(x)e^{iS_0(x)/\varepsilon}, \quad u_t|_{t=0} = 0, \quad \varepsilon \to 0. \tag{1}$$

Here the function $c(y,x)\colon \mathbb{R}^2 \to \mathbb{R}$ ($y = (x-x_0)/\varepsilon^\alpha$) is smooth and strictly positive, and $c(y,x) \to c^\pm(x)$ as $y \to \pm\infty$ faster than any power of y, together with all its derivatives. The last condition reflects the localized nature of the inhomogeneity; we assume that the functions $c^\pm(x)$ are also smooth and positive. We assume that S_0, φ^0 are smooth functions, with φ^0 compactly supported, $\partial S_0/\partial x|_{\mathrm{supp}\varphi^0} \neq 0$, and the initial wave packet is located outside the localized inhomogeneity, i.e., $x|_{\mathrm{supp}\varphi^0} < 0$. The main result is a description of the scattering of the packet as it passes through the point x_0, the support of the inhomogeneity. The absence of focal points allows us to write simple analytical formulas for the asymptotics and to find an equation describing the transmission coefficient of the wave through the inhomogeneity. In our case, a reflected wave packet is absent for $\alpha < 1$ and appears for $\alpha \geq 1$.

Many works are devoted to equations with rapidly varying coefficients; introducing the fast variable $y = (x-x_0)/\varepsilon^\alpha$ transforms the right-hand side of the wave operator into an operator of the form

$$\frac{1}{\varepsilon^2}c^2(y,x)\Big(\varepsilon\frac{\partial}{\partial x} + \varepsilon^{1-\alpha}\frac{\partial}{\partial y}\Big)^2,$$

whose leading ε-symbol (after multiplication by ε^2) is the operator $-c^2(x,y)(p - i/\varepsilon^{1-\alpha}\partial/\partial y)^2$. The scheme of semiclassical asymptotics for equations with operator-valued symbols has been repeatedly applied to equations with rapidly varying coefficients (see, e.g., [1], [2], [3]); the key point of this scheme is the use of the eigenvalues of the symbol as classical Hamiltonians. In our case, the symbol is an operator with a small parameter, and its spectrum contains a continuous component. A general semiclassical theory for such a situation is absent; to describe the asymptotics, we use the considerations given in [4], as well as techniques developed for constructing soliton-like asymptotics of nonlinear equations (see, e.g., [5]). The asymptotics of the solution are expressed in terms of a "reference scattering problem" for an ordinary differential equation. Note that the parameter in front of the derivative $\partial/\partial y$ in the symbol tends to zero for $\alpha < 1$, to infinity for $\alpha > 1$, and equals one for $\alpha = 1$. The first situation corresponds to the semiclassical limit in the reference problem; in this case, scattering is determined by a transport equation in the "fast" variable y. The second situation corresponds to a small scattering potential; in this case, the solution of the reference problem can be represented as a series in regular perturbation theory; however, since the "effective energy" of scattering is also small, the series consists of non-smooth functions and its terms contain powers of the square root of the small parameter in front of the potential. Finally, the third situation (previously analyzed in [6]) corresponds to a reference problem containing no small parameter; the exact solution of this problem (in particular, the reflection and transmission coefficients) determines the scattering of semiclassical wave packets. The result of this work is an asymptotic series for the solution of the Cauchy problem (1). Since the asymptotic series looks different for $\alpha > 1$ and $\alpha < 1$, we will consider these two cases separately.

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Tue, Jun 23
10:50–11:35

Asymptotic eigenfunctions of the Laplace operator in an ellipse, generated by Birkhoff billiards with caustics in the form of hyperbolas

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It is well known that Birkhoff billiards in an ellipse are divided into two groups. The corresponding trajectories from the first group lie between co-focal ellipses, with the inner ellipse representing the caustic for such trajectories. The trajectories from the second group lie between two caustics in the form of hyperbolas. Both type of billiards generate asymptotic eigenfunctions of boundary value problems for the Laplace operator in an ellipse. The asymptotic eigenfunctions corresponding to the first type of billiards were constructed by V. F. Lazutkin in the form of the Maslov canonical operator, and are presented in particular in the book (V. F. Lazutkin, Springer-Verlag, 1993). In a recent paper by S. Y. Dobrokhotov, V. E. Nazaikinsky, A. V. Tyurin and A. V. Tsvetkova (Russian Journal of Mathematical Physics, 2025), these functions were presented in the form of Airy functions of a complex argument. Also an approach for constructing asymptotic eigenfunctions for boundary value problems based on non-compact Lagrangian manifolds was proposed there. In this talk, we discuss this approach for construction of effective formulas for asymptotic eigenfunctions in the ellipse that correspond to the second type of Birkhoff billiards, which have caustics in the form of hyperbolas.

This work was done together with A. V. Tyurin, and A. V. Tsvetkova. The part of S. Yu Dobrokhov work was supported by State Assignment (with the State registration number of the Assignment 124012500442-3) and the part of A. V. Tyurin, and A. V. Tsvetkova was supported by the grant of Theoretical Physics and Mathematics Advancement Foundation "BASIS" (grant 24-7-2-34).

Wed, Jun 24
12:50–13:20

Homogenization of hyperbolic equations with corrector

Mark A. Dorodnyi

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In $L_2(\mathbb{R}^d)$, consider the elliptic differential operator $A_\varepsilon = -\operatorname{div} g(\mathbf{x}/\varepsilon)\nabla$, $\varepsilon > 0$. Here $g(\mathbf{x})$ is a $\mathbb{Z}^d$-periodic, positive definite, bounded $(d\times d)$-matrix-valued function with real entries. We study the behavior of the solution of the Cauchy problem $u_\varepsilon(\mathbf{x},\tau)$, $\mathbf{x}\in\mathbb{R}^d$, $\tau\in\mathbb{R}$, $\varepsilon\to 0$, for the hyperbolic equation

$$\partial_\tau^2 u_\varepsilon(\mathbf{x},\tau) = -(A_\varepsilon u_\varepsilon)(\mathbf{x},\tau), \qquad u_\varepsilon(\mathbf{x},0)=\phi(\mathbf{x}), \qquad \partial_\tau u_\varepsilon(\mathbf{x},0)=\psi(\mathbf{x}). \quad (1)$$

A principal term of approximation for the solution of (1) was obtained in [1, 2]. Introduce the effective operator $A_0 = -\operatorname{div} g^0\nabla$ and the corresponding effective problem

$$\partial_\tau^2 u_0(\mathbf{x},\tau) = -(A_0 u_0)(\mathbf{x},\tau), \qquad u_0(\mathbf{x},0)=\phi(\mathbf{x}), \qquad \partial_\tau u_0(\mathbf{x},0)=\psi(\mathbf{x}).$$

Here g^0 is a positive constant effective matrix. The following error estimate was obtained:

$$\|u_\varepsilon(\cdot,\tau)-u_0(\cdot,\tau)\|_{L_2(\mathbb{R}^d)} \le C(1+|\tau|)^{1/2}\varepsilon\Big(\|\phi\|_{H^{3/2}(\mathbb{R}^d)} + \|\psi\|_{H^{1/2}(\mathbb{R}^d)}\Big).$$

This talk is devoted to obtaining an $L_2(\mathbb{R}^d)$-approximation with an error $O(\varepsilon^2)$ with correctors taken into account. In [3], it was shown that such an approximation in terms of the threshold characteristics at the lower edge of the spectrum of the operator A_1 can be obtained only for the Cauchy problem with the initial data from a special class:

$$\partial_\tau^2 \tilde{u}_\varepsilon(\mathbf{x},\tau) = -(A_\varepsilon \tilde{u}_\varepsilon)(\mathbf{x},\tau), \qquad \tilde{u}_\varepsilon(\mathbf{x},0)=\tilde{\phi}_\varepsilon(\mathbf{x}), \qquad \partial_\tau \tilde{u}_\varepsilon(\mathbf{x},0)=\psi(\mathbf{x}),$$

where $\tilde{\phi}_\varepsilon(\mathbf{x}) = \phi(\mathbf{x}) + \varepsilon\sum_{j=1}^d \Psi_j(\mathbf{x}/\varepsilon)\partial_j\phi(\mathbf{x})$; Ψ_j is the weak $\mathbb{Z}^d$-periodic solution of the cell problem

$$\operatorname{div} g(\mathbf{x})(\nabla\Psi_j(\mathbf{x})+\mathbf{e}_j)=0, \qquad \int_{(0,1)^d}\Psi_j(\mathbf{x})\,d\mathbf{x}=0, \quad j=1,\dots,d;$$

and $\{\mathbf{e}_j\}_{j=1}^d$ is the canonical basis of $\mathbb{R}^d$. The following result was obtained:

$$\|\tilde{u}_\varepsilon(\cdot,\tau)-u_0(\cdot,\tau)-K_\varepsilon(\cdot,\tau)\|_{L_2(\mathbb{R}^d)} \le C(1+|\tau|)\varepsilon^2(\|\phi\|_{H^3(\mathbb{R}^d)} + \|\psi\|_{H^2(\mathbb{R}^d)}). \quad (2)$$

Here $K_\varepsilon(\mathbf{x},\tau) = \varepsilon\sum_{j=1}^d \Psi_j(\mathbf{x}/\varepsilon)\partial_j u_0(\mathbf{x},\tau)$ is the corrector.

