Book of Abstracts

Construction of a potential for given essential spectrum of the Schrödinger operator

Wed, Jun 22
17:30–18:00

Grigoriy Agafonkin

Moscow State University

We consider the singular self-adjoint Shrödinger operator in $L_2([0,+\infty))$ formally defined as

$$H = -\frac{d^2}{dx^2} + \sum_{k=1}^{+\infty} a_k \delta_{x_k},$$
$$D(H) = \Big\{ u \in W_2^2([0,+\infty) \setminus \{x_k,\ k \in \mathbb{N}\}) \cap C([0,+\infty))\colon u(0) = 0 \Big\},$$

where $a_k \in \mathbb{R}$, x_k is an increasing sequence of positive real numbers and δ_{x_k} denotes the Dirac delta function supported at x_k. It is proved that for every closed semi-bounded set $S \subset \mathbb{R}$ one can always choose the values of parameters a_k and x_k such that the essential spectrum of the operator H coincides with the set S. The potential constructed by our method has asymptotically increasing antiderivative, and the estimates of its growth are also given.

The same technique can also be applied to the problem for the operator in $L_2([0,+\infty))$ of the form

$$L = -\frac{d^2}{dx^2} + \sum_{k=1}^{+\infty} a_k \chi_{[x_{k-1},x_k]},$$
$$D(L) = \Big\{ u \in W_2^2([0,+\infty))\colon u(0) = 0 \Big\}$$

(where χ_A stands for the characteristic function of the set A); in both cases the boundary condition can be chosen Neumann instead of Dirichlet without affecting the main result.

The work is supported by Moscow Mathematical Center of Fundamental and Applied Mathematics.

Homogenization of the parabolic equation with periodic coefficients at the edge of a spectral gap

Fri, Jun 24
17:00–17:30

Elena Aksenova

St. Petersburg State University

In $L_2(\mathbb{R})$, consider a second-order elliptic differential operator A_ε, $\varepsilon > 0$, of the form $A_\varepsilon = -\frac{d}{dx} g(x/\varepsilon) \frac{d}{dx} + \varepsilon^{-2} p(x/\varepsilon)$ with periodic coefficients. For small ε, we study the behavior of the semigroup $e^{-A_\varepsilon t}$, $t > 0$, cut by the spectral projection of the operator A_ε for the interval $[\varepsilon^{-2}\nu, +\infty)$. Here $\varepsilon^{-2}\nu$ is the right edge of a spectral gap for the operator A_ε. We obtain approximation for the "cut semigroup" in the operator norm in $L_2(\mathbb{R})$ with error $O(\varepsilon)$, and also a more accurate approximation with error $O(\varepsilon^{-2})$ (after singling out the factor $e^{-t\nu/\varepsilon^2}$).

We rely on the spectral approach to homogenization theory. The results are applied to homo genization of the Cauchy problem $\partial_t v_\varepsilon = -A_\varepsilon v_\varepsilon$, $v_\varepsilon(x,0) = f_\varepsilon(x)$, with the initial data f_ε from a special class.

The talk is based on the joint work [1] with Akhmatova, Sloushch, and Suslina.

[1] A. R. Akhmatova, E. S. Aksenova, V. A. Sloushch and T. A. Suslina (2021): Homogenization of the parabolic equation with periodic coefficients at the edge of a spectral gap, Complex Variables and Elliptic Equations, DOI: 10.1080/17476933.2021.1947259.

Sun, Jun 26
12:20–13:10

Asymptotic eigenfunctions of wave equation with boundary degeneracy in close to integrable case

Anatoly Yu. Anikin

Ishlinsky Institute for Problems in Mechanics

We deal with short-wave asymptotic eigenfunctions (quasi-modes) for wave operator with variable coefficients and degeneracy on the boundary of a domain. We are interested in quasi-modes localized in regions having non-empty intersection with the domain boundary.

Due to the degeneracy, the trajectories of the associated Hamiltonian system go to infinity at finite time. However, there is a phase space extension procedure, which allows to continue trajectories after they reach the boundary. For some class of domains and depth functions we show that the Hamiltonian system is integrable, and that the extended phase space is foliated into invariant tori. We present constructive formulas for asymptotic solutions associated with such tori.

Then we discuss the perturbation of this integrable problem. In this case, we propose an efficient algorithm for constructing asymptotic solutions of this type based on solving the transport equation on a chosen Diophantine torus of the unperturbed problem.

Finally, we discuss a problem of non-linear standing coastal waves, and propose an algorithm for constructing solutions based on an asymptotic analog of the Carrier-Greenspan transform.

The talk is based on the joint work with V. V. Rykhlov, as well as the earlier joint works with S. Yu. Dobrokhotov, V. E. Nazaikinskii, and A. V. Tsvetkova.

Wed, Jun 22
11:20–12:10

Quantum graphs with small edges: analyticity of resolvents and spectra

Denis I. Borisov

Institute of Mathematics, Ufa Federal Research Center RAS

We consider an arbitrary metric graph, to which we glue a graph with edges of lengths proportional to ε, where ε is a small positive parameter. On such graph, we introduce a general self-adjoint second order differential operator $\mathcal{H}_\varepsilon$ with varying coefficients subject to general vertex conditions; all coefficients in differential expression and vertex conditions are supposed to be holomorphic in ε. Under certain rather general assumption we show that that certain parts of the resolvent of $\mathcal{H}_\varepsilon$ are analytic in ε and we show how to find effectively all coefficients in their Taylor series. This allows us to represent the resolvent of $\mathcal{H}_\varepsilon$ by an uniformly converging Taylor-like series and its partial sums can be used for approximating the resolvent up to an arbitrary power of ε. Then we discuss the behavior of the spectrum of the considered operator and prove that it converges to the spectrum of a certain limiting operator on a graph with no small edges. For the eigenvalues converging to the isolated eigenvalues of the limiting operator, we prove that they are also analytic in ε and show how to find their Taylor expansions.

