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Ivan Alexeev: Stable random variables with a complex index α: The case of $|\alpha - 1/2| < 1/2$.

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In this paper, we construct complex-valued random variables that satisfy the usual stability condition, but for a complex parameter α such that $|\alpha - 1/2| < 1/2$. The characteristic function of the obtained random variables is found and limit theorems for sums of independent random variables are proved. The corresponding Levy processes and semigroups of operators corresponding to these processes are constructed.

Marija Cuparić: On the asymptotic efficiency of recent characterization based exponentiality tests of L^2 and L^∞ type

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Based on the joint work with Bojana Milošević (University of Belgrade, Faculty of Mathematics) and Marko Obradović (University of Belgrade, Faculty of Mathematics)

Recently, several powerful characterization based tests for exponentiality have been proposed. The test statistics of those tests represent weighted L^2 and L^∞ distances between appropriate V-empirical integral transforms of random variables that appear in the characterization. It turns up that those tests are very powerful and efficient competitors to commonly used tests. Here we present those tests, and their limiting behaviour under the null hypothesis and under fixed alternative distributions. Those results are generalized to the case of V- and U- statistics with estimated parameters. A significant part of the talk will be dedicated to wide comparison study from the Bahadur efficiency perspective.

References

[1] M. Cuparić, B. Milošević and M. Obradović. New consistent exponentiality tests based on V-empirical Laplace transforms with comparison of efficiencies. *Preprint, arXiv:1904.00840*, 2019

[2] M. Cuparić, B. Milošević and M. Obradović. Asymptotic distribution of certain degenerate V- and U-statistics with estimated parameters. *Preprint, arXiv:2012.11020*, 2020.

[3] M. Cuparić, B. Milošević and M. Obradović. New L^2-type exponentiality tests. *SORT*, 43 (1): 25–50, 2019.

[4] M. Cuparić, B. Milošević and M. Obradović. New class of supremum-type exponentiality tests based on V-empirical Laplace transforms and Puri-Rubin characterization. *Proceedings of the 21st European Young Statisticians Meeting*, 15–19, 2019.

[5] M. Cuparić, B. Milošević, Ya. Yu. Nikitin and M. Obradović. Some consistent exponentiality tests based on Puri-Rubin and Desu characterizations. *Applications of Mathematics*, 65 (3): 245–255, 2020.

Vladimir Dmitriev: Martingale control variate for reinforcement learning

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Based on the work with Denis Belomestny and Sergey Samsonov (Higher School of Economics, International Laboratory of Stochastic Algorithms and High-Dimensional Inferences) Policy gradient algorithms belong to reinforcement learning and are used to learn optimal or suboptimal policies. In PG setup policy is chosen to be parametric conditional distributions. After collecting trajectory samples of agent, the stochastic gradient ascent is performed in the space of the policy parameters. Several techniques are proposed to reduce variance of the stochastic gradient [1]. Control variates are common choice for variance reduction tasks, associated with Markov decision processes [2]. Standard control variates depend on states and being subtracted from the estimate of state-action value function [3]. Such variates are usually associated with baselines. Standard choice of baseline is an estimate of state value function [4]. In contrast, construction of state-action dependent control variates is complicated. Well known approaches are Stein control variates [5] or Q-prop [6].
We propose to exploit martingale decomposition [7] to design the procedure to construct state-action dependent control variate for arbitrary policy gradient algorithm. Martingale decomposition could be applied to MDP if Markov process has the dynamics of the specific type [7], where new element of Markov chain is a deterministic function of previous element and i.i.d sequence. Such dynamics describes wide range of Markov chains. Our technique takes trajectory samples, which should additionally include data about noise in the system [7]. The algorithm proceeds as follows - after computation of advantage estimates it subtracts sample estimates of few terms of the martingale decomposition. The standard policy gradient step is performed with the shifted advantages. Terms of martingale decomposition are approximated with ANNs. Each step of policy gradient the weights of control variate approximations are updated with an appropriate loss. Our technique is tested on several continuous control tasks and shows the increase in performance of the algorithms after martingale control variate augmentation.

