Linear algebraic groups and related structures

On occasion of the 70 anniversary of Nikolai Gordeev

Leonhard Euler International Mathematical Institute

St Petersburg, 11—12 November 2021

https://indico.eimi.ru/event/437/

Schedule of talks (Moscow Time, CET+2)

Thursday. November 11th:

10:45 – 11:00 Welcome

11:00 – 11:50 A.Vershik (PDMI RAS), "A new look at groups generated by involutions"

Abstract. We construct a general theory of finite groups generated by reflections based on the notion of numberings of partially ordered sets. The classical theory of Coxeter groups (for the case of the symmetric group) corresponds to a special choice of an ordered set ---the simplest Young diagram (n, 1). From a formal viewpoint, we replace the defining relation (σ_i · σ_{i + 1})^ 3 = Id, where σ_i, i = 1,2 ... k, are involutions, by the relation (σ_i · σ_{i + 1})^6 = Id leaving the commutation condition unchanged: (σ_i · σ_j)^2 = Id for | i-j |> 1, with the following extra condition: the group generated by the adjacent involutions σ_i and \sigma_{i+1} is a finite product of the groups of orders 2 and 3.
Such symmetry groups (finite and countable) naturally arise in combinatorics and their classification does not seem hopeless. For example, if the ordered set is a finite Young diagram, then hypothetically we do not go beyond the Coxeter groups. The characteristic of our approach is that groups are considered as special subgroups of symmetric groups. Particularly interesting are infinite such groups and their representations.

12:00-12:50 V.Popov (MI RAS) "Group varieties and group structures of algebraic groups"

Abstract. The talk is aimed to discuss to what extent the group variety of a connected algebraic group or the group manifold of a connected real Lie group determines its group structure.

13:00 -13:50 C.De Concini, (Univ. Roma I) "Paving Springer fibers. The Case of E7".

Abstract. In the paper De Concini, C.; Lusztig, G.; Procesi, C. Homology of the zero-set of a nilpotent vector field on a flag manifold. J. Amer. Math. Soc. 1 (1988), no. 1, 15-34, it was proven the so-called Springer fiber B_n for any nilpotent n element in a complex simple Lie algebra g has homological properties that suggest that B_n should have a paving by affine spaces.
This last statement was proved to hold in the case in which g is classical but remained open for exceptional groups in types E7 and E8.
In a joint project with Maffei we are trying to fill this gap. At this point our efforts has been successful in type E7 and "almost" in type E8 where one is reduced to show it only in one case.
The goal of the talk is to survey the problem and give an idea on how to show our new results.

Break

16:00-16:50 E.Bayer-Fluckiger, (EPF Lausanne), "Automorphisms of K3 surfaces and isometries of lattices"

Abstract. We show that every Salem number of degree 4,6,8,12, 14, 16 and 18 is the dynamical degree of an automorphism of a complex K3 surface, and give necessary and sufficient conditions for this in degrees 10 and 18.

17:00-17:50 A. Lubotzky, (Weizmann Inst.), "Stability, non-approximated groups, and high-dimensional expanders "

Abstract. Several well-known open questions, such as: "are all groups sofic or hyperlinear?" , have a common form: can all groups be approximated by asymptotic homomorphisms into the symmetric groups Sym(n) (in the sofic case) or the unitary groups U(n) (in the hyperlinear case)? In the case of U(n), the question can be asked with respect to different metrics and norms.

We answer, for the first time, some of these versions, showing that there exist finitely presented groups that are not approximated by U(n) with respect to the Frobenius (=L_2) norm and many other norms.

The strategy is via the notion of "stability": Some higher dimensional cohomology vanishing phenomenon is proven to imply stability. Using the Garland method (a.k.a. high dimensional expanders as quotients of Bruhat-Tits buildings), it is shown that some non-residually-finite groups are stable and hence cannot be approximated. These groups are central extensions of some lattices in p-adic Lie groups (constructed via a p-adic version of a result of Deligne).

All notions will be explained. Based on joint works with M. De Chiffre, L. Glebsky and A. Thom and with I. Oppenheim .

18:00-18:50 V.Chernousov, (Univ. Alberta) "On conjugacy of Cartan subalgebras in extended affine Lie algebras and classification of torsors over Laurent polynomial rings"

*Abstract***.** In the talk we will present results on a problem of conjugacy of Cartan subalgebras for a class of infinite dimensional Lie algebras called extended affine Lie algebras and how this problem intertwinds with the classification of torsors over Laurent polynomial rings. Joint work with P. Gille, E. Neher, A. Pianzola, U. Yahorau.

19:15 Informal meeting, greetings, virtual refreshments…

Friday, November, 12th, Moscow Time

9:00 -09:50 A. Merkurjev, (UCLA), "Degree two negligible cohomology of finite groups"

Abstract. Abstract: Let G be a finite group and let M be an abelian group viewed as a G-module with trivial G-action. Fix a field F. A cohomology class c in H^n(G,M) is called negligible over F if for every field extension L/F and every continuous group homomorphism of the absolute Galois group of L to G the class c belongs to the kernel of the induced homomorphism H^n(G,M) --> H^n(L,M).
We determine all negligible cohomology classes c in H^2(G,M).
This is a joint work with M.Gherman.