The work is supported by Russian Science Foundation (20-11-19995).

Scattering by microinhomogeneous traps

Fri, Jun 24
12:20–13:10

Gregory A. Chechkin

Moscow State University & Institute of Mathematics, Ufa Federal Research Center RAS & Institute of Mathematics and Mathematical Modeling, Almaty

The vibrations of a body with a hole are studied under the assumption that the body has a micro-inhomogeneous internal structure (see Figure 1). We assume that inside the body there is a large number of periodically arranged small inclusions, which can partially reflect acoustic waves. We study the asymptotic behavior of the spectrum of such a problem as a small parameter tends to zero, which characterizes the size of the holes and the period of the microstructure.

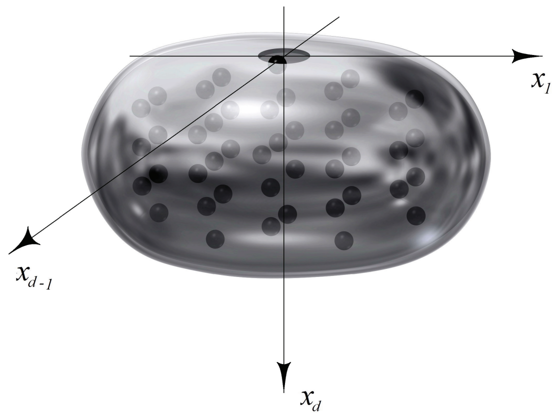


Figure 1: Helmholtz resonator with microinhomogeneous structure

This work is based on the joint research with R. R. Gadyl'shin and A. L. Piatnitski. The author is partially supported by RSF (project 20-11-20272).

Lagrangian manifolds from Kepler trajectories and global asymptotics in the form of the Airy function for the scattering problem on the repulsive Coulomb potential

Thu, Jun 23
12:20–13:10

Sergey Dobrokhotov

Ishlinsky Institute for Problems in Mechanics

The exact solution of the scattering problem for the Schrödinger equation with a repulsive Coulomb potential is well known and is expressed in terms of a degenerate hypergeometric function (see Faddeev, L. D., Merkuriev, S. P.: Quantum Scattering Theory for Several Particle Systems. Kluwer, Dordrecht, 1993). We construct a semiclassical asymptotic solution to this problem and show that this asymptotics is based on Lagrangian manifolds woven from Kepler trajectories, and is globally expressed in terms of the Airy function of a complex argument having the form of an algebraic function.

This is a joint work with Sergey Levin and Anton Tolchennikov. This work was supported by Russian Science Foundation, project 21-11-00341.

Fri, Jun 24
16:30–17:00

Homogenization of nonstationary periodic Maxwell system in the case of constant permeability

Mark Dorodnyi

St. Petersburg State University

We study the Cauchy problem for the nonstationary Maxwell system in the case where the magnetic permeability is given by a constant positive matrix μ, and the dielectric permittivity is given by the rapidly oscillating (as $\varepsilon \to 0$) matrix $\eta^\varepsilon(\mathbf{x}) \coloneqq \eta(\mathbf{x}/\varepsilon)$:

$$\begin{cases} \partial_t \mathbf{E}_\varepsilon(\mathbf{x},t) = (\eta^\varepsilon(\mathbf{x}))^{-1} \operatorname{curl} \mathbf{H}_\varepsilon(\mathbf{x},t), & \operatorname{div} \eta^\varepsilon(\mathbf{x}) \mathbf{E}_\varepsilon(\mathbf{x},t) = 0, \quad \mathbf{x} \in \mathbb{R}^3,\ t \in \mathbb{R}; \\ \partial_t \mathbf{H}_\varepsilon(\mathbf{x},t) = -\mu^{-1} \operatorname{curl} \mathbf{E}_\varepsilon(\mathbf{x},t), & \operatorname{div} \mu \mathbf{H}_\varepsilon(\mathbf{x},t) = 0, \quad \mathbf{x} \in \mathbb{R}^3,\ t \in \mathbb{R}; \\ \mathbf{E}_\varepsilon(\mathbf{x},0) = (P_\varepsilon \mathbf{f})(\mathbf{x}),\ \mathbf{H}_\varepsilon(\mathbf{x},0) = \boldsymbol{\phi}(\mathbf{x}), & \mathbf{x} \in \mathbb{R}^3. \end{cases}$$

Here a symmetric matrix-valued function $\eta(\mathbf{x})$ is periodic with respect to some lattice, positive definite, and bounded. Next, $\boldsymbol{\phi} \in L_2(\mathbb{R}^3;\mathbb{C}^3)$, $\operatorname{div} \mu\boldsymbol{\phi}(\mathbf{x}) = 0$, $\mathbf{f} \in L_2(\mathbb{R}^3;\mathbb{C}^3)$ and P_ε is the orthogonal projection of the space $L_2(\mathbb{R}^3;\mathbb{C}^3;\eta^\varepsilon)$ onto the subspace

$$\{\mathbf{u} \in L_2(\mathbb{R}^3;\mathbb{C}^3) \colon \operatorname{div} \eta^\varepsilon(\mathbf{x})\mathbf{u}(\mathbf{x}) = 0\}.$$

It is well known that the electric and magnetic fields $\mathbf{E}_\varepsilon$ and $\mathbf{H}_\varepsilon$ weakly converge to the fields $\mathbf{E}_0$ and $\mathbf{H}_0$, which are the solutions of the homogenized Maxwell system with a *constant effective* dielectric permittivity η^0.