References

[1] Siguad O., Garcia F.. Markov Decision Processes in Artificial Intelligence 2013.

[2] Belomestny, D., Iosipoi, L., Moulines, E., Naumov, A., Samsonov, S., . Variance reduction for dependent sequences with applications to Stochastic Gradient MCMC 2020.

[3] Puterman M.. Markov Decision Processes: Discrete Stochastic Dynamic Programming. 1994.

[4] Schulman J., Moritz, P., Sergey, L., Michael, J.,Abbeel, P.. High-Dimensional Continuous Control Using Generalized Advantage Estimation 2016.

[5] Liu, H., Feng, Y., Mao, Y., Zhou, D., Peng, J., Liu, Q. Action-depedent Control Variates for Policy Optimization via Stein's Identity. 2017.

[6] Gu, S., Lillicrap, T., Ghahramani, Z., Turner, R., Levine, S. Q-Prop: Sample-Efficient Policy Gradient with An Off-Policy Critic. 2017.

[7] Belomestny, D., Moulines, E., Samsonov, S. Variance reduction for MCMC methods via martingale representations 2020.

Mariia Dospolova: Discrete intrinsic volumes

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For a convex lattice polytope $P \subset \mathbb{R}^d$ of dimension d with vertices in $\mathbb{Z}^d$, denote by $L(P)$ its discrete volume which is defined as the number of integer points inside P. The classical result due to Ehrhart says that for a positive integer n, the function $L(nP)$ is a polynomial in n of degree d whose leading coefficient is the volume of P. In particular, $L(nP)$ approximates the volume of nP for large n.
In convex geometry, one of the central notion which generalizes the volume is the intrinsic volumes. The main goal of this talk is to introduce their discrete counterparts. In particular, we show that for them the analogue of the Ehrhart result holds, where the volume is replaced by the intrinsic volume.
We also introduce and study a notion of Grassmann valuation which generalizes both the discrete volume and the solid-angle valuation introduced by Reeve and Macdonald.

References

[1] E. Ehrhart, *Sur les polyèdres rationnels homothétiques à n dimensions*, C. R. Acad. Sci. Paris, 254 (1962), pp. 616–618.

[2] D. Amelunxen and M. Lotz. Intrinsic volumes of polyhedral cones: a combinatorial perspective. *Discrete & Computational Geometry*, 58(2):371–409, 2017.

[3] M. Beck and S. Robins. *Computing the continuous discretely.* Undergraduate Texts in Mathematics. Springer, New York, second edition, 2015. Integer-point enumeration in polyhedra, With illustrations by David Austin.

[4] R. P. Stanley, *Combinatorial reciprocity theorems*, Advances in Math., 14 (1974), pp. 194–253.

[5] B. Grünbaum. Grassmann angles of convex polytopes. *Acta Math.*, 121:293–302, 1968.

[6] K. Jochemko and R. Sanyal. Combinatorial positivity of translation-invariant valuations and a discrete hadwiger theorem. *Journal of the European Mathematical Society*, 20(9):2181–2208, 2018.

[7] F. Götze, Z. Kabluchko, and D. Zaporozhets. Grassmann angles and absorption probabilities of Gaussian convex hulls. Preprint at http://arxiv.org/abs/1911.04184, 2019.

[8] J. E. Reeve. On the volume of lattice polyhedra. *Proc. london Math. Soc. (3)*, 7:378–395, 1957.

[9] I. G. Macdonald. The volume of a lattice polyhedron. *Proc. Cambridge Philos. Soc.*, 59:719–726, 1963.