In view of this result, the following question arises: can one find an $L_2(\mathbb{R}^d)$-approximation with an error $O(\varepsilon^2)$ for the solution of the Cauchy problem

$$\partial_\tau^2 \check{u}_\varepsilon(\mathbf{x},\tau) = -(A_\varepsilon \check{u}_\varepsilon)(\mathbf{x},\tau), \quad \check{u}_\varepsilon(\mathbf{x},0) = \varepsilon \sum_{j=1}^{d} \Psi_j(\mathbf{x}/\varepsilon)\partial_j \phi(\mathbf{x}), \quad \partial_\tau \check{u}_\varepsilon(\mathbf{x},0) = 0? \tag{3}$$

In [4], this question was studied in the one-dimensional case ($d = 1$). For the solution of (3) an effective approximation $\check{u}_\varepsilon^{\mathrm{eff}}$ was found:

$$\|\check{u}_\varepsilon(\cdot,\tau) - \check{u}_\varepsilon^{\mathrm{eff}}(\cdot,\tau)\|_{L_2(\mathbb{R})} \leq C(1+|\tau|)\varepsilon^2 \|\phi\|_{H^4(\mathbb{R})}.$$

The contribution to the effective approximation $\check{u}_\varepsilon^{\mathrm{eff}}$ comes from all edges of the "periodic" spectral gaps of the operator A_1. This result together with (2) yields an $L_2(\mathbb{R})$-approximation with an error $O(\varepsilon^2)$ for the solution of the original problem (1) in the one-dimensional case.

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Fri, Jun 26
15:00–15:25

Homogenization of the two-dimensional Dirac equation with periodic coefficients

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The talk concerns the homogenization theory of periodic differential operators. In $L_2(\mathbb{R}^2;\mathbb{C}^2)$ a two-dimensional Dirac operator with periodic rapidly oscillating singular magnetic potential and matrix potential is considered:

$$\mathcal{D}_\varepsilon = \left(D_1 - \frac{1}{\varepsilon}A_1\left(\frac{x}{\varepsilon}\right)\right)\sigma_1 + \left(D_2 - \frac{1}{\varepsilon}A_2\left(\frac{x}{\varepsilon}\right)\right)\sigma_2 + V\left(\frac{x}{\varepsilon}\right), \qquad \operatorname{Dom}\mathcal{D}_\varepsilon = H^1(\mathbb{R}^2;\mathbb{C}^2), \varepsilon > 0. \tag{1}$$

Here σ_j, $j = 1, 2$, are the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{2}$$

It is assumed that the magnetic potential $\mathbf{A}(x) = (A_1(x), A_2(x))^t$ satisfies the following conditions:

$$A_j(x+n) = A_j(x), \;\; x \in \mathbb{R}^2, n \in \mathbb{Z}^2, \quad A_j = \bar{A}_j \in L_{p,\mathrm{loc}}(\mathbb{R}^2), \;\; p > 2, j = 1, 2, \tag{3}$$

and the matrix potential $V(x)$ satisfies the following conditions:

$$V(x) = V^*(x) = \begin{pmatrix} V_{11}(x) & V_{12}(x) \\ V_{21}(x) & V_{22}(x) \end{pmatrix}, \quad V(x+n) = V(x), \;\; x \in \mathbb{R}^2, n \in \mathbb{Z}^2, \tag{4}$$

$$V_{ij} \in L_{p,\mathrm{loc}}(\mathbb{R}^2), \;\; p > 2, i, j = 1, 2. \tag{5}$$

Under those conditions the operator $\mathcal{D}_\varepsilon$ is self-adjoint. It is also assumed that the gauge conditions hold:

$$\frac{\partial A_1}{\partial x_1} + \frac{\partial A_2}{\partial x_2} = 0, \qquad \int_\Omega A_j(x)\,dx = 0, \quad j = 1, 2. \tag{6}$$

Here $\Omega = [0,1)^2$ is the cell of the lattice $\mathbb{Z}^2$.

We introduce necessary notation. Let $\varphi \in W_p^1(\mathbb{R}^2)$ be the $\mathbb{Z}^2$-periodic solution of the following problem:

$$\begin{cases} \nabla\varphi(x) = (A_2(x), -A_1(x))^t, \\ \int_\Omega \varphi(x)\,dx = 0. \end{cases} \tag{7}$$

Define $\mathbb{Z}^2$-periodic functions

$$\omega_\pm(x) = e^{\pm\varphi(x)}, \qquad \widehat{\omega}_\pm(x) = \frac{\omega_\pm(x)}{\|\omega_\pm(x)\|_{L_2(\Omega)}}. \tag{8}$$

Our first result is an approximation of the resolvent $(\mathcal{D}_\varepsilon - iI)^{-1}$ for small $\varepsilon > 0$.

Theorem ([1]). *Under conditions* (3), (4), (5), (6) *the following estimate holds:*

$$\|(\mathcal{D}_\varepsilon - iI)^{-1} - \Xi^\varepsilon(\mathcal{D}^0 - iI)^{-1}\Xi^\varepsilon\|_{L_2(\mathbb{R}^2)\to L_2(\mathbb{R}^2)} \le C\varepsilon, \qquad \varepsilon > 0. \tag{9}$$

Here $\mathcal{D}^0 = \gamma(D_1\sigma_1 + D_2\sigma_2) + V^0$ *is the effective operator,*

$$\begin{gathered} \Xi(x) = \begin{pmatrix} \widehat{\omega}_-(x) & 0 \\ 0 & \widehat{\omega}_+(x) \end{pmatrix}, \qquad \Xi^\varepsilon(x) = \Xi\left(\frac{x}{\varepsilon}\right); \\ V^0 = \int_\Omega \Xi(x)V(x)\Xi(x)\,dx; \\ \gamma = \int_\Omega \widehat{\omega}_+(x)\widehat{\omega}_-(x)\,dx = \|\omega_+\|^{-1}_{L_2(\Omega)}\|\omega_-\|^{-1}_{L_2(\Omega)}. \end{gathered} \tag{10}$$

The constant C depends only on p, $\|\mathbf{A}\|_{L_p(\Omega)}$ and $\|V\|_{L_p(\Omega)}$.

Our second result is an approximation of the solution of a two-dimensional Dirac equation for small $\varepsilon > 0$:

$$i\frac{\partial u_\varepsilon(x,t)}{\partial t} = \mathcal{D}_\varepsilon u_\varepsilon(x,t), \qquad u_\varepsilon(x,0) = \Xi^\varepsilon(x)v(x), \tag{11}$$

where $v \in L_2(\mathbb{R}^2;\mathbb{C}^2)$ is a given function. Define the *effective Dirac equation*:

$$i\frac{\partial u_0(x,t)}{\partial t} = \mathcal{D}^0 u_0(x,t), \qquad u_0(x,0) = v(x). \tag{12}$$

Theorem. *If $v \in H^2(\mathbb{R}^2;\mathbb{C}^2)$, then the following estimate holds:*

$$\|u_\varepsilon(\cdot,t) - \Xi^\varepsilon u_0(\cdot,t)\|_{L_2(\mathbb{R}^2)} \le (C_1 + C_2|t|)\varepsilon\|v\|_{H^2(\mathbb{R}^2)}, \qquad t \in \mathbb{R}, \varepsilon > 0. \tag{13}$$

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Wed, Jun 24
15:00–15:45

On the minima of the first band function of a periodic polyharmonic operator

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Consider in $L_2(\mathbb{R}^d)$ an operator

$$H = (-\Delta)^l + V(x),$$

where l is a natural number, and the potential V is assumed to be periodic with respect to a lattice Γ. The spectrum of H has a band-gap structure,

$$\operatorname{spec}(H) = \bigcup_{j=1}^{\infty} \lambda_j(\overline{\tilde{\Omega}}).$$

Here $\tilde{\Omega}$ is a Brillouin zone of the lattice Γ, and $\lambda_j(k)$ are the eigenvalues of the operators $H(k)$ acting on the cell Ω with quasiperiodic boundary conditions that depend on the quasimomentum k. We are interested in the structure of the bottom of the spectrum of H, i.e. the behaviour of the function λ_1 near its minimum. It is well known that if $l = 1$ then the minimum of λ_1 in $\overline{\tilde{\Omega}}$ is attained at the unique point $k = 0$. It turns out that for $l > 1$ this is not the case. We show that under assumption $d \le 2l + 1$ and for all sufficiently small potential V, $V(x) \not\equiv \text{const}$, the minimum of the first band function λ_1 is attained at least at two points.

Wed, Jun 24 17:50–18:20

Embedding constants in Sobolev spaces with admissible boundary conditions

Tatiana Garmanova

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For a finite set of continuous linear functionals $\mathcal{U}=\{F_1,\ldots,F_m\}$, $F_i : W_1^n[0,1]\to\mathbb{R}$, we consider the space

$$W^n_{p,\mathcal{U}}[0,1] := \Big\{ f \in W^n_p[0,1] \;\Big|\; f \in \bigcap_{i=1}^{m} \ker F_i \Big\}.$$

The main objects of study are the sharp embedding constants for the embeddings $W^n_{p,\mathcal{U}_1}[0,1] \hookrightarrow W^k_{\infty,\mathcal{U}_2}[0,1]$,

$$\Lambda_{n,k,p,\infty,\mathcal{U}} := \sup_{f\in W^n_{p,\mathcal{U}_1}[0,1]\setminus\{0\}} \frac{\|f^{(k)}\|_{L_\infty[0,1]}}{\|f^{(n)}\|_{L_p[0,1]}}, \qquad p\in[1,\infty], 0\le k\le n-1.$$

Different choices of $\mathcal{U}$ cover a wide range of boundary conditions. To date such constants have been studied only in isolated cases – particular values of the parameters under specially chosen boundary conditions (primarily the Dirichlet conditions).

