30th St.Petersburg Summer Meeting in Mathematical Analysis
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N. Abuzyarova (Bashkir State University).

Properties of spectra of differentiation invariant subspace in the Schwartz space.

Let E be the Schwartz space of infinitely differentiable functions on the real line. We consider its closed differentiation invariant subspace W with discrete spectrum L which is weakly synthesizable. It means that W is the closed span of its residual subspace and the set ExpW of all exponential monomials contained in W. Among all weakly synthesizable subspaces W, there are nice ones which are equal to the direct (algebraical and topological) sum of the residual part of W and the closed span of ExpW. Is a given weakly synthesizable subspace W equal to such a direct sum or not? An answer is obtained in terms of characteristics and (or) properties of the spectrum L.

A. Aleman (University of Lund).

Composition of analytic paraproducts.

For a fixed analytic function g in the unit disc, we consider the analytic paraproducts induced by g, which are defined by $T_g f(z) = \int_0^z f(\zeta)g'(\zeta)\,d\zeta$, $S_g f(z) = \int_0^z f'(\zeta)g(\zeta)\,d\zeta$, together with the multiplication operator $M_g f(z) = f(z)g(z)$. The boundedness of these operators on various spaces of analytic functions on the unit disc is well understood. The original motivation for this work is to understand the boundedness of compositions (products) of two of these operators, for example T_g^2, T_gS_g, M_gT_g, etc. The talk intends to present a general approach which yields a characterization of the boundedness of a large class of operators contained in the algebra generated by these analytic paraproducts acting on the classical weighted Bergman and Hardy spaces in terms of the symbol g. In some cases it turns out that this property is not affected by cancellation, while in others it requires stronger and more subtle restrictions on the oscillation of the symbol g than the case of a single paraproduct. *(Joint work with C. Cascante, J. Fàbrega, D. Pascua and J.A. Peláez.)*

G. Amosov (Steklov Mathematical Institute of RAS).

On perturbations of semigroups based on operator-valued measures.

Suppose that $S_t : X \to X, t \geq 0$, is a C_0-semigroup on the Banach space X. Consider on the half-axis $\mathbb{R}_+$ the covariant operator-valued measure $\mathcal{M}$ taking values in operators on X and satisfying the property

$$S_t \circ \mathcal{M}(B) = \mathcal{M}(B+t), \qquad t \geq 0,$$

for all measurable subsets $B \subset \mathbb{R}_+$.

Theorem. Suppose that $\mathcal{M}$ is semi-absolutely continuous in the following sense:

$$\lim_{t\to+0} \overline{\mathcal{M}}([T, T+t)) = 0, \qquad \forall T \geq 0,$$

where

$$\overline{\mathcal{M}}(E) = \sup\Big\{\Big|\sum_{i=1}^n \mathcal{M}(E_i)x_i\Big| : x_j \in X,\ |x_j| \leq 1,\ E \supset E_j \subset \mathbb{R},\ E_j \text{ are disjoint}\Big\}$$

for all measurable $E \subset \mathbb{R}_+$. Then the solution to the integral equation

$$\breve{S}_t = S_t + \int_0^t \mathcal{M}(ds) \circ \breve{S}_{t-s}$$

produce a C_0-semigroup $\breve{S} = \breve{S}_t,\ t \geq 0$. The generator of $\breve{S}$ acts as

$$x \mapsto \lim_{t\to+0} \frac{S_t - I + \mathcal{M}([0,t))}{t} x$$

with the domain consisting of exactly those x for which the strong limit exists.

Meaningful examples of covariant measures can be obtained from orbits of unitary groups in an infinite-dimensional space [1].

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[1] G.G. Amosov, A.S. Mokeev, A.N. Pechen, Noncommutative graphs based on finite-infinite system couplings: Quantum error correction for a qubit coupled to a coherent field, Phys. Rev. A **103** (2021), no. 4, 042407, 17 pp.

(Joint work with E. L. Baitenov.)

N. Arcozzi (University of Bologna).

The Dirichlet space on the bi-disc.

The Dirichlet space on the bi-disc can be informally defined as the tensor product $\mathcal{D}(\mathbb{D}^2) = \mathcal{D}(\mathbb{D}) \otimes \mathcal{D}(\mathbb{D})$ of two copies of the classical holomorphic Dirichlet space. Multipliers and Carleson measures for the space were recently characterized, and the results have been extended to the three-disc, but not to higher powers. Underlying all this there is a new multi-parameter potential theory which is still in its infancy, and many basic problems await an answer. The talk reports on work by several authors: Pavel Mozolyako, Karl-Mikael Perfekt, Giulia Sarfatti, Irina Holmes, Alexander Volberg, Georgios Psaromiligkos, Pavel Zorin-Kranich, and the speaker.

V. Beloshapka (Lomonosov Moscow State University).

The quadric models in CR geometry.

Over the last 30 years, it has been assumed that the graded Lie algebra of infinitesimal holomorphic automorphisms of quadric model CR manifold has no nontrivial graded components of weight greater than two. Recently it turned out that it is not true. There exist certain enigmatic "special" quadrics, whose graded components go further. The author is going to speak about his recent advances in the understanding of this phenomenon. The main results are obtained with the help of distributions, the Fourier transform and the fundamental principle of Ehrenpreis.

Y. Belov (St. Petersburg State University).

Gabor analysis for rational functions.