Denote by $\mathbf{D}_\varepsilon = \eta^\varepsilon \mathbf{E}_\varepsilon$, $\mathbf{D}_0 = \eta^0 \mathbf{E}_0$, $\mathbf{B}_\varepsilon = \mu \mathbf{H}_\varepsilon$, $\mathbf{B}_0 = \mu \mathbf{H}_0$ the corresponding electric displacement vectors and the magnetic inductions. Our main results are:

- Let $\boldsymbol{\phi}, \mathbf{f} \in H^2(\mathbb{R}^3;\mathbb{C}^3)$, and $\operatorname{div} \mu\boldsymbol{\phi} = 0$. Then for $t \in \mathbb{R}$ and $\varepsilon > 0$ we have

$$\begin{aligned} \|\mathbf{H}_\varepsilon(\cdot,t) - \mathbf{H}_0(\cdot,t)\|_{L_2(\mathbb{R}^3)} &\leqslant C(1+|t|)\varepsilon(\|\boldsymbol{\phi}\|_{H^2(\mathbb{R}^3)} + \|\mathbf{f}\|_{H^2(\mathbb{R}^3)}), \\ \|\mathbf{B}_\varepsilon(\cdot,t) - \mathbf{B}_0(\cdot,t)\|_{L_2(\mathbb{R}^3)} &\leqslant C(1+|t|)\varepsilon(\|\boldsymbol{\phi}\|_{H^2(\mathbb{R}^3)} + \|\mathbf{f}\|_{H^2(\mathbb{R}^3)}). \end{aligned}$$

- Let $\mathbf{f} \in H^3(\mathbb{R}^3;\mathbb{C}^3)$, and $\boldsymbol{\phi} = 0$. Then for $t \in \mathbb{R}$ and $\varepsilon > 0$ we have

$$\begin{aligned} \|(\mathbf{E}_\varepsilon(\cdot,t) - \mathbf{E}_\varepsilon(\cdot,0)) - (\mathbf{1} + \Sigma^\varepsilon)(\mathbf{E}_0(\cdot,t) - \mathbf{E}_0(\cdot,0))\|_{L_2(\mathbb{R}^3)} &\leqslant C|t|(1+|t|)\varepsilon\|\mathbf{f}\|_{H^3(\mathbb{R}^3)}, \\ \|(\mathbf{D}_\varepsilon(\cdot,t) - \mathbf{D}_\varepsilon(\cdot,0)) - (\mathbf{1} + \widetilde{\Sigma}^\varepsilon)(\mathbf{D}_0(\cdot,t) - \mathbf{D}_0(\cdot,0))\|_{L_2(\mathbb{R}^3)} &\leqslant C|t|(1+|t|)\varepsilon\|\mathbf{f}\|_{H^3(\mathbb{R}^3)}. \end{aligned}$$

Here Σ^ε and $\widetilde{\Sigma}^\varepsilon$ are the so-called correctors of zero order. These results are sharp with respect to the norm type as well as with respect to the dependence on t. However, these estimates can be improved under some additional assumptions:

- Let $\boldsymbol{\phi}, \mathbf{f} \in H^{3/2}(\mathbb{R}^3;\mathbb{C}^3)$, and $\operatorname{div} \mu\boldsymbol{\phi} = 0$. Then for $t \in \mathbb{R}$ and $\varepsilon > 0$ we have

$$\|\mathbf{H}_\varepsilon(\cdot,t) - \mathbf{H}_0(\cdot,t)\|_{L_2(\mathbb{R}^3)} \leqslant C(1+|t|)^{1/2}\varepsilon(\|\boldsymbol{\phi}\|_{H^{3/2}(\mathbb{R}^3)} + \|\mathbf{f}\|_{H^{3/2}(\mathbb{R}^3)}),$$
$$\|\mathbf{B}_\varepsilon(\cdot,t) - \mathbf{B}_0(\cdot,t)\|_{L_2(\mathbb{R}^3)} \leqslant C(1+|t|)^{1/2}\varepsilon(\|\boldsymbol{\phi}\|_{H^{3/2}(\mathbb{R}^3)} + \|\mathbf{f}\|_{H^{3/2}(\mathbb{R}^3)}).$$

- Let $\mathbf{f} \in H^{5/2}(\mathbb{R}^3;\mathbb{C}^3)$, and $\boldsymbol{\phi} = 0$. Then for $t \in \mathbb{R}$ and $\varepsilon > 0$ we have

$$\|(\mathbf{E}_\varepsilon(\cdot,t) - \mathbf{E}_\varepsilon(\cdot,0)) - (\mathbf{1} + \Sigma^\varepsilon)(\mathbf{E}_0(\cdot,t) - \mathbf{E}_0(\cdot,0))\|_{L_2(\mathbb{R}^3)} \leqslant C|t|(1+|t|)^{1/2}\varepsilon\|\mathbf{f}\|_{H^{5/2}(\mathbb{R}^3)},$$
$$\|(\mathbf{D}_\varepsilon(\cdot,t) - \mathbf{D}_\varepsilon(\cdot,0)) - (\mathbf{1} + \widetilde{\Sigma}^\varepsilon)(\mathbf{D}_0(\cdot,t) - \mathbf{D}_0(\cdot,0))\|_{L_2(\mathbb{R}^3)} \leqslant C|t|(1+|t|)^{1/2}\varepsilon\|\mathbf{f}\|_{H^{5/2}(\mathbb{R}^3)}.$$

The talk is based on the joint work with Tatiana Suslina. Supported by Russian Science Foundation (project 17-11-01069).

Microlocal analysis of internal waves in 2D aquaria

Thu, Jun 23
16:30–17:20

Semyon Dyatlov

MIT

For a bounded smooth planar domain Ω, we study the forced evolution problem for the 4th order PDE

$$(\partial_t^2 \Delta + \partial_{x_2}^2)u(t,x) = f(x)\cos(\lambda t), \qquad t \geq 0, x \in \Omega, \tag{1}$$

with homogeneous initial conditions and Dirichlet boundary condition on $\partial\Omega$. This is motivated by concentration of fluid velocity on attractors for stratified fluids in effectively 2-dimensional aquaria, first observed experimentally in 1997.