Definition. A set $\mathcal{U} = \{F_1,\ldots,F_m\}$ of continuous linear functionals on $W_1^n[0,1]$ is called admissible if: (1) $F_1,\ldots,F_m$ are linearly independent over $W^n_p[0,1]$, $\forall p\in[1,\infty]$; (2) the system $F_1,\ldots,F_m$ is complete on the space $\mathcal{P}_{n-1}$ of polynomials of degree at most $n-1$. The space $W^n_{p,\mathcal{U}}[0,1]$ is then called a Sobolev space with admissible boundary conditions.

Let $\varphi(f) = f^{(n)}$ and $V_{p,\mathcal{U}} := \varphi(W^n_{p,\mathcal{U}}[0,1]) \subset L_p[0,1]$. We denote its annihilator by

$$\mathcal{F}_{\mathcal{U}} := V^{\perp}_{p,\mathcal{U}} = \Big\{ h\in L_{p'}[0,1] \;\Big|\; \int_0^1 f^{(n)}(x)h(x)\,dx = 0,\ \forall f\in W^n_{p,\mathcal{U}}[0,1] \Big\},\ \ \frac{1}{p}+\frac{1}{p'}=1.$$

Theorem. *For an admissible set $\mathcal{U}$,* $\dim \mathcal{F}_{\mathcal{U}} = |\mathcal{U}| - n$.

Theorem. *For an admissible $\mathcal{U}$ and any $a\in[0,1]$ there exists $q^{(n)}_{n,k}(\,\cdot\,,a)\in L_{p'}[0,1]$ such that*

$$f^{(k)}(a) = \int_0^1 f^{(n)}(x) q^{(n)}_{n,k}(x,a)\,dx, \qquad \forall f\in W^n_{p,\mathcal{U}}[0,1].$$

We say that the boundary conditions $\{\mathcal{U},\mathcal{U}_0\}$ are consistent if there is a continuous embedding $W^n_{p,\mathcal{U}}[0,1] \hookrightarrow W^k_{\infty,\mathcal{U}_0}[0,1]$. The central result reduces the computation of the embedding constant to a best-approximation problem.

Theorem. *Let* $\{\mathcal{U}, \mathcal{U}_0\}$ *be a consistent pair of admissible boundary conditions. Then*

$$\Lambda_{n,k,p,\infty,\mathcal{U}} = \sup_{a\in[0,1]} \inf_{u\in\mathcal{F}_{\mathcal{U}}} \|q^{(n)}_{n,k}(\,\cdot\,,a) + u(\cdot)\|_{L_{p'}[0,1]}.$$

Fri, Jun 26
16:20–16:45

Asymptotics of the Kelvin wedge for water waves generated by a moving localized disturbance

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The mathematical description of ship waves – surface water waves generated by a moving disturbance – goes back to the classical works of Kelvin (e.g., [1]), who modelled the disturbance as an external pressure applied to the free surface. Kelvin showed that the wave field is confined within a characteristic wedge (the Kelvin wedge). The asymptotics of the wave field near the wedge boundaries in the case of infinite depth were later derived by Ursell [2] for a point disturbance.

In the present talk, we revisit this classical problem for a surface pressure localized near a moving point, and derive effective asymptotic formulas for the free-surface elevation. The results consist of two parts.

In the first part, we study the asymptotics of the free-surface elevation far from a uniformly moving source for both finite and infinite constant depth. We obtain explicit analytical expressions describing the wave field inside the Kelvin wedge and near its boundaries.

In the second part, we consider the general case of a curvilinear source trajectory and slowly varying depth. The asymptotics of the free-surface elevation are expressed via the Maslov canonical operator [3], which reconstructs the wave field from the geometric data of the problem (rays, wavefronts, and amplitudes). Although the resulting expressions are not given in closed form, they provide a complete geometric description of the wave field, including a generalization of the Kelvin wedge to the case of variable depth and curvilinear motion (the "curvilinear wedge").

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Fri, Jun 26
13:05–13:30

On the completeness of root functions for systems of first-order ordinary differential equations

Alexey Kosarev

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We consider the spectral problem on the interval $[0,1]$ for a first-order $n \times n$ system of ordinary differential equations with two-point boundary conditions

$$y' - B(x)y = \lambda A(x)y, \qquad U_0 y(0) + U_1 y(1) = 0,$$

where λ is the complex spectral parameter, $A(x) = \operatorname{diag}\{a_1(x), \dots, a_n(x)\}$, and U_0, U_1 are constant $n \times n$ matrices. The entries of the matrices $A(x)$ and $B(x)$ are assumed to be complex-valued and integrable. Our main focus is on the case in which the diagonal entries of A are non-collinear, that is, they cannot be written in the form $a_k(x) = \alpha_k \psi(x)$ for some common function ψ.

In the first part of the talk, using asymptotic representations of the fundamental matrix of solutions for large $|\lambda|$, we introduce the notions of regularity and almost regularity of the problem in a sector and along a ray, as well as in a double sector and along two lines. We also introduce the definiteness condition in a sector Λ: one can choose a permutation σ of the set $\{1, \dots, n\}$ and an integer $s \in \{0, 1, \dots, n\}$ such that

$$\operatorname{Re}(\lambda a_{\sigma(1)}(x)) \le \dots \le \operatorname{Re}(\lambda a_{\sigma(s)}(x)) \le 0 \le \operatorname{Re}(\lambda a_{\sigma(s+1)}(x)) \le \dots \le \operatorname{Re}(\lambda a_{\sigma(n)}(x))$$

for all $\lambda \in \Lambda$ and for a.e. $x \in [0,1]$. These concepts naturally generalize the classical Birkhoff–Tamarkin regularity conditions and make it possible to deal with a wider class of systems.

In the second part, we present results on the completeness of the system of root functions. The main result states that if the definiteness condition holds and the boundary value problem under consideration is regular or almost regular either in a double sector or along two lines in the complex plane, then the system of its eigenfunctions and associated functions is complete in the space $(L_p[0,1])^n$ for $1 \le p < \infty$.

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Fri, Jun 26
10:50–11:35

Time-dependent Schrödinger equation for harmonic oscillator in the Aharonov–Bohm magnetic field

Ari Laptev

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We construct an approximation of the kernel of the solution of the time dependent Schrödinger equation whose Hamiltonian is a 2D harmonic oscillator in Aharonov–Bohm magnetic field. The main tools used here were established in the paper of A. Laptev and I. M. Sigal [2] and also in [1], where the authors considered a class of Fourier Integral Operators with global complex phases approximating the fundamental solutions (propagators) for time-dependent Schrödinger equations. For the example considered in this paper we are able to find the main term in the approximation of the kernel that equals a version of the Mehler formula.

This is a joint paper [3] with my PhD student Jiyu Fan at Imperial College London.

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Fri, Jun 26
10:00–10:45

On the eigenfunctions of the discrete and essential (continuous) spectrum in the problem of scattering of three identical quantum particles on a line with point interaction in pairs

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The report discusses a quantum Hamiltonian, namely, a semi-bounded self-adjoint operator, which is related to the problem of scattering of three identical particles on a line with point interaction in pairs, or, in other words, with a delta-functional singular potential of interaction. The potential support coincides with a symmetric star graph, which is the union of three straight lines passing through one point on a two-dimensional plane. Using symmetry, we show that such a model is explicitly solvable. This means that the eigenfunctions of the discrete spectrum and the generalized eigenfunctions of the essential (absolutely continuous) spectrum are calculated explicitly, that is, in quadratures.

Our approach is related to the Fourier transform of functions on a cyclic symmetry group, incomplete separation of variables, reduction to a functional equation of the 'Maryland model' type and its solution. Some physical interpretation of asymptotics of the eigenfunctions from the point of view of the diffraction theory is proposed.

The work is supported by the grant of the Russian Science Foundation 22-11-00070-P.

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Wed, Jun 24 15:50–16:35

On the asymptotics of solutions to systems of second-order differential equations and its applications

Karakhan Mirzoev[†], Tatiana Safonova[‡]

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Let $P(x)$ and $Q(x)$ be matrix-functions on the set $I \coloneqq [1; +\infty)$ such that $P(x)$ is a non-degenerate matrix-function, $P^{-1}(x) = (p_{ij}(x))$ and $Q(x) = (q_{ij}(x))$ $(i, j = 1, 2, \ldots, n)$ are Hermitian matrix-functions of order n, $n \in \mathbb{N}$, defined and measurable on I, and the functions $p_{ij}(x)$ and $q_{ij}(x)$ are locally summable on I $(p_{ij}, q_{ij} \in L^1_{\text{loc}}(I))$.

The listed conditions allow us to define the quasi-derivatives of a given locally absolutely continuous vector-function $y = (y_1(x), y_2(x), \ldots, y_n(x))^t$ $(y \in AC_{\text{loc}}(I)$, t is the transpose symbol), using the matrices P and Q, by setting

$$y^{[0]} \coloneqq y, \qquad y^{[1]} \coloneqq Py', \qquad y^{[2]} \coloneqq (y^{[1]})' - Qy,$$

and a quasi-differential expression, by setting

$$l[y](x) \coloneqq -y^{[2]}(x) = -(Py')' + Qy, \qquad x \in I.$$

(in the definition of $y^{[2]}$, it is assumed that $y^{[1]} \in AC_{\text{loc}}(I)$).

The expression $l[y]$ defines a minimal closed symmetric operator L_0 in the Hilbert space of quadratically integrable n-component vector-functions $\mathcal{L}^2_n(I)$.

This work is devoted to the asymptotics of solutions at infinity of the equation

$$l[y](x) \coloneqq \lambda y, \tag{1}$$

where λ is a fixed complex number.