Let g be a function in $L^2(\mathbb{R})$. By G_Λ, $\Lambda \subset \mathbb{R}^2$, we denote the system of time-frequency shifts of g, $G_\Lambda = e^{2pii\omega x}g(x-t)_{(t,\omega)\in\Lambda}$. A typical model set Λ is the rectangular lattice $\Lambda_{\alpha,\beta} := \alpha\mathbb{Z}\times\beta\mathbb{Z}$. One of the Gabor analysis basic problems is to describe the frame set of g, i.e. all pairs α, β such that $G_{\Lambda_{\alpha,\beta}}$ is a frame in $L^2(\mathbb{R})$. It follows from the general theory that $\alpha\beta \le 1$ is a necessary condition (we assume $\alpha, \beta > 0$, of course). Do all such α, β belong to the frame set of g?

Until 2011, only few such functions g (up to translation, modulation, dilation and the Fourier transform) were known. In 2011, K. Grochenig and J. Stockler extended this class by including the totally positive functions of finite type (uncountable family yet depending on a finite number of parameters) and later added the Gaussian finite type totally positive functions. We suggest a different approach to the problem and prove that all Herglotz rational functions with imaginary poles also belong to the class under consideration. This approach also gives new results for general rational functions. In particular, we are able to confirm Daubechies' conjecture for rational functions and irrational densities. *(Joint work with Yu. Lyubarskii and A. Kulikov.)*

A. Bikchentaev (Kazan Federal University).

On measurable operators affiliated to semifinite von Neumann algebras.

Let $\mathcal{M}$ be a von Neumann algebra of operators on a Hilbert space $\mathcal{H}$ and τ be a faithful normal semifinite trace on $\mathcal{M}$. Let t_τ be the measure topology on the $*$-algebra $S(\mathcal{M},\tau)$ of all τ-measurable operators. We define three t_τ-closed classes $\mathcal{P}_1$, $\mathcal{P}_2$ and $\mathcal{P}_3$ of $S(\mathcal{M},\tau)$ with $\mathcal{P}_1 \cup \mathcal{P}_3 \subset \mathcal{P}_2$ and investigate their properties.

If an operator $T \in S(\mathcal{M},\tau)$ is p-hyponormal for $0 < p \le 1$, then T lies in $\mathcal{P}_1$; if an operator T lies in $\mathcal{P}_k$, then UTU^* belongs to $\mathcal{P}_k$ for all isometries U from $\mathcal{M}$ and $k = 1, 2, 3$; if an operator T from $\mathcal{P}_1$ admits the bounded inverse T^{-1} then T^{-1} lies in $\mathcal{P}_1$. If a bounded operator T lies in $\mathcal{P}_1 \cup \mathcal{P}_3$ then T is normaloid. If an $T \in S(\mathcal{M},\tau)$ is hyponormal and T^n is τ-compact operator for some natural number n, then T is both normal and τ-compact. If an operator T lies in $\mathcal{P}_1$ then T^2 belongs to $\mathcal{P}_1$. If $\mathcal{M} = \mathcal{B}(\mathcal{H})$ and $\tau = \mathrm{tr}$ is the canonical trace, then the class $\mathcal{P}_1$ (resp., $\mathcal{P}_3$) coincides with the set of all paranormal (resp., $*$-paranormal) operators on $\mathcal{H}$. Let $A, B \in S(\mathcal{M},\tau)$ and A be p-hyponormal with $0 < p \le 1$. If AB is τ-compact then A^*B is τ-compact [1]. We also investigate some properties of the Kalton–Sukochev uniform majorization in $S(\mathcal{M},\tau)$ [2].

The work performed under the development program of Volga Region Mathematical Center (agreement No. 075-02-2021-1393).

[1] A. Bikchentaev, Paranormal measurable operators affiliated with a semifinite von Neumann algebra. II, Positivity **24** (2020), no. 5, 1487–1501.

[2] A. Bikchentaev, F. Sukochev, Inequalities for the block projection operators, J. Funct. Anal. **280** (2021), no. 7, 108851, 18 pp.

E. Doubtsov (St. Petersburg Department of V.A. Steklov Mathematical Institute).

Calderón–Zygmund operators on RBMO.

Let μ be an n-dimensional finite positive measure on $\mathbb{R}^m$. We obtain a $T1$ condition sufficient for the boundedness of Calderón–Zygmund operators on RBMO(μ), the regular BMO space of Tolsa. (Joint work with A.V. Vasin.)

K. Dyakonov (ICREA and Universitat de Barcelona).

Functions with small and large spectra as (non)extreme points in subspaces of H^∞.

Let Λ be a subset of $\mathbb{Z}_+ := 0, 1, 2, \ldots$, and let $H^\infty(\Lambda)$ denote the space of bounded analytic functions f on the disk whose coefficients $\hat{f}(k)$ vanish for $k \notin \Lambda$. Assuming that either Λ or $\mathbb{Z}_+ \setminus \Lambda$ is finite, we determine the extreme points of the unit ball in $H^\infty(\Lambda)$.

S. Favorov (Karazin's Kharkiv national university).

Discrete temperate distributions in Euclidean spaces.