The behavior of solutions to (1) is intimately tied to the *chess billiard* map on the boundary $\partial\Omega$, which depends on the forcing frequency λ. Under the natural assumption that the chess billiard b has the Morse–Smale property, we show that as $t \to \infty$ the singular part of the solution u concentrates on the attractive cycle of b. The proof combines various tools from microlocal analysis, scattering theory, and hyperbolic dynamics. Joint work with Jian Wang and Maciej Zworski.

Asymptotic data and Stokes data for the tt*-Toda equations, and some relations with physics

Thu, Jun 23
10:00–10:50

Martin Guest

Waseda University

We give a concrete example of an “integrable” nonlinear p.d.e., a version of the 2D Toda equations, whose solutions can be parametrized by asymptotic data and also by Stokes data. The p.d.e. was first studied by the physicists Cecotti and Vafa, and by Dubrovin. Certain special solutions are related to Frobenius manifolds (such as quantum cohomology or unfoldings of singularities). The equations have been solved in a series of joint works with Its and Lin, using a combination of methods from integrable systems and p.d.e. theory. The explicit nature of the data leads to relations with geometry and physics; we describe some of these briefly.

[1] M. A. Guest, A. R. Its, and C.-S. Lin, *Isomonodromy aspects of the tt* equations of Cecotti and Vafa III. Iwasawa factorization and asymptotics*, Commun. Math. Phys. 374 (2020) 923-973 (arXiv:1707.00259)

[2] M. A. Guest, and N.-K. Ho, *Kostant, Steinberg, and the Stokes matrices of the tt*-Toda equations*, Selecta Math. 25 (2019) article 50 (arXiv: 1802.01126)

[3] M. A. Guest, *Topological-antitopological fusion and the quantum cohomology of Grassmannians*, Jpn. J. Math. 16 (2021) 155-183 (arXiv:2012.01123)

[4] M. A. Guest, and T. Otofuji, *Positive energy representations of affine algebras and Stokes matrices of the affine Toda equations*, (arXiv:2109.00765)

Sat, Jun 25
11:20–11:50

New integral representations of the Maslov canonical operator with complex phases

Alexander I. Klevin

Ishlinsky Institute for Problems in Mechanics

The canonical Maslov operator with complex phases (the complex germ theory, see e.g. [1]) allows us to construct asymptotic solutions of a wide class of linear partial differential and pseudodifferential equations with a small parameter in the form of oscillating functions localized in the vicinity of surfaces of various dimensions (for example, asymptotics in the form of Gaussian wave packets or Gaussian wave beams). The main geometric object in such problems is a vector bundle over the isotropic manifold in the phase space and with planes in the complexified phase space (a complex germ) as fibers. Asymptotics are represented in an effective complex WKB form in the neighborhood of (regular) points that is diffeomorphically projectable from the isotropic manifold into the configuration space, and in the form of oscillating integrals with a complex phase function in the neighborhood of singular points. Similar to those recently proposed in [2] for the real canonical operator new representations of the canonical operator with complex phases are constructed. New representations allow us to avoid the transition to not very effective in practical applications the momentum-position coordinate system, which is usually necessary to do when using the canonical operator in the standard form. The applied result is to obtain simpler expressions for practical calculations. In some cases an effective representation of asymptotic solutions in the form of special functions is possible.

The reported study was funded by RFBR, project number 20-31-90111.

[1] V. P. Maslov, *The Complex WKB Method for Nonlinear Equations I: Linear Theory*, Birkhäuser, Basel (1994).

[2] S. Yu. Dobrokhotov, V. E. Nazaikinskii, A. I. Shafarevich, New integral representations of the Maslov canonical operator in singular charts, *Izvestiya: Mathematics*, **81**(2), 286–328 (2017).

Asymptotics for fundamental system of solutions of $n \times n$ first order system of differential equations

Thu, Jun 23
17:30–18:00

Alexey P. Kosarev

Moscow State University

We deal with a $n \times n$ system of differential equations of the form

$$\mathbf{y}' = \lambda A(x)\mathbf{y} + B(x)\mathbf{y}, \ \ x \in [0,1], \tag{1}$$

where $A(x) = \operatorname{diag}\{\gamma_1(x), \ldots, \gamma_n(x)\}$, $\gamma_i(x) \neq \gamma_j(x)$ for $i \neq j$, $\gamma_i(x) \neq 0$, $i, j = 1, \ldots, n$. All elements of matrices A and B are at least summable and complex-valued, λ is the complex parameter. This equation includes eigenvalue problems for the Dirac operator ($n = 2m, \gamma_1 = \cdots \gamma_m = i, \gamma_{m+1} = \cdots = \gamma_{2m} = -i$) and for the system of telegraph equations ($n = 2, \gamma_1 = \gamma_2$), also such problems arise in studies of polynomial spectral pencils.

Our objective is to obtain asymptotic behavior for fundamental system of solutions (1) assuming minimal requirments on the smoothness of the coefficients. Let define matrices

$$E(x,\lambda) = \operatorname{diag}\Big\{e^{\lambda \int_0^x \gamma_1 dt}, \ldots, e^{\lambda \int_0^x \gamma_n dt}\Big\}, \qquad M(x) = \operatorname{diag}\Big\{e^{\int_0^x b_{11} dt}, \ldots, e^{\int_0^x b_{nn} dt}\Big\}.$$

Theorem. *Assume that elements of matrices $A(x), B(x)$ belong to the space $W_1^k[a,b]$ ($k \geq 1$), $\arg\{\gamma_j(x) - \gamma_i(x)\} = h_{ij}, i, j = 1, \ldots, n$, then in each sector Π of the complex plane within which none of the quantities $\operatorname{Re}\{\lambda(\gamma_j(x) - \gamma_i(x))\}$ change sign there exists a fundamental solution matrix $Y(x,\lambda)$ of the equation (1), which has the representation*

$$Y(x,\lambda) = M(x)(I + R(x,\lambda))E(x,\lambda), \tag{2}$$

where $R(x,\lambda)$ is a holomorphic matrix function in the sector Π for sufficiently large $|\lambda|$ and $R(x,\lambda)$ admits the representation

$$R(x,\lambda) = \frac{R^1(x)}{\lambda} + \cdots + \frac{R^k(x)}{\lambda^k} + o(1)\lambda^{-k},$$

where $o(1)$ is an infinitesimal function uniformly in $x \in [0,1]$ as $\lambda \to \infty$, $\lambda \in \Pi$.