Note that equation (1) is equivalent to the system of first-order differential equations:

$$\mathbf{y}' = (F - \Lambda)\mathbf{y}, \tag{2}$$

where $\mathbf{y} = (y_1, y_2, \ldots, y_n, y_1^{[1]}, y_2^{[1]}, \ldots, y_n^{[1]})^t$,

$$F = \begin{pmatrix} O & P^{-1} \\ Q & O \end{pmatrix}, \qquad \Lambda = \begin{pmatrix} O & O \\ \lambda E & O \end{pmatrix},$$

and O and E are the zero and identity matrices of order n, respectively.

Next, suppose that the matrices $P^{-1}(x)$ and $Q(x)$ are represented as

$$P^{-1}(x) = x^{-\nu-2}(P_0 + P_1(x)), \qquad Q(x) = x^{\nu}(Q_0 + Q_1(x)),$$

where $\nu > 0$, P_0 and Q_0 are numerical matrices, $P_1(x)$ and $Q_1(x)$ satisfy the conditions

$$\int_1^{+\infty} \frac{\ln^r(x)}{x}|P_1(x)|dx < +\infty, \qquad \int_1^{+\infty} \frac{\ln^r(x)}{x}|Q_1(x)|dx < +\infty,$$

and $r+1$ is the maximum multiplicity of the characteristic root of the numerical matrix

$$A = \begin{pmatrix} 1/2E & P_0 \\ Q_0 & -(\nu+1/2)E \end{pmatrix}.$$

The listed conditions allow us to prove an analogue of S. A. Orlov's well-known theorem (see [1]) – to find the asymptotics of the fundamental system of solutions to equation (1), from which, in particular, the following theorem follows.

Theorem. *Let the matrices $P(x)$ and $Q(x)$ satisfy the conditions listed above. Then the maximum number of linearly independent solutions of the equation* (1) *belonging to $\mathcal{L}_n^2(I)$ is equal to the number of roots of the polynomial $\mathcal{F}(z,\nu) := \det(A - zE)$ lying in the region $\operatorname{Re} z < 0$ and does not depend on λ. Moreover, the spectrum of any self-adjoint extension of the operator L_0 is discrete.*

Next, let $n = 2$. In this case, the polynomial $\mathcal{F}(z,\nu)$ has the form

$$\mathcal{F}(z,\nu) = \left[\left(z+\frac{\nu}{2}\right)^2 - \left(\frac{\nu+1}{2}\right)^2\right]^2 - \left[\left(z+\frac{\nu}{2}\right)^2 - \left(\frac{\nu+1}{2}\right)^2\right] \times \operatorname{sp}(P_0Q_0)$$
$$+ \det P_0 \times \det Q_0.$$

We note that the polynomial $\mathcal{F}(z,\nu)$ is an arbitrary quadratic trinomial in $(z+\nu/2)^2$, so the number of roots of this polynomial lying in the left half-plane, by choosing the elements of the matrices P_0 and Q_0, can be made to be any of the numbers 2, 3 and 4.

Similar results in the scalar case were obtained in [2] and [3].

This work was financially supported by the Russian Science Foundation, project 25-11-00304.

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Tue, Jun 23
12:35–13:20

Lefschetz formula on end-periodic manifolds

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The classical Atiyah–Bott formula [1] expresses the Lefschetz number of a geometric endomorphism of an elliptic complex of (pseudo)differential operators on a closed smooth manifold as the sum of contributions of fixed points of the corresponding diffeomorphism, assuming that all fixed points are nondegenerate. These contributions explicitly depend only on the endomorphism itself but not on the operators forming the complex. This formula was generalized in [2] to the case of manifolds with conical singularities (or, equivalently, with cylindrical ends), where the contributions of interior fixed points are supplemented by those of fixed cylindrical ends (which already depend on the operators of the complex themselves). In recent decades, a number of papers devoted to elliptic theory on manifolds with periodic ends (e.g., see [3]–[6] and references therein) have appeared in the literature. In this talk, we present a Lefschetz formula for asymptotically periodic geometric endomorphisms of asymptotically periodic elliptic differential complexes on such manifolds. The results will be published in the forthcoming paper [7]. In the special case of manifolds with cylindrical ends, this formula covers (and strengthens) the results in [2].

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Thu, Jun 25
10:50–11:35

Fractional powers of operators as traces of operator-valued curves

Alexander I. Nazarov

St. Petersburg Department of Steklov Institute of Mathematics RAS & St. Petersburg State University

We relate non integer powers $\mathcal{L}^s$, $s > 0$, of a given (unbounded) positive self-adjoint operator $\mathcal{L}$ in a real separable Hilbert space $\mathcal{H}$ with a certain differential operator of order $2\lceil s\rceil$, acting on even functions $\mathbb{R} \to \mathcal{H}$. This extends the results by Caffarelli–Silvestre and Stinga–Torrea regarding the characterization of fractional powers $0 < s < 1$ of differential operators via an extension problem.

The talk is based on the paper [1] joint with Roberta Musina.

References

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Mon, Jun 22
15:00–15:45

Asymptotic structure of the spectra of water-waves in meromictic lakes and the Black sea

Sergei A. Nazarov

Institute of Mechanical Engineering Problems RAS

There are only 53 extraordinary meromictic (many-layered) lakes on the globe as well as the Black Sea, the biggest two-layered basin in the world. We examine surface and interface water-waves in such a pond $\Omega \cup \Gamma \cup \Omega_+^\varepsilon$ filled by two immiscible weighty liquids in the upper thin (the relative thickness $\varepsilon > 0$ is a small parameter) and lower massive heavier volumes $\Omega_+^\varepsilon = \omega \times (0, \varepsilon)$ and $\Omega_-^\varepsilon = \Omega \subset \mathbb{R}_-^3$. These waves are described through the following Steklov–Neumann spectral problem for the Laplace operator Δ_x

$$\begin{gathered} -\rho_\pm \Delta_x v_\pm^\varepsilon(x) = 0, \qquad x = (y, z) \in \Omega_\pm^\varepsilon, \\ \rho_\pm \partial_n v_\pm^\varepsilon(x) = 0, \qquad x \in \Sigma_\pm^\varepsilon, \\ \rho_+ \partial_z v_+^\varepsilon(x) = \rho_+ \lambda^\varepsilon v_+^\varepsilon(x), \qquad x \in \Gamma^\varepsilon, \\ \rho_+ \partial_z v_+^\varepsilon(x) - \rho_+ \lambda^\varepsilon v_+^\varepsilon(x) = \rho_- \partial_z v_-^\varepsilon(x) - \rho_- \lambda^\varepsilon v_-^\varepsilon(x), \;\; \partial_z v_+^\varepsilon(x) = \partial_z v_+^\varepsilon(x), \;\; x \in \Gamma. \end{gathered}$$

Here, λ^ε is the spectral parameter, $v_\pm^\varepsilon$ are the velocity potentials and $\rho_- > \rho_+ > 0$ are the densities. Furthermore, $\Gamma^\varepsilon = \omega \times \{\varepsilon\}$ and

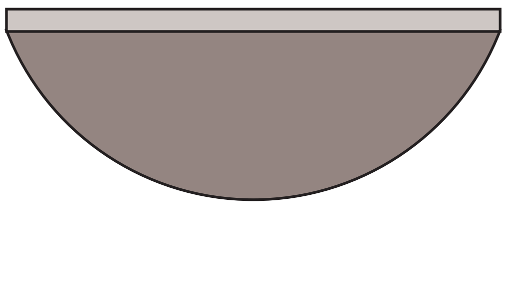
$$\Gamma = \Big\{ x = (y, z) \in \mathbb{R}^2 \times \mathbb{R} \colon y = (y_1, y_2) \in \omega, z = 0 \Big\}$$

are the free surface and interface while ∂_n is the outward normal derivative at the walls and the bottom $\Sigma_-^\varepsilon \cup \Sigma_+^\varepsilon$ but $\partial_n = \partial_z$ at Γ^ε.

Asymptotic analyses as $\varepsilon \to +0$ lead to different limit problems in the low-frequency $\lambda^\varepsilon = O(\varepsilon)$ and the mid-frequency $\lambda^\varepsilon = O(1)$ ranges of the spectrum

$$0 = \lambda_1^\varepsilon = \lambda_2^\varepsilon < \lambda_3^\varepsilon \le \lambda_4^\varepsilon \le \cdots \le \cdots \le \lambda_k^\varepsilon \le \cdots \to +\infty.$$

Those are the planar Neumann spectral problem in the cross-section $\omega \subset \mathbb{R}^2$ and the spatial Steklov–Neumann problem in the lower bulk $\Omega \subset \mathbb{R}_-^3$, respectively.



Moreover, the derived limit problems prescribe absolutely different behaviors of the corresponding water-wave modes in the low- and mid-frequency ranges. Obtained asymptotic formulas explain quite many miscellaneous interesting, useful and catastrophic, phenomena occurring in the meromictic ponds. Mentioning several effects of meromixis, the most attention is paid to events in Arctic and Pacific Oceans within Crimean War 1853–1856.

Mon, Jun 22
17:00–17:45

Analytic Schur multipliers and applications to functions of perturbed dissipative operators

Vladimir V. Peller

St. Petersburg State University

I am going to speak about recent joint results with A. B. Aleksandrov.

It will be announced that if a function of two variables is a Schur multiplier and it is an analytic function of two variables, then it can be represented in the form of a Haagerup tensor products of bounded functions that are analytic in one variable.

This result will be used to obtain representations of the operators of the form $f(M) - f(L)$, where L and M are maximal dissipative operators such that $M - L$ is a relatively bounded perturbation of L. This leads to sharp estimates for the norms $\|f(M) - f(L)\|$. In the case when $M - L$ is a relatively trace class perturbation of L an analogue of the Lifshits–Krein trace formula will be obtained.