Let $f = \sum_{\lambda\in\Lambda}\sum_k p_k(\lambda)D^k\delta_\lambda$ be a temperate distribution on $\mathbb{R}^d$ with uniformly discrete support Λ and uniformly discrete spectrum (that is $\operatorname{supp}\hat{\mathrm{f}}$). We prove that under conditions

$$0 < c \le \sum_k |p_k(\lambda)| \le C < \infty$$

the support Λ is a finite union of cosets of full-rank lattices. The optimality of the above estimates is discussed. The result generalizes the corresponding one for discrete measures [1]. For its proof we use some properties of almost periodic distributions and a local version of Wiener's Theorem on trigonometric series.

[1] S.Yu. Favorov, Large Fourier Quasicrystals and Wiener's Theorem, J. Fourier Anal. Appl. **25**, no. 2, (2019), 377–392.

K. Fedorovskiy (Lomonosov Moscow State University).

Dirichlet problem for second order elliptic PDE, and one related approximation problem.

First, we plan to discuss the Dirichlet problem for second order homogeneous elliptic equations with constant complex coefficients in domains on the complex plane. We will present and discuss the following result: every Jordan domain in $\mathbb{C}$ with $C^{1,\alpha}$-smooth boundary, $\alpha \in (0,1)$, is not regular with respect to the Dirichlet problem for any not strongly elliptic equation of a specified type. Next, we will touch the problem on uniform approximation of functions on compact sets in the complex plane by polynomial solutions of such equations. We present some recent results and open questions related to this problem and its links with the Dirichlet problems under consideration. *(Joint work with A. Bagapsh and M. Mazalov.)*

A. Gasparyan (PSI RAS).

Inequalities for hypercubic functionals. Generalized Chebyshev inequalities.

By hypercubic functionals we mean the expressions of type

$$G_p{}^{(\sigma)}(\Phi, F) = \sum_{k \in B_2^p} (-1)^{(\sigma,k)} \Phi(k, F), \Phi(\overline{k}, F),$$

where $\sigma \in B_2^p = \{0,1\}^p$, $\Phi :\, B_2^p \times \mathcal{F} \to \mathbb{R}$ (or $\mathbb{C}$), and $\mathcal{F} \subseteq V^p$, V is a functional space. In the very particular case $p = 2$, this family of functionals contains Binet–Cauchy, Chebyshev, Cauchy–Bunyakovsky–Schwarz, Newton, Alexandrov and some other type functionals, for which well-known identities and inequalities hold. In this talk, I plan to familiarize colleagues with identities and inequalities established for the hypercubic functionals defined above. Also, certain applications will be discussed.

W. Green (Washington University in St. Louis).

Dominating Sets in Bergman Spaces on Domains in $\mathbb{C}^n$.

We obtain local estimates (also called propagation of smallness, or Remez-type inequalities) for analytic functions in several variables. Using Carleman estimates, we obtain a three sphere-type inequality, where the outer two spheres can be any sets satisfying a boundary separation property, and the inner sphere can be any set of positive Lebesgue measure. We apply this local result to characterize the dominating sets for Bergman spaces on strongly pseudo-convex domains and give a sufficient condition on more general domains.

E. Kalita (Institute of Applied Mathematics and Mechanics).

Nonlinear elliptic equations with subcoercive operators.

Let X, Y be separable reflexive Banach spaces, $X \subset Y$ densely. Let $A : X \to X^*$ be monotone operator (dissipative, in the linear case), coercive in the norm of Y, that is $(Au, u)/|u|_Y \to \infty$ as $|u|_Y \to \infty,\ u \in X$.

We introduce a notion of solution of the equation $Au = f$ for $u \in Y,\ f \in Y^*$ in such a situation. If $\exists\, V \subset X$ dense (and so $V \subset Y$ dense) with $Av \in Y^*$ for $v \in V$, then our solution coincides with the solution in sense of monotonic extension of the operator A.

We study elliptic equations of nonstrictly divergent form $\operatorname{div}^t A(x, D^s u) = f(x)$, $s \neq t$, $x \in \mathbb{R}^n$, under degenerate Cordes–type condition, and we obtain existence and uniqueness results in a more general situation than known before.

V. Kapustin (St. Petersburg Department of Steklov Mathematical Institute).

A canonical system related to the Riemann zeta function.

We present a de Branges space and the associated canonical system related to the Riemann zeta function.

I. Kayumov (Kazan Federal University).

On linear extremal problems in the classes of nonvanishing bounded analytic functions.

I will talk about linear extremal problems in the classes of nonvanishing bounded analytic functions defined in the unit disk centered at the origin. It turns out that the extremal values in the class of nonvanishing bounded analytic functions can be estimated with the sharp constant $2/e$ from below via the extremal values in the whole class of bounded analytic functions.

B. Khabibullin (Bashkir State University).

The Nevanlinna characteristic and integral inequalities with maximal radial characteristic for meromorphic functions and differences of subharmonic functions.