The matrix functions R^m can be represented explicitly:

$$q_{ij} := b_{ij} e^{\int_0^x b_{jj}(t) - b_{ii}(t) dt}, \qquad r_{ij}^1 = \frac{q_{ij}}{\gamma_j - \gamma_i}, \ i \neq j, \qquad r_{ii}^1 = \int_0^x \sum_{\substack{s=1,\\ s\neq i}}^n q_{is}(t) r_{si}^1(t) dt,$$

$$r_{ij}^{m+1} = \frac{-\frac{d}{dx} r_{ij}^m + \sum\limits_{\substack{s=1,\\ s\neq i}}^n q_{is} r_{sj}^m}{\gamma_j - \gamma_i}, \ i \neq j, \qquad r_{ii}^{m+1} = \int_0^x \sum_{\substack{s=1,\\ s\neq i}}^n q_{is}(t) r_{si}^{m+1}(t) dt, \ m = 1, \ldots, k-1.$$

The report is based on the joint work with A. A. Shkalikov.

Wed, Jun 22
15:10–16:00

Calogero type bounds in two dimensions

Ari Laptev

Imperial College London & St. Petersburg University

For a Schrödinger operator on the plane $\mathbb{R}^2$ with electric potential V and Aharonov–Bohm magnetic field we obtain an upper bound on the number of its negative eigenvalues in terms of the $L^1(\mathbb{R}^2)$-norm of V. Similar to Calogero's bound in one dimension, the result is true under monotonicity assumptions on V. Our proof method relies on a generalisation of Calogero's bound to operator-valued potentials.

Wed, Jun 22
12:20–13:10

Systems of functional-difference equations with characteristic parameter and their applications

Mikhail A. Lyalinov

St. Petersburg State University

In this work we construct eigenfunctions of a Schrödinger operator in a half-plane with the Neumann boundary condition. The singular potential of the operator is the Dirac δ-function having its support on two half-lines with the same origin located on the boundary of the half-plane. These kind of problems gives rise in quantum scattering of three one-dimentional particles and in diffraction theory.

We study negative eigenvalues and eigenfunctions of the corresponding self-adjoint operator. We make use of an integral representation of the Kontorovich–Lebedev type for the solutions and reduce the problem of description of the spectrum and of the eigenfunctions to a system of functional-difference equations with a characteristic (spectral) parameter. The system is then studied by means of reduction to an integral equation with a selfadjoint integral operator-matrix that is interpreted as a perturbation of the so-called Mehler integral matrix operator. We then consider sufficient conditions of existence of the discrete component in the spectrum of the latter perturbation. Birman–Schwinger principle is exploited to describe conditions when the discrete component of the spectrum is finite. The corresponding results are applied to description of the eigenfunctions of the Schrödinger operator in hand represented by the the Kontorovich–Lebedev integrals.

We are based on the Kontorovich–Lebedev integrals in construction of the eigensolutions, however, reduction to an alternative Sommerfeld-type representaion enables us to find asymptotics of the eigenfunctions at large distances.

Thu, Jun 23
15:10–16:00

Lagrangian manifolds and asymptotic solutions of differential and pseudodifferential equations with localized right-hand sides

Vladimir E. Nazaikinskii

Ishlinsky Institute for Problems in Mechanics

We construct asymptotic solutions as $h \to 0$ of the (pseudo)differential equation

$$\widehat{\mathcal{H}}\psi = f, \qquad f = V\Big(\frac{x-\xi}{h}\Big), \quad \widehat{\mathcal{H}} = \mathcal{H}\Big(\overset{2}{x}, -ih\overset{1}{\frac{\partial}{\partial x}}, h\Big), \quad x = (x_1, \dots, x_n) \in \mathbb{R}^n, \quad (1)$$

where the function $V(y)$ rapidly decays at infinity and the principal symbol $H(x,p) = \mathcal{H}(x,p,0)$ of the operator $\widehat{\mathcal{H}}$ is real-valued. Equation (1) is closely related to the asymptotics of the Green function of linear differential and pseudodifferential equations (including the Helmholtz equation), which, in various settings, was studied by Babich, Buldyrev, Duistermaat, Hörmander, Keller, Kucherenko, Maslov, Melrose, Shatalov, Sternin, Uhlmann, and many others.

The right-hand side of Eq. (1) can be represented in the form $f = \mathcal{K}^h_{\Lambda_0} A_0$, where A_0 is the Fourier transform of V and $\mathcal{K}^h_{\Lambda_0}$ is the Maslov canonical operator on the Lagrangian manifold $\Lambda_0 = \{(x,p) \in \mathbb{R}^{2n} \mid x = \xi\}$. An asymptotic solution of (1) will be constructed under the following conditions on the function $H(x,p)$: (I) $H(\xi,p) \geq C|p|^{-N}$ with some constants $C, N > 0$ for sufficiently large $|p|$; (II) the trajectories of the Hamiltonian system $\dot{x} = H_p$, $\dot{p} = -H_x$ with initial data on $L_0 = \operatorname{char} H \cap \Lambda_0$ (where $\operatorname{char} H = \{(x,p) \mid H(x,p) = 0\}$ is the characteristic set of $\widehat{\mathcal{H}}$) are infinitely extendible to the right; (III) all these trajectories forever leave each ball $|x| \leq R$ for sufficiently large $t \geq T = T(R)$. It suffices to consider two cases (and then use a partition of unity). (a) Let $\operatorname{supp} A_0 \cap L_0 = \varnothing$; then problem (1) has an asymptotic solution of the form $\psi \sim \mathcal{K}^h_{\Lambda_0}(A_0/H|_{\Lambda_0})$. (b) Let A_0 be compactly supported with $\operatorname{supp} A_0 \cap L_0 \neq \varnothing$. Instead of (II), assume momentarily that *all* trajectories of the Hamiltonian system are infinitely extendible. Then the Cauchy problem