Wed, Jun 24
10:00–10:45

Orlicz-type spaces generated by convolution type integral functionals

Andrey L. Piatnitski

Higher School of Modern Mathematics MIPT & The Arctic University of Norway

The talk will focus on the main properties of Orlitz-type functional spaces generated by integral functionals of the form

$$F(u) = \int_{\Omega\times\Omega} \frac{a(x-y)b(x,y)}{p(x,y)} |u(x) - u(y)|^{p(x,y)}\, dx\, dy \tag{1}$$

or of more general form

$$F(u) = \int_{\Omega\times\Omega} a(x-y)\varphi(|u(x) - u(y)|, x, y)\, dx\, dy; \tag{2}$$

here Ω is a regular domain in $\mathbb{R}^d$ or the whole $\mathbb{R}^d$, $d \geq 1$, and the convolution kernel $a(\cdot)$ is a non-negative integrable function in $\mathbb{R}^d$. If F is given by (1), we assume that $b(x, y)$ is a positive bounded measurable function, and $p(x, y)$ satisfies the inequality

$$1 < p_- \leq p(x, y) \leq p_+.$$

In the case of F given by (2) we impose appropriate growth and convexity assumptions on the function $\varphi(\xi, x, y)$.

We define the functional spaces

$$\mathcal{L} = \Big\{ u \in L^1_{\rm loc}(\mathbb{R}^d) \colon F(u) + \|u\|^{p_-}_{L^{p_-}} < +\infty \Big\}$$

equipped with the Luxemburg norm and then study the properties of these spaces. In particular, we show that $\mathcal{L}$ is a separable Banach space and that, under natural assumptions on φ, $C_0^\infty(\Omega)$ is dense in $\mathcal{L}$. We also characterize the dual spaces.

This is a joint work with Denis Borisov (Ufa).

Mon, Jun 22
17:50–18:20

Existence of non-radial extremal functions for Hardy–Sobolev inequalities in non-convex cones

Nikita Rastegaev

St. Petersburg Department of Steklov Institute of Mathematics RAS

The symmetry breaking is obtained for Neumann problems for equations with p-Laplacian generated by the Hardy–Sobolev inequality:

$$\begin{cases} -\Delta_p u = \dfrac{u^{q-1}}{|x|^{(1-\sigma)q}} & \text{in } \Sigma_D, \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Sigma_D, \\ u > 0 & \text{in } \Sigma_D. \end{cases}$$

Here $1 < p < n$, $n \geq 3$, $0 < \sigma \leq 1$, $q = p^*_\sigma = np/(n - \sigma p)$, $D \subset \mathbb{S}^{n-1}$ and

$$\Sigma_D = \{xt \colon x \in D, t \in (0, +\infty)\} \subset \mathbb{R}^n$$

is a non-convex cone. Such problems have obvious radial solutions – Talenti–Bliss functions of $|x|$. However, under a certain restriction on the first Neumann eigenvalue $\lambda_1(D)$ of the Beltrami–Laplace operator on D:

$$\lambda_1(D) < (1 - \alpha)(n - 1 - \alpha(p - 1)), \qquad \alpha = (1 - \sigma)\frac{q}{p},$$

we prove this radial solution cannot be the extremal function, therefore the minimizer must be non-radial.

In the case $p = 2$, $\sigma = 1$ this problem was investigated in [1].

This research was supported by the Theoretical Physics and Mathematics Advancement Foundation “BASIS”.

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Tue, Jun 23
17:00–17:25

On formulas for the sums of some Euler–Zager multiplicative series

Tatiana Safonova

Northern (Arctic) Federal University

The following formulas are well known

$$\frac{\sin \pi z}{\pi z} = 1 - \frac{(\pi z)^2}{3!} + \frac{(\pi z)^4}{5!} + \cdots = \prod_{n=1}^{+\infty}\Big(1 - \frac{z^2}{n^2}\Big).$$

Following L. Euler (1734), it is easy to extract the identity from them

$$\sum_{1\le j_1<j_2<\ldots<j_k<+\infty} \prod_{l=1}^{k} \frac{1}{j_l^2} = \frac{\pi^{2k}}{(2k+1)!},$$

which is the solution of the famous Basel problem when $k = 1$.

The goal of this work is to generalize this equality, namely, to obtain similar formulas for sums of the form

$$\mathcal{U}_{2nk} = (-1)^{(n+1)k} \sum_{1\le j_1<j_2<\cdots<j_k<+\infty} \prod_{l=1}^{k} \frac{1}{(2j_l)^{2n}},$$

$$\mathcal{V}_{2nk} = (-1)^{(n+1)k} \sum_{1\le j_1<j_2<\cdots<j_k<+\infty} \prod_{l=1}^{k} \frac{1}{(2j_l-1)^{2n}}$$

for $n \in \mathcal{N}$ and $k = 0, 1, \ldots$ ($\mathcal{U}_0 := 1$, $\mathcal{V}_0 := 1$).

The method of this work is based on the spectral theory of ordinary differential operators on a segment.

Let $n \ge 2$ be a fixed natural number. The cardinality of the set of all possible mappings from the set $\{1, 2, \ldots, n-1\}$ to the set $\{0, 1\}$ is 2^{n-1}. Let us number the elements of this set with symbols m_s and define the numbers $a_s (= a_s(n))$ by

$$a_s := \sum_{j=1}^{n-1} (-1)^{m_s(j)} \varepsilon^j, \qquad s = 1, 2, \ldots, 2^{n-1},$$

where $\varepsilon := e^{i\pi/n}$, and i is the imaginary unit (in the case $n = 1$, we assume that $a_1 = 0$). The following theorem is valid.

Theorem. *For $n = 1, 2, \ldots$, the following equalities hold*

$$\mathcal{U}_{2nk} = \frac{-i^{n+1}\pi^{2nk}}{2^{2nk+n-1}(2nk+n)!} \sum_{s=1}^{2^{n-1}} (-1)^{\nu(s)} (1 + a_s)^{2nk+n}, \qquad k = 0, 1, \ldots,$$

$$\mathcal{V}_{2nk} = \frac{\pi^{2nk}}{2^{2nk+n-1}(2nk)!} \sum_{s=1}^{2^{n-1}} (1 + a_s)^{2nk}, \qquad k = 1, 2, \ldots,$$

where the numbers $\nu(s)$ are such that if $n=1$, then $\nu(1)=0$, and if $n\geq 2$, then $\nu(s)=0$ when the number of terms with a plus sign in a_s is even, and $\nu(s)=1$ otherwise.

The symbol $\zeta(N_1,N_2,\ldots,N_k)$ is used to denote the sum

$$\zeta(N_1,N_2,\ldots,N_k)s := \sum_{1\leq i_1<i_2<\cdots<i_k<+\infty}\frac{1}{i_1^{N_1}i_2^{N_2}\cdots i_k^{N_k}},$$

where $N_1,N_2,\ldots,N_k$ are natural numbers and $N_1>1$, and call them k-multiple values of the Euler–Zagier zeta function. Similarly, we define the k-fold values of the Dirichlet lambda function by setting

$$\lambda(N_1,N_2,\ldots,N_k) := \sum_{1\leq i_1<i_2<\cdots<i_k<+\infty}\frac{1}{(2i_1-1)^{N_1}\cdots(2i_k-1)^{N_k}}.$$

If $N_1=N_2=\cdots=N_k=N$, then these sums will be denoted by the symbols $\zeta(\{N\}_k)$ and $\lambda(\{N\}_k)$, respectively. The following statement follows from the theorem.

Corollary. *For $n,k=1,2,\ldots$, the following equalities hold*

$$\zeta(\{2n\}_k)=\frac{(-1)^{(n+1)k+1}i^{n+1}\pi^{2nk}}{2^{n-1}(2nk+n)!}\sum_{s=1}^{2^{n-1}}(-1)^{\nu(s)}(1+a_s)^{2nk+n},$$

$$\lambda(\{2n\}_k)=\frac{(-1)^{(n+1)k}\pi^{2nk}}{2^{2nk+n-1}(2nk)!}\sum_{s=1}^{2^{n-1}}(1+a_s)^{2nk}.$$

In addition, the following equalities hold for the sums on the right-hand sides of these identities for $n,j=1,2,\ldots$

$$\sum_{s=1}^{2^{n-1}}(-1)^{\nu_s}(1+a_s)^{2nj+n}$$

$$=\frac{(-1)^{n+j-1}(2i)^{n-1}n!}{j!}\begin{vmatrix} f_{11} & 1 & 0 & \cdots & 0\\ f_{22} & f_{21} & 2 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ f_{j-1,j-1} & f_{j-1,j-2} & f_{j-1,j-3} & \cdots & j-1\\ f_{j,j} & f_{j,j-1} & f_{j,j-2} & \cdots & f_{j1}\end{vmatrix}$$

and

$$\sum_{s=1}^{2^{n-1}}(1+a_s)^{2nj}=\frac{(-1)^{j}2^{n-1}}{j!}\begin{vmatrix} \varphi_{11} & 1 & 0 & \cdots & 0\\ \varphi_{22} & \varphi_{21} & 2 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \varphi_{j-1,j-1} & \varphi_{j-1,j-2} & \varphi_{j-1,j-3} & \cdots & j-1\\ \varphi_{j,j} & \varphi_{j,j-1} & \varphi_{j,j-2} & \cdots & \varphi_{j1}\end{vmatrix},$$

where

$$f_{kr} = \frac{(-1)^{nr}(2nk+n)!}{\pi^{2nr}(2n(k-r)+n)!}\zeta(2nr), \qquad \varphi_{kr} = \frac{(-1)^{nr}2^{2nr}(2nk)!}{\pi^{2nr}(2n(k-r))!}\lambda(2nr)$$

for $k = 1, 2, \ldots, j$ and $r = 1, 2, \ldots, k$.