Let f be a meromorphic function on the complex plane $\mathbb{C}$ with the maximum function $M(r, f)$ of its modulus on the circles centered at zero of radius r. A number of classical, well-known and widely used results allow us to estimate from above the integrals of the positive part of the logarithm $\ln M(t, f)$ over subsets $E \subset [0, r]$ via the Nevanlinna characteristic $T(r, f)$ and the linear Lebesgue measure of the set E. We give much more general estimates for the Lebesgue–Stieltjes integrals of $\ln^+ M(t, f)$ over the increasing integration function of m. Our results are established immediately for the differences of subharmonic functions on closed circles centered at zero, i.e., δ-subharmonic functions, but they are new for meromorphic functions on $\mathbb{C}$ and contain all the previous results on this topic as a very special and extreme case. The only condition in our main theorem is the Dini condition for the modulus of continuity of the integration function m. This condition is, in a sense, necessary. Thus, our results, to a certain extent, complete the study of upper estimates for the integrals of the radial maximum growth characteristics of arbitrary meromorphic and δ-subharmonic functions through the Nevanlinna characteristic with its versions and through quantities associated with the integration function m such as the Hausdorff h-measure or h-content, and the d-dimensional Hausdorff measure of the support of non-constancy for m.

D. Khammatova (Kazan Federal University).

On Landau's type estimates for coefficients of univalent functions.

We estimate the sum of coefficients for functions with restrictions on the pre-Schwarzian derivative. We obtain an estimate, which is sharp up to a constant and find upper and lower bounds for that constant. Also, we estimate the sum of the first three coefficients for all functions f such that $\log f'(z)$ is bounded with respect to the Bloch norm.

E. Korotyaev (St. Petersburg State University).

Two-sided estimates for band lengths of Laplacians on periodic graphs.

We consider Laplacians on periodic discrete graphs. Their spectrum consists of a finite number of bands. We obtain two-sided estimates for the total length of the spectral bands of the Laplacian in terms of geometric parameters of the graph. Moreover, we consider Schroedinger operators with periodic potentials on periodic discrete graphs. We obtain two-sided estimates for the total length of the spectral bands of the operators in terms of geometric parameters of the graph and the potential. The proof is based on the Floquet theory and the trace formulas for fiber operators. In particular, we show that these estimates are sharp. It means that these estimates become identities for specific graphs and potentials. *(Joint work with N. Saburova.)*

A. Lunyov (Facebook, Inc.).

Stability of spectral characteristics of boundary value problems for 2×2 Dirac type systems.

Boundary value problems associated in $L^2([0,1];\mathbb{C}^2)$ with the following 2×2 Dirac type equation

$$L_U(Q)y = -iB^{-1}y' + Q(x)y = \lambda y, \ \ B = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}, \ \ b_1 < 0 < b_2, \ \ y = \mathrm{col}(y_1, y_2),$$

with a potential matrix $Q \in L^p([0,1];\mathbb{C}^{2\times 2})$, $p \geq 1$, and subject to the regular boundary conditions $Uy := U_1, U_2 y = 0$ has been investigated in numerous papers. If $b_2 = -b_1 = 1$, then this equation is equivalent to the one-dimensional Dirac equation.

In this talk, we present recent results concerning the stability property under the perturbation $Q \to \widetilde{Q}$ of different spectral characteristics of the corresponding operator $L_U(Q)$ obtained in our recent preprint [2]. Our approach to the spectral stability relies on the existence of triangular transformation operators for Dirac system with $Q \in L^1$, which was established in [1].

Assuming boundary conditions to be strictly regular, let $\Lambda_Q = \lambda_{Q,n_{n\in\mathbb{Z}}}$ be the spectrum of $L_U(Q)$. It happens that the mapping $Q \to \Lambda_Q - \Lambda_0$ sends $L^p([0,1];\mathbb{C}^{2\times 2})$ into the weighted space $\ell^p((1+|n|)^{p-2})$ as well as into $\ell^{p'}$, $p' = p/(p-1)$. One of our main results is the Lipschitz property of this mapping on the compact sets in $L^p([0,1];\mathbb{C}^{2\times 2})$, $p \in [1,2]$. The proof of the inclusion into the weighted space $\ell^p((1+|n|)^{p-2})$ involves, as an important ingredient, an inequality that generalizes the classical Hardy–Littlewood inequality for Fourier coefficients. A similar result is proved for the eigenfunctions of $L_U(Q)$, using the deep Carleson–Hunt theorem for the "maximal" Fourier transform. Certain modifications of these spectral stability results are also proved for the balls in $L^p([0,1];\mathbb{C}^{2\times 2})$, $p \in [1,2]$.

[1] A.A. Lunyov and M.M. Malamud, On the Riesz basis property of root vectors system for 2×2 Dirac type operators, J. Math. Anal. Appl. **441** (2016), 57–103; arXiv:1504.04954.

[2] A.A. Lunyov and M.M. Malamud, Stability of spectral characteristics and Bari basis property of boundary value problems for 2×2 Dirac type systems, arXiv:2012.11170.

V. Muller (Mathematical Institute, Czech Academy of Sciences).

High order isometric liftings and dilations.

We show that a Hilbert space bounded linear operator has an m-isometric lifting for some integer $m \geq 1$ if and only if the norms of its powers grow polynomially. In analogy with unitary dilations of contractions, we prove that such operators also have an invertible m-isometric dilation. *(Joint work with C. Badea and L. Suciu.)*

M. Malamud (Peoples Friendship University of Russia).

On singular spectrum of N-dimensional perturbations (to the Aronszajn–Donoghue–Kac theory).

