LOCALIZATION FOR QUASIPERIODIC OPERATORS: SOME LITERATURE RECOMMENDATIONS

Note: the list is by no means exhaustive; it is just a starting point.

General spectral theory (spectral theorem, etc.):

- G. Teschl, Mathematical Methods in Quantum Mechanics.
- M. Reed, B. Simon, Methods of Modern Mathematical Physics, Volume 1.
- M. Birman, M. Solomyak, Spectral Theory of Self-Adjoint Operators in Hilbert Space.

Schnol's theorem (specific version for our purposes)

- R. Han, *Schńol's theorem and the spectrum of long range operators*, Proc. Amer. Math. Soc. **147** (2019), 2887 – 2897. See references for the original papers by Schnol.

Introduction to ergodic Schrödinger operators

- H. Cycon, R. Froese, W. Kirsch, B. Simon, Schrödinger operators, with application to quantum mechanics and global geometry, Texts and Monographs in Physics. Berlin etc. Springer-Verlag (1987). Chapters 9,10; also Chapter 6 (RAGE theorem).
- Book by D. Damanik and J. Fillman is in progress.
- D. Damanik, *Lyapunov exponents and spectral analysis of ergodic Schrödinger operators: a survey of Kotani theory and its applications.* Spectral theory and mathematical physics: a Festschrift in honor of Barry Simon's 60th birthday, 539 – 563, Proc. Sympos. Pure Math. **76**, Part 2, Amer. Math. Soc., Providence, RI, 2007.
- P. Deift, B. Simon, *Almost periodic Schrödinger operators III. The absolutely continuous spectrum in one dimension*, Comm. Math. Phys. **90** (1983), 389 – 411.

Anderson model (random operators): there are many proofs of localization in 1D, including two recent papers which contain a lot of references. I also include the standard book on the multi-dimensional case. Make sure to check references therein for full history.

- V. Bucaj, D. Damanik, J. Fillman, V. Gerbuz, T. VandenBoom, F. Wang, Z. Zhang, *Localization for the one-dimensional Anderson model via positivity and large deviations for the Lyapunov exponent*, Trans. Amer. Math. Soc. 372 (2019), 3619 – 3667.
- S. Jitomirskaya, X. Zhu, *Large Deviations of the Lyapunov Exponent and Localization for the 1D Anderson Model*, Comm. Math. Phys. 370 (2019), 311 – 324.
- M. Aizenman, S. Warzel, *Random Operators: Disorder Effects on Quantum Spectra and Dynamics*, AMS, 2016.

There are probably 100 or so papers on the almost Mathieu operator. The following papers contain most of references and state-of-the-art results:

- A. Avila, *The absolutely continuous spectrum of the almost Mathieu operator*, preprint, `https://arxiv.org/abs/0810.2965`, 2008.
- S. Jitomirskaya, *Metal-insulator transition for the almost Mathieu operator*, *Ann. of Math.* **150**, (1999), 1159 – 1175.
- S. Jitormiskya and W. Liu, *Universal hierarchical structure of quasiperiodic eigenfunctions*, Ann. Math. **187** (2018), no. 3, 721 – 776.

Results from complex analysis. Specifically, Riesz representation theorem.

- Levin, B. Ya., Lectures on Entire Functions, Translations of Mathematical Monographs, AMS, 1996.

Perturbative methods for quasiperiodic operators (KAM-type methods, etc).

- Chulaevsky V.A., Sinai Y.G., *Anderson localization for the 1-D discrete Schrödinger operator with two-frequency potential.* Commun. Math. Phys. 125, no. 1 (1989), 91 – 112.
- Dinaburg E.I., Sinai J.G., *The one-dimensional Schrödinger equation with quasiperiodic potential*, Funct. Anal. Appl. 9 (1975), no. 4, 8 – 21 (in Russian).
- Fröhlich J., Spencer T., Wittwer P., *Localization for a class of one-dimensional quasiperiodic Schrödinger operators.* Commun. Math. Phys. 132 (1990), no. 1, 5 – 25.
- Sinai Y.G., *Anderson localization for one-dimensional difference Schrödinger operator with quasiperiodic potential.* J. Stat. Phys. 46 (1987), no. 5 – 6, 861 – 909.

Bourgain's method: book. Contains a lot of material and references. The exposition is very condensed.

- Bourgain J., Green's Function Estimates for Lattice Schrödinger Operators and Applications, Annals of Mathematics Studies 158, Princeton University Press, 2005.

Bourgain's method, one-dimensional results. The first two papers (Bourgain – Goldstein, Bourgain–Jitomirskaya) were used to prepare the minicourse.

- Bourgain J., Goldstein M., On nonperturbative localization with quasi-periodic potential, Ann. Math. 152 (2000), 835 – 879.
- Bourgain J., Jitomirskaya S., *Anderson localization for the Band Model*, Geometric Aspects of Functional Analysis, 67 – 79. Part of Lecture Notes in Mathematics, vol. 1745.
- Goldstein M., Schlag W., Voda M., *On the spectrum of multi-frequency quasiperiodic Schrödinger operators with large coupling*, Inv. Math. 217 (2019), no. 2, 603 – 701.
- Schlag, W., *An introduction to multiscale techniques in the theory of Anderson localization.*, `https://arxiv.org/abs/2104.14248`.

Bourgain's method, higher dimensional results.

- Bourgain J., Goldstein M., Schlag W, *Anderson Localization for Schrödinger operators on* $\mathbb{Z}^2$ *with quasi-periodic potential*, Acta Math. 188 (2002), 41 – 86.
- Bourgain J., *Anderson localization for quasi-periodic lattice Schrödinger operators on* $\mathbb{Z}^d$*, d arbitrary*, Geom. Funct. Anal. 17 (2007), 682 – 706.
- Jitomirskaya S., Liu W., Shi Y., *Anderson localization for multi-frequency quasi-periodic operators on* $\mathbb{Z}^d$, Geom. Funct. Anal. 30 (2020), no. 2, 457–481.
- Liu W., *Quantitative inductive estimates for Green's functions of non-self-adjoint matrices*, Analysis & PDE to appear, `https://arxiv.org/abs/2007.00578`.

Semialgebraic sets.

- S. Basu, R. Pollack and Roy M., Algorithms in Real Algebraic geometry. Springer, Berlin (2006).
- J. Bochnak, M. Coste and M. Roy., Real Algebraic Geometry. Springer, Berlin (1998).
- Binyamini G., Novikov D., *Complex cellular structures*, Ann. Math. 190 (2019), 145 – 248.