Thus, from these equalities and the equivalences of the corollary, it can be deduced that

$$\zeta(\{2n\}_1) = \zeta(2n), \qquad \lambda(\{2n\}_1) = \lambda(2n),$$
$$\zeta(\{2n\}_2) = \frac{1}{2!}\Big(\zeta^2(2n) - \zeta(4n)\Big), \qquad \lambda(\{2n\}_2) = \frac{1}{2!}\Big(\lambda^2(2n) - \lambda(4n)\Big), \qquad \text{etc.}$$

Fri, Jun 26
15:30–15:55

Homogenization of elliptic fourth-order operators with periodic coefficients

Igor N. Safronov

St. Petersburg State University

In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider a fourth-order differential operator

$$\begin{aligned}\mathcal{A}_\varepsilon &= b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D}) \\ &+ \sum_{j,k,l=1}^{d} \Big(D_l D_k f^{(1)}_{jkl}(\mathbf{x}/\varepsilon) D_j + D_j (f^{(1)}_{jkl}(\mathbf{x}/\varepsilon))^* D_k D_l \Big) \\ &+ \sum_{j,k=1}^{d} \Big(D_k D_j f^{(2)}_{jk}(\mathbf{x}/\varepsilon) + (f^{(2)}_{jk}(\mathbf{x}/\varepsilon))^* D_j D_k \Big) \\ &+ \sum_{j,k=1}^{d} \Big(D_k f^{(3)}_{jk}(\mathbf{x}/\varepsilon) D_j + D_j (f^{(3)}_{jk}(\mathbf{x}/\varepsilon))^* D_k \Big) \\ &+ \sum_{j=1}^{d} \Big(D_j f^{(4)}_{j}(\mathbf{x}/\varepsilon) + (f^{(4)}_{j}(\mathbf{x}/\varepsilon))^* D_j \Big) \\ &+ f^{(5)}(\mathbf{x}/\varepsilon) + (f^{(5)}(\mathbf{x}/\varepsilon))^* + \sigma Q_0(\mathbf{x}/\varepsilon), \qquad \varepsilon > 0.\end{aligned}$$

Here $g(\mathbf{x})$ is a Hermitian $(m \times m)$-matrix-valued function in $\mathbb{R}^d$ such that $g, g^{-1} \in L_\infty$, $g(\mathbf{x}) > 0$, and $g(\mathbf{x})$ is periodic with respect to some lattice Γ. Next, $b(\mathbf{D})$ is a second-order differential operator; its symbol $b(\boldsymbol{\xi})$ is an $(m \times n)$-matrix-valued homogeneous function of degree 2 of $\boldsymbol{\xi} \in \mathbb{R}^d$ such that $\operatorname{rank} b(\boldsymbol{\xi}) = n$, $\boldsymbol{\xi} \neq 0$. We assume that $m \geq n$.

The coefficients $\{f^{(j)}_{\dots}(\mathbf{x})\}_{j=1}^{5}$ are Γ-periodic matrix-valued functions of size $n \times n$ such that $f^{(j)}_{\dots} \in L_{p^{(j)}}(\Omega)$. The corresponding exponents $p^{(j)}$ are determined depending on the dimension d according to the provided Table 1. In addition, the operator involves a bounded and positively definite $n \times n$ matrix-valued function $Q_0(\mathbf{x})$, as well as two parameters: $\varepsilon \in (0, \varepsilon_{\max}]$ and $\sigma > \sigma_0$, where $\varepsilon_{\max}$ and σ_0 are some explicit constants.

Table 1. *Admissible ranges for the exponents* $p^{(j)}$.

d	1 or 2	3 or 4	≥ 5
$p^{(1)}$	> 2	$\geq d$	$\geq d$
$p^{(2)}$	> 2	> 2	$\geq d/2$
$p^{(3)}$	> 1	$\geq d/2$	$\geq d/2$
$p^{(4)}$	> 1	$> 2d/(d+2)$	$\geq d/3$
$p^{(5)}$	> 1	> 1	$\geq d/4$

Strictly speaking, the operator $\mathcal{A}_\varepsilon$ is defined via a closed, positive definite quadratic form. The goal is to find an approximation of the resolvent $\mathcal{A}_\varepsilon^{-1}$ for small values of ε. The main result is formulated in the following theorem.

Theorem. *For $0 < \varepsilon \le \varepsilon_{\max}$, the following estimates hold:*

$$\|\mathcal{A}_\varepsilon^{-1} - \mathring{\mathcal{A}}^{-1}\|_{L_2(\mathbb{R}^d;\mathbb{C}^n)\to L_2(\mathbb{R}^d;\mathbb{C}^n)} \le C_1\varepsilon,$$
$$\|\mathcal{A}_\varepsilon^{-1} - \mathring{\mathcal{A}}^{-1} - \varepsilon^2\mathcal{K}_\varepsilon\Pi_\varepsilon\|_{L_2(\mathbb{R}^d;\mathbb{C}^n)\to H^2(\mathbb{R}^d;\mathbb{C}^n)} \le C_2\varepsilon.$$

The effective operator $\mathring{\mathcal{A}}$ is a fourth-order elliptic differential operator with constant coefficients, defined via the solutions of auxiliary problems on the cell. The corrector is a composition of the auxiliary smoothing operator Π_ε and the operator $\mathcal{K}_\varepsilon$, which is a pseudodifferential operator of order (-2) and is given in terms of $b(\mathbf{D})$, $\mathbf{D}$, $\mathring{\mathcal{A}}^{-1}$ and certain rapidly oscillating functions (solutions of the auxiliary problems).

The constants C_1 and C_2, as well as the threshold σ_0 and the upper bound $\varepsilon_{\max}$ for the parameters σ and ε, can be written out explicitly in terms of the problem data.

The method of investigation is based on the operator-theoretic approach developed by M. Sh. Birman and T. A. Suslina in [1]. An operator of arbitrarily high order, but without lower-order terms, was studied in [2], where similar estimates were obtained. An operator of the second order, but with lower-order terms, was considered in [3].

References

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Tue, Jun 23
15:50–16:35

Radiation principle for a two-dimensional acoustic diffraction grating

Oleg V. Sarafanov

St. Petersburg State University

We consider a two-dimensional reflection grating that occupies a “semi-plane” with smooth periodical boundary. The grating is described by a non-homogeneous elliptic boundary value problem for the stationary equation of acoustics with smooth coefficients varying periodically along the boundary of the grating with the same period. The problem is supposed to be formally selfadjoint with respect to the Green formula, and, far from the boundary, its coefficients stabilize to smooth periodic functions.

The radiation principle is the theorem on the existence of a solution to a boundary value problem in a grating such that the main part of its asymptotics contains only incoming or only outgoing waves. The report discusses the definition of incoming and outgoing waves in a grating, formulates the principle of radiation, and calculates the wave amplitudes in the asymptotics of the solution.

This work is supported by the Russian Science Foundation project 25-21-00533.

Tue, Jun 23
15:00–15:45

On the index of elliptic operators on manifolds with cylindrical ends

Anton Savin

RUDN University

Index theory of elliptic operators on manifolds with singularities has been studied in many papers. The case of Dirac-type operators has been studied most thoroughly. For general manifolds with singularities, conditions ensuring the Fredholm property are known, and some index formulas have been obtained. The popularity of this case is explained by the fact that, on the one hand, Dirac-type operators are used in most applications of index theory, and, on the other hand, the techniques of local index theory are applicable specifically to Dirac operators, allowing index calculations without using topological methods.

Significantly fewer papers are devoted to elliptic theory on manifolds with singularities for general operators (i.e., general differential operators or pseudodifferential operators). In many cases, calculi of the corresponding operators are known, Fredholm solvability of the operators is obtained (see, for example, works by Kondrat'ev, Melrose and Piazza, Plamenevskii and Senichkin, Egorov and Schulze, Lauter and Nistor, Nazaikinskii and Savin and Sternin). In particular, C^*-algebras of pseudodifferential operators and their symbols have been constructed, and the K-groups of these algebras have been calculated.

However, the index problem remains open even in the case of manifolds with the simplest singularities. The point here is that, in contrast with the classical Atiyah–Singer index theory on a closed smooth manifold, the algebra of symbols of pseudodifferential operators on manifolds with singularities is essentially noncommutative. Therefore the topological index is no longer defined by standard topological methods (these methods are inapplicable in this situation), but within the framework of noncommutative geometry [1], when topological invariants for elements of essentially noncommutative algebras are constructed. In this framework, in [2], an index formula for elliptic pseudodifferential operators on two-dimensional surfaces with conical points was obtained, and in [3], a method for obtaining an index formula on infinite cylinders of the form $\mathbb{R} \times \mathbb{T}^n$ was indicated. Here, it is necessary to note the close connection between elliptic theory on manifolds with singularities and on noncompact manifolds. In particular, the theory on the infinite cylinder $\mathbb{R} \times \mathbb{T}^n$ is related to the theory on the suspension of the torus $\mathbb{T}^n$, considered as a manifold with two conical points.