The main results of the Aronszajn–Donoghue–Kac theory are extended to the case of n-dimensional (in the resolvent sense) perturbations $\widetilde{A}$ of an operator $A_0 = A_0^*$ defined on a Hilbert space $\mathfrak{H}$. Applying technique of boundary triplets, we describe singular continuous and point spectra of extensions A_B of a simple symmetric operator A acting in $\mathfrak{H}$ in terms of the Weyl function $M(\cdot)$ of the pair $\{A, A_0\}$ and a boundary n-dimensional operator $B = B^*$. Assuming that the multiplicity of singular spectrum of A_0 is maximal, we establish that the singular parts $E^s_{A_B}$ and $E^s_{A_0}$ of the spectral measures E_{A_B} and E_{A_0} of the operators A_B and A_0, respectively, are mutually singular. We also obtain estimates of the multiplicity of point and singular continuous spectra of selfadjoint extensions of A.

Applying this result to direct sums $A = A^{(1)} \oplus A^{(2)}$, we generalize and clarify the Kac theorem on multiplicity of singular spectrum of Schroedinger operator on the line. Applications to differential operators will be also discussed. The talk is based on results announced in [1].

[1] M.M. Malamud, On singular spectrum of finite-dimensional perturbations (toward the Aronszajn–Donoghue–Kac Theory), Doklady Math. **100** (2019), no. 1, 358—362.

K. Malyutin (Kursk State University).

Interpolation by Jones-type series in spaces of analytic functions in the half-plane.

Let $\mathcal{A}_+$ denote a space of analytic functions in the upper half-plane $\mathbb{C}_+ = \{z : \operatorname{Im} z > 0\}$, where $\mathcal{A}_+$ is one of the following spaces: 1) the space functions of finite order $\rho > 1$, 2) the space functions of finite order $\rho > 0$ and of normal type, 3) the space of bounded functions H^∞. Let $D = \{a_n, q_n\}_{n=1}^\infty$ be a divisor (i.e., a set of distinct complex numbers $\{a_n\}_{n=1}^\infty \subset \mathbb{C}_+$ with limit points on the real axis, together with their integer multiplicities $\{q_n\}_{n=1}^\infty \subset \mathbb{N}$). In the space $\mathcal{A}_+$, the following interpolation problem is considered:

$$F^{(k-1)}(a_n) = b_{n,k}, \quad k = 1, 2, \ldots, q_n, \quad n \in \mathbb{N}, \quad F \in \mathcal{A}_+.$$

We find a criterion for the interpolation of the divisor D in $\mathcal{A}_+$ in terms of canonical products and in terms of the Nevanlinna measure $\mu_+(G) = \sum_{a_n \in G} q_n \sin(\arg a_n)$ defined by the interpolation nodes. The solution to the problem is constructed in the form of a Jones-type interpolation series. This method is also used to solve interpolation problems in spaces of meromorphic functions with a given growth in the half-plane.

D. Mokeev (Higher School of Economics).

Inverse resonance problem for Dirac operators on the half-line.

We consider massless Dirac operators with compactly supported potentials on the half-line. We solve the inverse problems in terms of Jost function and scattering matrix, including characterization. We study resonances as zeros of the Jost function and prove that they uniquely determine a potential of the Dirac operator. We also estimate the forbidden domain for the resonances and determine asymptotics of the resonance counting function. Finally, we show how these results are applied to canonical systems. *(Joint work with Evgeny Korotyaev.)*

I. Musin (Institute of Mathematics with Computer Centre of Ufa Scientific Centre of RAS).

On the Fourier–Laplace transform of functionals on a space of ultradifferentiable functions on a convex compact.

Classes of ultradifferentiable functions are classically defined by imposing growth conditions on the derivatives of the functions. Following this approach, we consider the Fréchet–Schwartz space of infinitely differentiable functions with uniform bounds on their partial derivatives on the closure of a bounded convex domain of a multidimensional real space. The main aim is to obtain a Paley–Wiener–Schwartz type theorem relating the properties of linear continuous functionals on this space with the behaviour of their Fourier–Laplace transforms. Similar problems have been considered by M. Neymark, B. A. Taylor, M. Langenbruch, and A. V. Abanin. Also, we give applications of the theorem obtained to PDE and their systems.

R. Nasibullin (Kazan Federal University).

Hardy's inequalities for Jacobi weights.

We prove new one-dimensional Hardy type inequalities for Jacobi weights. Using these inequalities, we obtain Nehari–Pokornii type univalence conditions for analytic in the unite disk $\mathbb{D} = z \in \mathbb{C} : |z| < 1$ functions. The following theorem holds.

Theorem 1. Let f be a meromorphic function in $\mathbb{D}$. Assume that $n \in \mathbb{N}$, a_k and μ_k, $k = \overline{1,n}$, are positive real numbers and

$$|S_f(z)| \le \sum_{k=1}^{n} \frac{b_k A(\mu_k)}{(1-|z|^2)^{\mu_k}}, \quad z \in \mathbb{D},$$

where $b_k = \frac{2P_{2-\mu_k}}{A(\mu_k)} a_k$, $a_1 + a_2 + \ldots + a_n \le 1$, $0 \le \mu_1 \le \mu_2 \le \ldots \le \mu_n \le 2$, and

$$A(\mu) = \begin{cases} 2^{3\mu-1}\pi^{2(1-\mu)}, & 0 \le \mu \le 1, \\ 2^{3-\mu}, & 1 \le \mu \le 2. \end{cases}$$

Then the function f is univalent in $\mathbb{D}$.