$$-ih\frac{\partial v}{\partial t} + \widehat{\mathcal{H}} v = 0, \quad v|_{t=0} = f \tag{2}$$

has an asymptotic solution of the form $v(t) = \mathcal{K}^h_{\Lambda_t}(A|_{\Lambda_t})$, where $\Lambda_t = g^t_H(\Lambda_0)$ is the time t flow of Λ_0 and A is a solution (defined on $\Lambda = \bigsqcup_{t\geq 0} \Lambda_t$) of the transport equation along the trajectories of the Hamiltonian system. For a suitable cutoff function χ equal to unity in a neighborhood (shrinking as $t \to \infty$) of the set $\Lambda_+ = \bigsqcup_{t>0} g^t_H(L_0) \subset \Lambda$, the integral

$$\psi = \frac{i}{h}\int_0^\infty \mathcal{K}^h_{\Lambda_t}([\chi A]|_{\Lambda_t})\,dt, \tag{3}$$

being in fact over a finite interval for each x, converges and gives an asymptotic solution of problem (1). Under the weaker condition (II), A will only be defined in a neighborhood of Λ_+, but formula (3) remains valid provided the support of χ is sufficiently narrow. Further, if $H_p \neq 0$ on L_0, then Λ_+ is a smooth manifold, and the stationary phase method gives

$$\psi = e^{i\pi/4}\left(\frac{2\pi}{h}\right)^{1/2} \mathcal{K}^h_{\Lambda_+}(A|_{\Lambda_+}) \quad \text{outside a neighborhood of the point } x = \xi \tag{4}$$

(where Λ_+ is interpreted in an obvious way as a Lagrangian manifold in $\mathbb{R}^{2n}$).

Finally, note that the asymptotic solution is not unique, and the specific choice (3) is determined by an analog of the limiting absorption principle.

The talk is based on joint work with A. Anikin, S. Dobrokhotov, and M. Rouleux.

Fri, Jun 24
10:00–10:50

One-dimensional models of thin lattices and "parasite" eigenvalues

Sergei A. Nazarov

Institute of Mechanical Engineering Problems RAS

The Neumann spectral problem for the Laplace operator in a junction of thin tubes is usually associated with the one-dimensional Pauling model which consists of ordinary second-order differential equations on edges of the corresponding graph and the classical Kirchhoff transmission conditions at its vertices. This model takes into account lengths of the ligaments and shape of their cross-sections but ignores sized of nodes and disposition of the lattice fragments, for example, angles between ligaments. An elaborated model is introduced and reflects the above-mentioned geometrical features of the lattice through some integral characteristics of the junction zones and new transmission conditions of the Kirchhoff–Robin–Steklov type. Eigenpairs of this model provide two-term asymptotics of those in the original spatial problem while error estimates for eigenvalues and eigenfunctions are derived. The main issue of the improved model is a proper choice of the one-dimensional images of lattice's ligaments and the sizes of the spatial nodes. Avoiding rules of this choice leads to appearing of "parasite" eigenvalues of the one-dimensional model, that is, negative and with large modulo. Asymptotics of these eigenvalues is constructed as well.

The results were obtained under support of Russian Science Foundation, grant no. 22-11-00046.

Fri, Jun 24
11:20–12:10

Periodic homogenization of non-symmetric convolution type operators

Andrey Piatnitski

The Arctic University of Norway, campus Narvik & Institute for Information Transmission Problems RAS

The talk will focus on homogenization problem for nonlocal convolution type operators with integrable kernels in a periodic medium. It is assumed that the kernels satisfy the uniform coerciveness and natural moment conditions. However, the symmetry conditions are not imposed.

We consider the Cauchy problem for the corresponding parabolic operators and show that, under the diffusive scaling, the family of Cauchy problems for the rescaled equations admits homogenization in moving coordinates as the scaling parameter tends to zero.

Sun, Jun 26
11:20–12:10

Mathematical scattering theory in electromagnetic waveguides

Aleksandr Poretskii

St. Petersburg State University

The waveguide occupies a domain $G \subset \mathbb{R}^3$ having several cylindrical outlets to infinity and is described by the non-stationary Maxwell system with perfectly conductive boundary conditions. The dielectric permittivity and magnetic permeability of the filling medium are assumed to be positive-definite 3×3 matrix-valued smooth functions of

$x \in G$; at infinity in each cylindrical outlet the matrices converge at an exponential rate to matrices independent of the axial variable.

For the corresponding stationary problem with spectral parameter, the continuous spectrum eigenfunctions and the scattering matrix are defined. We calculate the wave operators, introduce the scattering operator and describe its connection with the scattering matrix. The proof is based on extending the Maxwell system to an equation of the form $i\partial_t \Psi(x,t) = \mathcal{A}(x, D_x)\Psi(x,t)$ with elliptic operator $\mathcal{A}(x, D_x)$. We associate to the equation an initial boundary value problem and develop the scattering theory for the problem. The information on the Maxwell system is derived from the results obtained for the extended problem.

[1] B. A. Plamenevskii, A. S. Poretskii, and O. V. Sarafanov, Doklady Physics, 2022, Vol. 66, No. 2, pp. 70–73.