Short break

10:30 – 11:20 A.Thom, (TU Dresden), "Dense images of word maps"

Abstract. Let w be a non-trivial element of the free group. For ε>0, we prove that there exists an integer N(ε,w) such that w(G) is ε-dense in G, where G is a finite simple group or compact Lie group of rank N(ε,w) endowed with its natural bi-invariant metric. This confirms metric versions of a conjectures by Shalev and Larsen.

11:30-12:20 I.Panin, (PDMI RAS), "On Grothendieck---Serre conjecture in mixed characteristic."

Abstract. We prove the conjecture for the group SL_{1,D} in the mixed characteristic smooth case.

12.30 -12:55 V.Petrov, (SPbU), "Isotropy of Tits construction"

Abstract. Tits construction produces a Lie algebra out of a composition algebra and an exceptional Jordan algebra. The type of the result is F_4, ${}^2E_6$, E_7 or E_8 when the composition algebra has dimension 1,2,4 or 8 respectively. Garibaldi and Petersson noted that the Tits index ${}^2E_6^{35}$ cannot occur as a result of Tits construction. Recently Alex Henke proved that the Tits index E_7^{66} is also not possible. We push the analogy further and show that Lie algebras of Tits index E_8^{133} don't lie in the image of the Tits construction. The proof relies on basic facts about symmetric spaces and our joint result with Garibaldi and Semenov about isotropy of groups of type E_7 in terms of the Rost invariant. This is a part of a work joint with Simon Rigby.

13:00 --13:25 A.Stavrova, (SPbU), "R-equivalence on reductive algebraic groups"

Abstract. We generalize Manin's notion of R-equivalence for algebraic varieties to schemes and use this generalization to solve the specialization problem for R-equivalence class groups of reductive groups in the equicharacteristic case. The talk is based on the joint work with Philippe Gille https://arxiv.org/abs/2107.01950**.**

Break

15:30 – 15:55 S.Sinchuk, (SPbU), "On the A1-fundamental groups of Chevalley groups"

Abstract. Let k be an arbitrary field. The aim of the talk is to give on overview of a recent preprint, in which it is shown that in the linear case ($\Phi=\rA_\ell$, $\ell \geq 4$) and even orthogonal case ($\Phi = \rD_\ell$, $\ell\geq 7$, $\mathrm{char}(k)\neq 2$) the unstable functor $\mathrm{K}_2(\Phi, R)$ possesses the $\mathbb{A}^1$-invariance property and therefore can be represented in the unstable $\mathbb{A}^1$-homotopy category $\mathcal{H}^{\mathbb{A}^1}_{k}$ as the fundamental group of the simply-connected Chevalley--Demazure group scheme $\mathrm{G}_\mathrm{sc}(\Phi,-)$. This invariance result can be considered as the $\K_2$-analogue of the geometric case of Bass--Quillen conjecture. With our technique we are also able to show for a semilocal regular k-algebra A that $\K_2(\Phi, A)$ embeds as a subgroup into the Milnor group $\K^\mathrm{M}_2(\mathrm{Frac}(A))$.

16:00 – 16.25 A.Lavrenov, (SPbU) **"Morava K-theory of orthogonal groups"**

Abstract. Let G be a Chevalley group, and A* denote an oriented cohomology theory in the sense of Levine-Morel (e.g., Grothendieck's K-theory, Chow group, etc.) Then the ring A*(G) is an interesting invariant of the group. In the talk I will explain how to compute this ring for the special orthogonal group G = SO_m and the Morava K-theory A* = K(n)*. The talk is based on a joint work with V. Petrov, P. Sechin, and N. Semenov.

16:30 – 16:55 P.Gvozdevsky, (SPbU), "Bounded reduction of orthogonal matrices over polynomial rings"

Abstract. The talk is based on the author's paper [1].In the paper [2], Vaserstein showed that for any coefficient ring C of finite Krull dimension and any r ≥ max(3, dim C +2), an arbitrary matrix from the special linear group over a polynomial ring g ∈ SLr(C[x1, . . . , xn]) can be reduced to the diagonal shape diag(g',1) where g' ∈ SL_{r−1}(C[x_1, . . . , x_n]), by a bounded number (namely n(21n − 79)/2 + 33nr + 4r − 4) of elementary operations. He also deduced from this the similar result for the symplectic group.

In the talk we state the similar result recently obtained by the author for the split orthogonal group. That is the last remaining case among the split classical groups. This result can be viewed as the effective version of the early surjective K_1-functor stability, proved by Suslin and Kopeiko in [3].

We also discuss the connection of such theorems with the proof of the Kazhdan property (T) for split groups over finitely generated rings.

References

[1] Gvozdevsky P. Bounded reduction of orthogonal matrices over polynomial rings //arXiv: math.GR/2106.12697v1 (2021). [2] Vaserstein L. N. Bounded reduction of invertible matrices over polynomial rings by addition operations // Unpublished. Preprint 2006:http://www.personal.psu.edu/lxv1/pm2.pdf [3] Suslin A. A., Kopeiko V. I. Quadratic Modules and Orthogonal Group over Polynomial Rings // J. Soviet Math. Vol. 20, no. 6. (1982) p. 2665–2691.

17:00 -17:25 E.Voronetsky, (SPbU), "Explicit presentation of relative Steinberg groups".

Abstract. Relative Steinberg groups are defined as crossed modules over the absolute Steinberg group with some generators and relations. We find their presentations as abstract groups.

!7: 30 - Free time, greetings, closing…