S. Nasyrov (Kazan Federal University).

Asymptotical behavior of the conformal modulus of doubly connected planar domain under unbounded stretching along the abscissa axis.

Conformal moduli of doubly connected domains and quadrilaterals play an important role in investigation of various problems of the theory of conformal and quasiconformal mappings. One of the simplest quasiconformal mappings is the stretching along the abscissa axis. In 2005, Prof. Vourinen suggested the problem of finding the asymptotics of the conformal modulus of a doubly connected planar domain under stretching it along the abscissa axis, as the coefficient of stretching tends to infinity. We discuss the problem in the cases of bounded and unbounded domains and, for some types of domains, we find the main term of the asymptotics. Our study is based on the methods of geometric functions of a complex variable, in particular, on certain results by Ahlfors and Warshavskii.

S. Novikov (Samara National Research University).

Equiangular tight frames as dictionaries in sparse representations.

Let $\mathbf{\Phi}$ be a $d \times n$ matrix with real or complex entries, and let the columns of $\mathbf{\Phi}$ be ℓ_2-normalized. Consider the following linear under-determined set of equations:

$$\mathbf{\Phi}\alpha = \mathbf{x}.$$

We refer to $\mathbf{x}$ as a signal to be processed, and α stands for its *representation*. The matrix $\mathbf{\Phi}$ is referred to as the dictionary, and its columns $\{\{\varphi\}_i\}_{i=1}^n$ are called *atoms*. Equiangular tight frames have an important advantage over other dictionaries. In particular, it's possible to calculate the spark for such dictionaries.

A. Osekowski (University of Warsaw).

A dual approach to Burkholder's estimates and applications.

A celebrated result of Burkholder from the 80's identifies the best constant in the L^p estimate for martingale transforms ($1 < p < \infty$). This result is a starting point for numerous extensions and applications in many areas of mathematics. Burkholder's proof exploits the so-called Bellman function method: it rests on the construction of a certain special function, enjoying appropriate size and concavity requirements. This special function is of interest on its own right and appears, quite unexpectedly, in the context of quasiconformal mappings and geometric function theory. There is a dual approach to the L^p bound, invented by Nazarov, Treil and Volberg in the 90's. It gives a slightly worse constant, but the alternative Bellman function plays an independent, significant role in harmonic analysis, as evidenced in many papers in the last 20 years.

The purpose of the talk is to show how to improve the latter approach so that it produces the best constant and to discuss a number of applications.

M. Peloso (Università degli Studi di Milano).

Boundedness of Bergman projections on homogeneous Siegel domains.

In this talk I will discuss the problem of boundedness of the Bergman projection on Bergman spaces on homogeneous Siegel domains of Type II. It was shown that in the case of tube domains over symmetric cone, that is, symmetric Siegel domains of Type I, the Bergman projection P may be bounded even if the operator P_+, having as integral kernel the modulus of the Bergman kernel, is unbounded. I will describe what is known in this case and then discuss the case of homogeneous Siegel domains of Type II. I will discuss equivalent conditions, such as characterization of boundary values, duality, Hardy-type inequalities. *(Joint work with M. Calzi.)*

S. Platonov (Petrozavovsk State University).

On the Hankel transform of functions from Nikol'skii type classes.

Let a function f belong to the Lebesgue class $L_p(\mathbb{R})$, $1 \le p \le 2$, and let $\hat{f}$ be the Fourier transform of f. The classical theorem of E. Titchmarsh states that if the function f belongs to the Lipschitz class $\mathrm{Lip}(r,p;\mathbb{R})$, $0 < r \le 1$, then $\hat{f}$ is in $L_q(\mathbb{R})$ for $\frac{p}{rp+p-1} < q \le \frac{p}{p-1}$. Using the methods of Fourier–Bessel harmonic analysis, we prove an analogue of this result for the Hankel transform of functions from Nikol'skii type function classes on the half-line $[0,+\infty)$.

H. Queffélec (University of Lille).

Weighted Hardy–Hilbert spaces of analytic functions and their composition operators.

Let $\mathbb{D}$ be the unit disk and $\beta = (\beta_n)_{n\ge 0}$ a sequence of positive numbers satisfying

$$\liminf_{n\to\infty} \beta_n^{1/n} \ge 1.$$

The associated Hardy space $H = H^2(\beta) \subset \mathcal{H}(\mathbb{D})$ is the set of analytic functions $f(z) = \sum_{n=0}^{\infty} a_n z^n$ such that

$$\|f\|^2 = \sum_{n=0}^{\infty} |a_n|^2 \beta_n < \infty.$$

Such are the Hardy, Bergman, Dirichlet, spaces ($\beta_n = 1, 1/(n+1), n+1$ respectively). In this talk, we will investigate sufficient, or necessary, conditions, on β for all composition operators $C_\varphi, C_\varphi(f) = f \circ \varphi$, to be bounded on H. Here, $\varphi : \mathbb{D} \to \mathbb{D}$ is analytic. We will provide a simple necessary and sufficient condition when β is (essentially) decreasing, meaning that

$$\sup_{m\ge n} \frac{\beta_m}{\beta_n} \le C < \infty.$$

(Joint work with P. Lefèvre, D. Li, L. Rodríguez-Piazza.)

O. Reinov (Saint Petersburg State University).

On factorization of nuclear operators through $S_{s,p}$-operators.