In this talk, we give an index formula for elliptic operators on manifolds with cylindrical ends of the form $\mathbb{R}_+ \times \mathbb{T}^n$. To this end, we first define the topological index of the problem as the sum of contributions to the index from: 1) the interior symbol (it is a function on the cotangent bundle of the manifold) and 2) the symbol at infinity (it is a pseudodifferential operator with a parameter in the Agranovich–Vishik sense on the torus $\mathbb{T}^n$). The first contribution is given by an Atiyah–Singer-type integral, and the second contribution is a modification of the Melrose eta-invariant. The greatest difficulty is the construction of the modified eta-invariant. This construction is based on a local algebraic index theorem for families. The proof of the index theorem follows the following scheme: first, we show that the index theorem is true on the cylinder $\mathbb{R} \times \mathbb{T}^n$ using simple deformations to translation-invariant operators, then, using topological K-theory, we show that on the manifolds under consideration, any elliptic operator stably satisfies the symmetry condition with twisting defined by the diffeomorphism of the torus $\mathbb{T}^n$, $x \mapsto -x$. The presence of symmetry enables us to apply the surgery method developed by Nazaikinskii and Sternin and reduce the index of the original operator to the indices of three auxiliary operators (on the twisted double of the original manifold, on the torus $\mathbb{T}^{n+1}$, and on the cylinder $\mathbb{R} \times \mathbb{T}^n$). The indices of these operators are calculated, and a direct comparison of the resulting expressions shows that we arrive precisely at the topological index defined earlier.

The results presented in the talk are obtained in the joint work with H. H. Abbas and K. N. Zhuikov [4]. This research was supported by grant 24-21-00336 from the Russian Science Foundation†.

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† `https://rscf.ru/project/24-21-00336/`

Mon, Jun 22
10:50–11:35

Local regularity theory of the Navier–Stokes equations and long-time behaviour of solutions to the Cauchy problem for the Stokes system with a certain drift

Gregory A. Seregin

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A necessary condition for the existence (or non-existence) of certain Type I blowups of weak solutions to the Navier–Stokes equations can be expressed as the Liouville-type theorem for the so-called mild bounded ancient solutions to the same equations. In the talk, a duality method reducing the above Liouville-type question to the long time behaviour of the solution to a certain Cauchy problem for the Stokes system with a slow decaying drift is going to be discussed.

Thu, Jun 25
10:00–10:45

Line bundles over Lagrangian manifolds, corresponding to asymptotic solutions of the wave equation with smoothed jump of the velocity

Andrei I. Shafarevich

Moscow State University

We study short-wave asymptotic solutions to the wave equations with smoothed jump of the velocity. We assume that the characteristic of the jump is of order of square root of the wavelength. It appears that such solutions can be described in terms of one-dimensional bundles over Lagrangian surfaces. Using such representation, we obtain complete asymptotic series for solution and compute asymptotics of coefficients of transition and reflection. This is a joint work with Anna Allilueva.

Wed, Jun 24
17:00–17:45

On a certain class of functions with wavelet representation and their application in spectral problems

Igor A. Sheipak

Moscow State University

We consider wavelet representation of some class of functions. In particular, the Weierstrass function and the Takagi–Landsberg family of functions allow such a representation. Special attention is paid to the functions of Takagi–van der Waerden family which is a special case of the more general de Rham curve. Furthermore, generalizations of this class are considered. It is shown that the wavelet representation of the Takagi–van der Waerden functions is directly related to affine self-similarity via lower triangular self-similarity matrices, but not all wavelet representations are affine self-similar.

The Hölder exponent of Takagi–van der Waerden functions is calculated. For some parameters of wavelet representation the asymptotics of eigenvalues for corresponding spectral problem for string equation, where weight is generalized derivative of Takagi–van der Waerden functions is obtained.

We now present our main result.

Consider a function of the form

$$T_{\mathfrak{d},n} := \sum_{k=0}^{\infty} d_k s(n^k x),$$

where $s(x) = \min_{n\in\mathbb{Z}} |x-n|$. We also consider a spectral problem for a generalized string of the form $-y'' = \lambda T'_{\mathfrak{d},n} y$, $x \in (0;1)$, $y(0) = y(1) = 0$.

Theorem. *If $\{d_k n^k\}_{k=0}^{\infty} \in l_2$, then $T'_{\mathfrak{d},n} \in L_1[0;1]$ and we have two series of eigenvalues (positive and negative) with asymptotics*

$$\lambda_m^{\pm} \sim \left(\frac{\pi m}{\int_0^1 \sqrt{(T'_{\mathfrak{d},n}(t))_{\pm}}\,dt}\right)^2, \qquad m \to \infty.$$

References

[1] V. A. Romanov and I. A. Sheipak, *Generalized family of Takagi functions in some spectral problems*, Algebra i Analiz, 38:1 2026, 235–246.

Mon, Jun 22
10:00–10:45

Direct and inverse problems for the Sturm–Liouville operator and its perturbations

Andrei A. Shkalikov

Moscow State University

In the first part of the talk, we will present a short overview of classical and recent results on the direct and inverse problems for the Sturm–Liouville operator on a finite interval.

In the second part, we consider the Sturm–Liouville operator perturbed by a convolution integral operator

$$L(q, M) = -y'' + q(x)y + \int_0^x M(x-t)y(t)\,dt,$$

assuming that q, M are complex valued and

$$q \in L_2(0,\pi) \qquad \text{and} \qquad M_0(x) = (\pi - x)M(x) \in L_2(0,\pi).$$

We associate three spectral problems generated by Dirichlet, Dirichlet–Neumann and Neumann boundary conditions for the operator $L(q, M)$. We obtain sharp asymptotic formulae for all these problems, which are new even in the case $M = 0$, and show that both functions q and M can be uniquely reconstructed from the three spectra of these problems, provided that the norms $\|q\|$ and $\|M\|$ are sufficiently small.

The talk is based on the joint work with V. N. Sivkin.

Wed, Jun 24
10:50–11:35

Homogenization of a $2p$-order elliptic operator with periodic coefficients in the energy norm

Vladimir Sloushch

St. Petersburg State University

In $L_2(\mathbb{R}^d;\mathbb{C}^n)$, we consider a strongly elliptic $2p$-order, $p \geq 2$, operator of the form $A_\varepsilon = b(\mathbf{D})^* g(\mathbf{x}/\varepsilon) b(\mathbf{D})$, where $\varepsilon > 0$ is a small parameter. Here $g(\mathbf{x})$ is a Hermitian $(m \times m)$-matrix-valued function; we assume that $g(\mathbf{x})$ is periodic with respect to some lattice, bounded and positive definite; $b(\mathbf{D})$, $\mathbf{D} = -i\nabla$, is a matrix differential operator given by $b(\mathbf{D}) = \sum_{|\alpha|=p} b_\alpha \mathbf{D}^\alpha$, b_α are $(m \times n)$-matrices. It is assumed that $m \geq n$ and the symbol $b(\boldsymbol{\xi}) = \sum_{|\alpha|=p} b_\alpha \boldsymbol{\xi}^\alpha$ has maximal rank for $\boldsymbol{\xi} \neq 0$. We obtain approximation for the resolvent $(A_\varepsilon + I)^{-1}$ in the "energy" norm, i. e., in the norm of operators acting from $L_2(\mathbb{R}^d;\mathbb{C}^n)$ to the Sobolev space $H^p(\mathbb{R}^d;\mathbb{C}^n)$:

$$\Big\| (A_\varepsilon + I)^{-1} - \Big((A^0 + I)^{-1} + \sum_{j=1}^{p-1} K_j(\varepsilon) \Big) \Big\|_{L_2 \to H^p} \leq C\varepsilon^p.$$

Here $A^0 = b(\mathbf{D})^* g^0 b(\mathbf{D})$ is the effective operator. The correctors $K_j(\varepsilon)$, $j = 1, \dots, p-1$, are of the order $O(\varepsilon^j)$, respectively; they contain rapidly oscillating factors. The effective matrix g^0 and the correctors are described in terms of the solutions of some auxiliary problems. For simplicity, in the talk we will limit ourselves to the case $p = 2$.

The talk is based on joint work with T. A. Suslina. A support from the Russian Science Foundation grant 22-11-00092-П is acknowledged.

Mon, Jun 22 12:50–13:20

Feynman–Kac formula for the Laplace operator with a zero-range potential in $\mathbb{R}^3$

Natalya V. Smorodina

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Consider the self-adjoint operator $-\Delta$ defined on the domain $W_2^2(\mathbb{R}^3)$. Next, consider its restriction to the subspace $\mathcal{D}_0 = \{f \in W_2^2(\mathbb{R}^3)\colon f(0) = 0\}$. The restricted operator has defect indices $(1,1)$.

Now, for each $k > 0$, we construct a self-adjoint extension $\mathcal{H}_k$ on the domain $\mathcal{D}(\mathcal{H}_k) = \mathcal{D}_0 \dotplus \mathcal{L}(\varphi_k)$, where $\mathcal{L}(\varphi_k)$ is the one-dimensional subspace spanned by the function $\varphi_k(\mathbf{x}) = e^{-kr}/r = e^{-k\|\mathbf{x}\|}/\|\mathbf{x}\|$.

On $\mathcal{D}(\mathcal{H}_k)$ we define the operator $\mathcal{H}_k$, by setting, for $f \in \mathcal{D}(\mathcal{H}_k)$ and $\mathbf{x} \neq 0$ $[\mathcal{H}_k f](\mathbf{x}) = -\Delta f(\mathbf{x})$. First the operator $\mathcal{H}_k$ appears in physical papers, one of the first mathematical paper where the properties of this operator were studied is [1].