Spectral estimates for the Birman–Schwinger operator in the critical case and Connes' integration against singular measures

Sun, Jun 26
15:10–16:00

Grigori Rozenblum

Chalmers University of Technology, Sweden & Euler International Mathematical Institute

We establish order sharp estimates of singular numbers for operators of the form $T = K^*PK$, where K is a negative order pseudodifferential operator in $\mathbb{R}^{\mathbf{N}}$ and $P = V\mu$ with measure μ singular with respect to the Lebesgue measure and V being a function on the support of μ. The order of K equals $-\mathbf{N}/2$, which characterizes the critical case. For a wide class of measures, including the Hausdorff measure on Lipschitz surfaces of arbitrary codimension, the asymptotics of eigenvalues is found. As a consequence, the noncommutative, Connes' integration against singular measures is defined.

A method for numerical study of surface waves scattering on an irregularity of the boundary in a diffraction grating

Sun, Jun 26
10:00–10:50

Oleg Sarafanov

St. Petersburg State University

We consider a two-dimensional "diffraction grating"; its boundary is supposed to be periodic outside a compact set. The shape of the periodic part of the boundary is chosen such that surface waves may propagate far from the irregularity. We suggest a method for numerical study of the surface waves scattering on the irregularity of the boundary.

Short-wave asymptotic solutions for evolution equations with singular coefficients

Sat, Jun 25
9:40–10:30

Andrei Shafarevich

Moscow State University

Shortwave asymptotics of solutions for a wide class of evolution equations with smooth coefficients are connected with geometric objects — Lagrangian surfaces or complex vector bundles over isotropic manifolds. If the coefficients of the equations contain singularities (or depend singularly on a small parameter of the problem), geometric objects are rebuilt on the sets corresponding to the carriers of these singularities. The paper discusses the form of asymptotic solutions and rearrangements of geometric objects for certain examples of evolutionary problems with singularities.

Wed, Jun 22
10:00–10:50

Half range problem in operator theory

Andrey A. Shkalikov

Moscow State University

Let $\mathcal{H}$ be Hilbert space with usual scalar product $(\cdot,\cdot)$ and it is also equipped by indefinite inner product $[x,y]=(Jx,y)$. Here J is the involution operator (fundamental symmetry):

$$J = J^* = P_+ - P_-, \qquad J^2 = 1,$$

where $P_\pm$ are orthoprojections.

The space $(\mathcal{H}, [\cdot,\cdot]) = \mathcal{K}$ is called Krein space (and Pontrjagin space Π_κ, provided that $\kappa = \min\{\kappa_+, \kappa_-\} < \infty$, where $\kappa_\pm = \mathrm{rank} P_\pm$.

A subspace $\mathcal{L}$ in $\mathcal{H}$ is said to be *nonnegative* if $[x,x] \geqslant 0$ for all $x \in \mathcal{L}$ and *maximal nonnegative* if there are no proper nonnegative extensions of $\mathcal{L}$.

A linear operator A is said to be self-adjoint (m-dissipative) in $\mathcal{K}$ if JA is self-adjoint (m-dissipative) in $\mathcal{H}$.

Problems (which are open even for bounded operators):

1 Does a self-adjoint or m-dissipative operator A in $\mathcal{K}$ possess a maximal nonnegative A-invariant subspace?
2 If yes, is there among these subspaces such that the restriction $A^+ = A|_{\mathcal{L}}$ onto this subspace has the spectrum only in the closed half-plane $\mathbb{C}^-$?
3 If yes, does the operator A^+ generate a C_0 or holomorphic semigroup?

These problems are closely connected with the following ones, which have impotent applications in concrete problems of mathematical physics:

1 Let $L(\lambda) = \lambda^2 A + \lambda B + C$, where the operator C is self-adjoint (generally unbounded) and A, B are symmetric, such that $\mathcal{D}(A) \supset \mathcal{C}$, $\mathcal{D}(B) \supset \mathcal{C}$. Find conditions which imply factorization

$$L(\lambda) = (\lambda - Z_1)A(\lambda - Z), \qquad Z_1 = -(AZ+B)A^{-1},$$

such that the spectrum of Z lies in the closed lower half-plane $\mathbb{C}^-$. What additional spectral properties one can find for the operator Z?
2 Let $\mathcal{L} = \begin{pmatrix} A & B \\ B^* & C \end{pmatrix}$ be a self-adjoint operator in $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$. Find conditions when $\mathcal{L}$ is similar to a diagonal operator in $\mathcal{H}$.

We shall present in the talk old and new results concerning these problems.

Regularity of solutions for the Coulomb multi-particle Schrödinger equation, and the one-particle density matrix

Wed, Jun 22
16:30–17:20

Alexander V. Sobolev

University College London

Regularity properties for multi-particle systems have been the focus of mathematical research since 1950's. Such results determine spectral properties of the one-particle density matrix which is the key object in the quantum-mechanical approximation schemes. In this talk I will give a short survey of recent regularity results with emphasis on sharp bounds for the eigenfunctions, and show how these bounds lead to the asymptotic formula for the eigenvalues of the one-particle density matrix.

Quasiclassical approximation for magnetic monopoles

Thu, Jun 23
11:20–12:10

Iskander A. Taimanov

Novosibirsk State University & Sobolev Institute of Mathematics

We expose a construction of the quasiclassical approximation for the eigenvalues of the magnetic Laplacian on a compact Riemannian manifold in the case when the magnetic field is not given by an exact 2-form. For that we generalize the canonical operator of Maslov which in this case takes values in sections of a nontrivial line bundle. The constructed approximation is demonstrated for the Dirac magnetic monopole on the two-dimensional sphere.

The talk is based on a joint work with Yu. A. Kordyukov.

Solution of the two-dimensional massless Dirac equation with a linear potential and localized right hand side

Sat, Jun 25
12:00–12:30

Anton A. Tolchennikov

Ishlinsky Institute for Problems in Mechanics

We consider the two-dimensional massless Dirac equation with a linear potential and localized right hand side:

$$x_1\sigma_0\psi + \sigma_1(-ih\psi_{x_1}) + \sigma_2(-ih\psi_{x_2}) = \psi^0\Big(\frac{x-x^0}{h}\Big),$$

where $x^0 = (-a, 0)$, $a > 0$, $\psi^0(x)$ — smooth, fast-decaying function, $h \ll 1$, σ_j — Pauli matrices. The solution must satisfy the absorption limit principle. The talk will be devoted to the construction of an asymptotic solution as $h \to 0$. Using the method of [1], we can construct an asymptotic solution outside a neighborhood of a singular line $x_2 = 0$. Earlier in the paper [2], the asymptotics of the fundamental solution for singular ray $x_2 = 0, x_1 > 0$ was obtained.