We consider the question of factorizations for products of $l_{r,q}$-nuclear and close operators through the operators of Schatten–Lorentz classes $S_{s,p}(H)$.

To get the sharpness of certain results, we apply the following generalization of a result by G. Pisier on convolution operators: Given a compact Abelian group G and $f \in C(G)$, the convolution operator $f* \cdot : M(G) \to C(G)$ can be factored through an $S_{s,p}$-operator if and only if the set $\hat{f}$ of Fourier coefficients of f is in $l_{r,q}$, where $1/q = 1/p + 1, 1/r = 1/s + 1$. If $q = r = 1$, we get the result of G. Pisier.

N. Shirokov (St. Petersburg State University and National Research University Higher School of Economics in SPb).

Polynomial approximation in a convex domain in $\mathbb{C}^n$ which is exponentially decreasing inside the domain.

Let $\Omega \subset \mathbb{C}^n$, $n \geq 2$, be a bounded convex domain with a C^2-smooth boundary. We suppose that Ω satisfies certain properties. The strictly convex, in the analytical sense, domains satisfy those properties. It is proved that for any function f holomorphic is Ω and smooth in $\bar{\Omega}$, there exist polynomials P_N, $\deg P_N \leq N$, such that $\left|f(z) - P_N(z)\right|$ has a polynomial decay for $z \in \partial\Omega$ and has an exponential decay as z is strictly inside Ω.

L. Slavin (University of Cincinnati).

A sharp BMO-BLO bound for the martingale maximal function.

We construct the exact Bellman function for the BMO-BLO action of the natural martingale maximal function for continuous-time martingales. (BLO stands for "bounded lower oscillation"; the natural maximal function is the one without the absolute value in the average). As consequences, we show that the BMO-BLO norm of the operator is 1 and also obtain a sharp weak-type inequality, which can be integrated to produce a broad range of sharp phi-estimates.

In an earlier work we found the corresponding Bellman function for alpha-regular discrete-time martingales, including the dyadic martingale. I will discuss the essential differences between the two cases. This is joint work with Adam Osękowski and Vasily Vasyunin.

M. Stepanova (Lomonosov Moscow State University).

An order over the set of analytic functions of two variables.

In the context of the theory of analytical complexity we can introduce a natural relation “to be not simpler” over the set of germs of analytic functions. The following question arises: is that relation an order relation (three axioms)? It turns out that the axiom of antisymmetry does not hold. We will give an example. Thus, this relation is only a preorder and turns into an order only after factorization.

S. Treil (Brown University).

Complex symmetric operators and inverse spectral problem for Hankel operators.

Hankel operators are bounded operators on ℓ^2 whose matrix is constant on diagonals orthogonal to the main one (i.e. its entries depend only on the sum of indices). Such operators connect many classical problems in complex analysis with problems in operator theory.

I'll be discussing the inverse spectral problem for such operators, i.e. the problem of finding a Hankel operator with prescribed spectral data. For non-selfadjoint operators the theory of the so-called complex symmetric operators gives a convenient way to present such spectral data.

It was discovered by P. Gerard and S. Grellier that the spectral data of a compact Hankel operator Γ and the reduced Hankel operator ΓS (where S is the forward shift in ℓ^2) completely determine the Hankel operator Γ. This turns out to be the case for general Hankel operators as well, i.e. the map from Hankel operators to the spectral data of Γ and of ΓS is injective. But what about surjectivity?

In the talk I'll discuss some positive results, as well as some counterexamples. Connections with Clark measures play an important role in the investigation, and will be discussed. *(Joint work with P. Gerard and A. Pushnitskii.)*

A. Tyulenev (Steklov Mathematical Institute of RAS).

Trace and extension theorems for Sobolev $W^1_p(\mathbb{R}^n)$-spaces. The case $p \in (1, n]$.

Let $S \subset \mathbb{R}^n$ be an arbitrary nonempty compact set such that the d-Hausdorff content $\mathcal{H}^d_\infty(S) > 0$ for some $d \in (0, n]$. For each $p \in (\max 1, n - d, n]$, we give an almost sharp intrinsic description of the trace space $W^1_p(\mathbb{R}^n)|_S$ of the Sobolev space $W^1_p(\mathbb{R}^n)$. Furthermore, for each $\varepsilon \in (0, \min p - (n - d), p - 1)$, we construct a new bounded linear extension operator $\mathrm{Ext}_{S,d,\varepsilon}$ mapping the trace space $W^1_p(\mathbb{R}^n)|_S$ to the space $W^1_{p-\varepsilon}(\mathbb{R}^n)$ such that $\mathrm{Ext}_{S,d,\varepsilon}$ is a right inverse operator for the corresponding trace operator. The construction of the operator $\mathrm{Ext}_{S,d,\varepsilon}$ does not depend on p and based on new delicate combinatorial methods.

I. Vasilyev (Jawaharlal Nehru University).

Hölder regularity of solutions to a non-local drift-diffusion equation, along a non-solenoidal BMO flow.

In this talk, we will show how to apply methods of the real harmonic analysis in order to prove the critical Hölder regularity of solutions to a critical non-local transport-diffusion equation, in case when the velocity field is in BMO and is not necessarily divergence free. Our proofs are inspired by some ideas of F. Nazarov and A. Kiselev. *(Joint work with F. Vigneron.)*

I. Videnskii (St. Petersburg State University).

An infinite product of extremal multipliers of a Hilbert space with Schwarz-Pick kernel.