For every $\varepsilon > 0$ define a function $L_\varepsilon(\mathbf{x})$, $\mathbf{x} \in \mathbb{R}^3$, by setting $L_\varepsilon(\mathbf{x}) = 1/\|\mathbf{x}\|$ for $\|\mathbf{x}\| > \varepsilon$ and $L_\varepsilon(\mathbf{x}) = (3\varepsilon^2 - \|\mathbf{x}\|^2)/2\varepsilon^3$ for $\|\mathbf{x}\| \in [0, \varepsilon]$. Further, define a martingale $S_\varepsilon(\mathbf{x}, t)$ by

$$S_\varepsilon(\mathbf{x}, t) = \exp\Big(\int_0^t \Big(\frac{\nabla L_\varepsilon(\mathbf{w}_\mathbf{x}(\tau))}{L_\varepsilon(\mathbf{w}_\mathbf{x}(\tau))}, d\mathbf{w}_\mathbf{x}(\tau)\Big) - \frac{1}{2}\int_0^t \frac{\|\nabla L_\varepsilon(\mathbf{w}_\mathbf{x}(\tau)\|^2}{(L_\varepsilon(\mathbf{w}_\mathbf{x}(\tau))^2}\, d\tau\Big).$$

Using the martingale $S_\varepsilon(\mathbf{x}, t)$ we construct the operator family P_k^t, by setting for $f \in \mathcal{D}(\mathcal{H}_k)$

$$[P_k^t f](\mathbf{x}) = \frac{1}{\|\mathbf{x}\|} \lim_{\varepsilon \to 0} \mathbb{E} g(\mathbf{w}_\mathbf{x}(t)) S_\varepsilon(\mathbf{x}, t),$$

where $g(\mathbf{x}) = f(\mathbf{x}) \cdot \|\mathbf{x}\|$.

Theorem. *For every $k > 0$ the family of operators P_k^t is a semigroup with generator $-\frac{1}{2}\mathcal{H}_k$. This semigroup extends by continuity to a semigroup of self-adjoint bounded operators in $L_2(\mathbb{R}^3)$ of the form $P_k^t = e^{-\frac{t}{2}\mathcal{H}_k}$.*

References

[1] F. A. Berezin and L. D. Faddeev, *Remark on the Schrödinger equation with a singular potential*, J. Dokl. Akad. Nauk SSSR, 137:5 (1961), 1011–1014.

Tue, Jun 23
10:00–10:45

Time-frequency analysis: eigenvalue asymptotics

Alex Sobolev[†], Alexey Derkach

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We study discrete spectrum of self-adjoint Weyl pseudodifferential operators with discontinuous symbols of the form $1_\Omega \phi$ where 1_Ω is the indicator of a domain in $\Omega \subset \mathbb{R}^2$, and $\phi \in C_0^\infty(\mathbb{R}^2)$ is a real-valued function. It was known that in general, the singular values s_k of such an operator satisfy the bound $s_k = O(k^{-3/4})$, $k = 1, 2, \dots$. We show that if Ω is a polygon, the singular values decrease as $O(k^{-1} \log k)$. In the case where Ω is a sector, we obtain an asymptotic formula that confirms the sharpness of the above bound.

Mon, Jun 22
15:50–16:35

WKB method: within and beyond its limits

Stanislav A. Stepin

Moscow State University

Under appropriate conditions equation

$$y''(x) - Q(x)\,y(x) = 0$$

is known to possess the solutions with WKB-asymptotics

$$Q(x)^{-1/4}\exp\Big(\pm\int_{x_0}^{x}\sqrt{Q(t)}\,dt\Big)$$

as $x \to \infty$. In this context the question naturally arises if the corresponding approach admits a suitable extension to a non-linear setting, especially for the equations

$$y''(x) - Q(x)\,|y(x)|^{\lambda-1}y(x) = 0$$

of Emden–Fowler type. Moreover, given a linear equation with $Q(x) = p(x)e^{x} + q(x)$ where p and q are polynomial, WKB-asymptotics prove to be a some kind of test for the integrability by quadratures property.

Mon, Jun 22
12:00–12:45

On perturbations of the spectra of $\mathcal{PT}$-symmetric Schrödinger operators

Iskander A. Taimanov

Moscow State University & Sobolev Institute of Mathematics, Novosibirsk

It is said that a one-dimensional Schrödinger operator

$$L = -\frac{d^2}{dx^2} + U(x)$$

is $\mathcal{PT}$-symmetric if

$$U(x) = \overline{U(-x)}.$$

If $U(x)$ is real-valued then the spectrum is also real-valued. We consider perturbations of such operators with small complex-valued $\mathcal{PT}$-potentials and the creation of the non-real-valued part of the spectrum under such perturbations.

The talk is based on the articles [1, 2].

References

[1] P. G. Grinevich and I. A. Taimanov, *On perturbations of the spectrum of a one-dimensional $\mathcal{PT}$-symmetric periodic Schrödinger operator*, Regular and Chaotic Dynamics, 30 (2025), 962–968.

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Fri, Jun 26
12:00–12:30

Operator separation of variables in adiabatic approximation and a Cauchy problem for wave equation in a regularly homogenized medium

Anton A. Tolchennikov[†], Sergey Yu. Dobrokhotov[‡], Vladimir E. Nazaikinskii[§]
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The talk is devoted to the homogenization method, which makes it possible to reduce equations with rapidly oscillating coefficients to equations with non-oscillating coefficients. This method is illustrated by the example of the wave equation. Namely, we consider a Cauchy problem for the wave equation with "regularly fast oscillating" speed

$$\frac{\partial^2 u}{\partial t^2} - \Big\langle \nabla, c^2\Big(\frac{\Theta(x)}{\mu}, x\Big)\nabla \Big\rangle u = 0, \qquad u|_{t=0} = V\Big(\frac{x}{h}\Big), \qquad u_t'|_{t=0} = 0,$$

where μ, h are small parameters, $c^2(y,x)$ is a function periodic in y. Moreover, in many cases we can assume that $c^2(y,x) = f_0(x) + \delta f_1(y,x)$ with $\langle f_1 \rangle = 0$ and δ being small parameter.

After the introduction of a fast variable and operator separation of variables (Peierls substitution) we obtain a homogenized equation of the type of the linearized Boussinesq equation.

Supported by the RSF grant 24-11-00213.

References

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Thu, Jun 25
11:40–12:10

Compactness, s-numbers and completeness of root vectors of a non-self-adjoint Schrödinger operator with a singular potential

Sergey N. Tumanov

Moscow State University

We consider the minimal operator L_0 in $L_2(\mathbb{R}_+)$ generated by the differential expression of the form

$$l(y) = -y'' + q(x)y, \tag{1}$$

where the complex-valued potential $q = u'$ is the derivative (in the distributional sense) of the function $u \in L_{2,\mathrm{loc}}(\mathbb{R}_+)$.

Extensions of such an operator can have compact resolvents and, therefore, a discrete spectrum. However, even for the simplest potentials, such as $q(x) = ix$, their root vectors does not form a Riesz basis, i.e., there is no similarity to self-adjoint or normal operators. At the same time, the system of root vectors can be complete in $L_2(\mathbb{R}_+)$.

We will show that a necessary condition for the compactness of the resolvent of regular extensions of L_0 (closed extensions with a nonempty resolvent set) is:

$$\forall d > 0 \qquad \lim_{x\to+\infty} \|q\|_{W_2^{-1}(x,x+d)} = +\infty,$$

and under additional requirements on q, this proves to be a criterion. It is natural to expect that the distribution of s-numbers is related to the growth pattern of $\|q\|_{W_2^{-1}(x,x+d)}$ in x and d, which we will also show.

In conclusion, the obtained estimates for the s-numbers allow us to establish a completeness theorem for root vectors.

Our study uses approaches that go back to the works of Molchanov, Lidskii, Ismagilov and Otelbaev.

Fri, Jun 26
16:50–17:15

Asymptotic methods in the problem of long nonlinear coastal wave dynamics from a localized source

Maria M. Votiakova

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The report discusses asymptotic methods in the problem of long coastal wave dynamics from a localized source. Asymptotic solutions to the Cauchy problem for the system of nonlinear shallow water equations are constructed, determined through the asymptotics of the linearized system [1].

Using the adiabatic approximation in the form of operator separation of variables [2], the two-dimensional problem is reduced to a one-dimensional one. The resulting reduced equation along the coastline surprisingly resembles the equation for the deep ocean: the square of the frequency in the dispersion relation is proportional to the wave vector to the first power.

The difference is that the equation for coastal waves contains a variable coefficient depending on the bottom slope angle in the principal term. Furthermore, the curvature of the coastline and quadratic terms in the expansion of the bottom function enter into the subprincipal symbol of the equation and affect the phase correction of the solution.

As a result of this work, simple explicit asymptotic formulas for coastal waves are obtained and their properties are investigated: dispersion and an analogue of Green's law for water waves.

The work was supported by the Russian Science Foundation (grant 24-11-00213).

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Wed, Jun 24
12:00–12:45

High-contrast periodic diffusion and sticky Brownian motion with resetting at the boundary

Elena A. Zhizhina

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We consider the homogenization problem for the diffusion operators in high-contrast periodic media:

$$A_\varepsilon u^\varepsilon(x) = \mathrm{div}(a^\varepsilon(x)\nabla u^\varepsilon)$$

with

$$a^\varepsilon(x) = \begin{cases} 1, & \text{if } x \in Y^\sharp \\ \varepsilon^2, & \text{if } x \in G^\sharp, \end{cases}$$

where $G \cup Y = \mathbb{T}^d$.

To describe the limit of A_ε under the diffusion scaling, we introduced in [1] an extended space, where the spatial component of the process is complemented by a component that describes the position within the period $\mathbb{T}^d$. We then investigated the scaling limit as $\varepsilon \to 0$ of the corresponding Markov processes on this extended space.

In my talk, I will present new results on the relationship between the limit process associated with diffusion in high-contrast periodic media and "sticky" Brownian motion in G with redistribution at the boundary. We will also discuss how to define "sticky" Brownian motion using the concept of local time.

References

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