This is joint work with S. Yu. Dobrokhotov and I. A. Bogaevsky. Supported by Russian Science Foundation under grant no 16-11-10282.

[1] A. Y. Anikin, S. Yu. Dobrokhotov, V. E. Nazaikinskii, M. Rouleux, The Maslov

canonical operator on a pair of Lagrangian manifolds and asymptotic solutions of stationary equations with localized right-hand sides, *Dokl. Math.*, **96**, 406–410 (2017).

[2] I. A. Bogaevsky, Fundamental solution of the stationary Dirac equation with a linear potential, *Theoretical and Mathematical Physics*, **205**, 1547–1563 (2020).

Sun, Jun 26
16:30–17:20

Compactness and Schatten class properties of Hankel operators on Fock spaces

Jani Virtanen

University of Reading & University of Helsinki

To characterize boundedness and compactness of Hankel operators H_f on weighted Fock spaces, we introduce the space IDA of locally integrable functions whose integral distance to holomorphic functions is finite. As an application, for bounded symbols f, we obtain a very general version of the classical compactness result of Berger and Coburn that states that H_f is compact if and only if $H_{\bar{f}}$ is compact. We also apply the results to the Berezin–Toeplitz quantization and consider extensions to the study of Schatten class properties of Hankel operators on the Fock space. Joint work with Zhangjian Hu.

Fri, Jun 24
15:10–16:00

Probabilistic approach in homogenization of high-contrast periodic models

Elena Zhizhina

Institute for Information Transmission Problems RAS

We consider a symmetric random walk on the lattice in a high contrast periodic medium that can be interpreted as a discrete approximation of a diffusion with high-contrast periodic coefficients. From the existing homogenization results it is known that under diffusive scaling the limit behaviour of this random walk need not be Markovian, the effective evolution equation contains a term that is non-local in time and represents the memory effect. In the paper [1] we proposed a trick that allows one to construct a limiting model in which there is no memory. The price of this is that we will work in some extended space: in addition to the coordinate of the random walk we introduce an extra variable that characterizes the position of the random walk inside the period. Then the limit dynamics of this two-component process is Markov, and therefore, completely given by its generator. We describe the limit process and observe that the memory effect appears under projection of the process on its first component. Recent results related to this construction will be discussed.

[1] A. Piatnitski, E. Zhizhina, Scaling limit of symmetric random walk in high-contrast periodic environment, Journal of Statistical Physics, 169(3) (2017).

High-frequency diffraction by a jump of curvature. A. V. Popov's case of tangential incidence

Sat, Jun 25
10:50–11:20

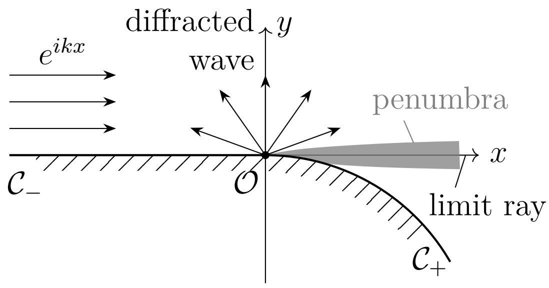
Ekaterina A. Zlobina

St. Petersburg State University

Two-dimensional problem of diffraction by contour $\mathcal{C}$ having jump of curvature at point $\mathcal{O}$ is considered. The contour is composed of the half-line $\mathcal{C}_-$ and a part of smooth curve $\mathcal{C}_+$ conjugating at $\mathcal{O}$. The plane wave e^{ikx} with large wavenumber $k \gg 1$ grazes along $\mathcal{C}_-$ and comes to $\mathcal{O}$ along tangential direction. The outgoing wave u^{out} is governed by the Helmholtz equation and the Neumann boundary condition:

$$u^{\text{out}}_{xx} + u^{\text{out}}_{yy} + k^2 u^{\text{out}} = 0, \qquad \partial_n u^{\text{out}}\Big|_{\mathcal{C}} = \begin{cases} 0, & x \le 0 \\ -\,\partial_n e^{ikx}\Big|_{\mathcal{C}_+}, & x > 0 \end{cases}.$$

Here, ∂_n denotes the derivative along the normal to a contour $\mathcal{C}$. The aim is to construct high-frequency asymptotic formulas for the wavefield in penumbra.



The problem has been previously addressed by Alexey Vladimirovich Popov [1] who, using a heuristic approach, derived an expression for the diffracted cylindrical wave arising at the non-smoothness point $\mathcal{O}$.

We have applied a Fock-type boundary layer technique [2,3]. This allows an asymptotic description of the total wavefield in boundary layers surrounding the limit ray via novel special functions different from the Fock ones [2,3]. In the area where $x > 0$, $y > 0$ and $y/x \gg 1$, an expression for the diffracted wave is obtained, which agrees with the one found in [1].

The talk is based on the joined work with Aleksei P. Kiselev. A support from the Russian Science Foundation grant 22-21-00557 is acknowledged.

[1] A. V. Popov, Backscattering from a line of jump of curvature, *in: Trudy V Vses. Sympos. Diffr. Raspr. Voln, Nauka, Leningrad, 1970*, 171–175 (1971).
[2] V. A. Fock, *Electromagnetic Diffraction and Propagation Problems*, Pergamon Press, Oxford, 1965.
[3] V. M. Babich, N. Ya. Kirpichnikova, *The Boundary Layer Method in Diffraction Problems*, Springer, Berlin, 1979.