For a reproducing kernel Hilbert space H on a set X, we define a distance $d(a, Z)$ between a point a and a subset Z. A space H has the Schwarz–Pick kernel if for every pair (a, Z), there exists an extremal multiplier of norm less than or equal to one, which vanishes at the set Z, and attains the value $d(a, Z)$ at the point a. For a normalized Hilbert space with Schwarz–Pick kernel and for a sequence of subsets that satisfies an abstract Blaschke condition, we prove that the associate Blaschke product of extremal multipliers converges in the norm of H.

R. Yulmukhametov (Institute of Mathematics with Computing Centre – Subdivision of the Ufa Federal Research Centre of Russian Academy of Science).

Riesz bases of normalized reproducing kernels in radial Hilbert spaces of entire functions.

We consider a reproducing kernel radial Hilbert space of entire functions and prove necessary and sufficient conditions for the existence of unconditional bases of reproducing kernels in terms of norms of monomials. The results obtained are applied to weighted Fock spaces.

"Analysis Day" on the occasion of 80-th anniversary of Nikolai Nikolski

(July 5, 2021, joint event with University of Bordeaux)

A. Bonami (Orleans University).

Inequalities involving Hardy spaces of Musielak type.

The space $H^{\log}$ has been introduced in relation with the product of functions $f \times g$ (in the distributional sense) such that f belongs to $H^1(\mathbb{R}^d)$ and g belongs to $BMO(\mathbb{R}^d)$. Since then, Hardy spaces of Musielak type have been the object of many studies, as well as generalizations of inequalities involving products. I will discuss some of them and characterize non-negative L^1 functions that belong to $H^{\log}(\mathbb{R}^d)$ and other Hardy spaces of Musielak type. *(Joint work in progress with S. Grellier and B. Sehba.)*

E. Malinnikova (Stanford University).

Landis' conjecture on the decay of solutions to Schroedinger equations on the plane.

We consider a real-valued function on the plane for which the absolute value of the Laplacian is bounded by the absolute value of the function at each point. In other words, we look at solutions of the stationary Schroedinger equation with a bounded potential. The question discussed in the talk is how fast such function may decay at infinity. We give the answer in dimension two, in higher dimensions the corresponding problem is open. *(Joint work with A. Logunov, N. Nadirashvili, and F. Nazarov.)*

V. Peller (Michigan State University, St. Petersburg State University).

Completely bounded Schur multipliers of Schatten–von Neumann class $S_p, 0 < p < 1$.

Gilles Pisier posed the problem of whether a matrix Schur multiplier of the Schatten–von Neumann class S_p for $1 < p < \infty$, $p \neq 2$, has to be completely bounded. We have proved that this is true in the case $0 < p < 1$. We also consider various sufficient conditions for an infinite matrix to be a Schur multiplier of S_p. In particular, we introduce a p-analog of the Haagerup tensor product of ℓ^∞ spaces. *(Joint work with A. B. Aleksandrov.)*

A. Poltoratski (University of Wisconsin).

Pointwise convergence of scattering data.

The scattering transform, appearing in the study of differential operators, can be viewed as an analog of the Fourier transform in non-linear settings. This connection brings up numerous questions on finding non-linear analogs of classical results of Fourier analysis. One of the fundamental results of linear analysis is a theorem by L. Carleson on pointwise convergence of the Fourier series. In this talk I will discuss convergence for the scattering data of a real Dirac system on the half-line and present an analog of Carleson's theorem for the non-linear Fourier transform.

K. Seip (Norwegian Institute of Science and Technology).

Contractive inequalities for Hardy spaces.

It has been recognized by many authors that contractive inequalities involving norms of H^p spaces can be particularly useful when the objects in question (like the norms and/or an underlying operator) lift in a multiplicative way from one (or few) to several (or infinitely many) variables. This has been my main motivation for looking more systematically at various contractive inequalities in the context of Hardy spaces on the d-dimensional torus. I will discuss results from recent studies of Hardy–Littlewood inequalities, Riesz projections, idempotent Fourier multipliers, and Hilbert points (which in one variable is another word for inner functions). We will see interesting phenomena occurring both in the transition from low to high dimension and from low to infinite dimension. *(Joint work with Sergei Konyagin, Herve Queffélec, and Eero Saksman and with Ole Fredrik Brevig and Joaquim Ortega-Cerda.)*

A. Volberg (Michigan State University).

Orthogonality in Banach spaces.

All spaces below are not Hilbert spaces. Given two finite dimensional subspaces L, K of a normed space X, we call K orthogonal to L if for every unit vector in K, the distance of this vector to L is 1. This usually does not mean that L is orthogonal to K. We consider the following questions: 1) Let E, F are two finite dimensional subspaces of a normed space X and let $dimF = dimE + m$. Can we always find a subspace K in F such that E is orthogonal to K? 2) Can we always find a subspace K in F such that K is orthogonal to E? 3) Can we always choose K of dimension m? 4) If not, what is the maximal possible dimension?

These questions seem to be considered 60-80 years ago, and in fact, some of them were answered (by Krein–Krasnoselski–Milman). But it looks like that some of these questions were overlooked...