Alexander Gushchin: The joint distributions of certain pairs of stochastic processes

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We study the sets of joint distributions of certain pairs of stochastic processes, for example, a martingale and its current maximum or an increasing process and its compensator. The emphasis is on extreme points of these sets. In particular, we introduce a subset of the set of joint distributions of an increasing process and its compensator that can be characterised by the property that every distribution in this subset is the only one in the considered set with given marginals. We study connections between different settings and also a connection with the Skorokhod embedding problem. We discuss several classical problems that can be solved in this way, as well as new results.

References

[1] D. A. Borzykh. On a property of joint terminal distributions of locally integrable increasing processes and their compensators. *Theory Stoch. Process.*, 23(39)(2):7–20, 2018.

[2] A. A. Gushchin. The joint law of terminal values of a nonnegative submartingale and its compensator. *Theory Probab. Appl.*, 62(2):216–235, 2018.

[3] A. A. Gushchin. On possible relations between an increasing process and its compensator in the non-integrable case. *Russian Math. Surveys*, 73(5):928–930, 2018.

[4] A. A. Gushchin. Single jump filtrations and local martingales. *Mod. Stoch., Theory Appl.*, 7(2):135–156, 2020.

[5] A. A. Gushchin. The joint law of a max-continuous local submartingale and its maximum. *Theory Probab. Appl.*, 65(4):545–557, 2021.

Nikita Karagodin: On the distribution of the last exit time over a slowly growing boundary for a Gaussian process

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Based on the joint work with Mikhail Lifshits (St.Petersburg State University, Department of Mathematics and Computer Sciences)

Consider a stationary Gaussian process with continuous trajectories and its "last exit time over a moving boundary", i.e. the last instant when the process hits a line $af(t)$ for a fixed function f and changing parameter a. After this instant, the process stays forever under the line. We are interested in the asymptotic distribution of the last exit time when the parameter a goes to zero.

In this work a limit theorem on the convergence of the distribution of the properly centered and scaled last exit time to a double exponential (Gumbel) law is proven.

A special case of this problem, for a particular process, emerged in recent works [1, 2] providing a mathematical study of a physical model (Brownian chain break).

The last exit time is a sufficiently popular object in the problems of economical applications such as studies of ruin probabilities. In risk theory the process

$$R(t) = u + af(t) - Y(t),$$

where $Y(t)$ is a centered Gaussian process with continious trajectories, represents a company balance. For instance, this could be used as the simplest model of an insurance company with a starting balance u, fixed income per time a (corresponding to a function $f(t) = t$) and stochastic expenses Y. In this setting the ruin time $\inf\{t : R(t) < 0\}$ is the first time preceeding the moment of the company balance going below 0 and the ultimate recovery time $\max\{t : R(t) \leq 0\}$ represents the moment after which the company balance will be always positive. In those settings, however, as a rule, one considers processes with stationary increments and trend is fixed, see [3, 4, 5], while in this work the ultimate recovery time for the case of the stationary process without starting balance and small trend is considered. As far as we know, the problem setting handling the small trend is new.

In order to succeed one has to choose between the variety of processes and the variety of boundaries. We consider weak assumptions on the correlation function of $Y(t)$ and strong assumptions on the boundary function $f(t)$, although covering the cases $f(t) = t^d, d > 0$ and $f(t) = \exp\{x^q\}, 0 < q < 1$. The case of the linear boundary is handled in [6].

References

[1] F.Aurzada, V.Betz, M.Lifshits, Breaking a chain of interacting Brownian particles. Ann. Appl. Probab. Published online: https://doi.org/10.1214/20-AAP1658.

[2] F.Aurzada, V.Betz, M.Lifshits, Universal break law for chains of Brownian particles with nearest neighbour interaction.

[3] K.Debicki, P.Liu, The time of ultimate recovery in Gaussian risk model. Extremes, 22(3), 499–521, 2019.

[4] J.Hüsler, Y.Zhang, On first and last ruin times of Gaussian processes. Statist. Probab. Lett., 78(10):1230–1235, 2008.

[5] Ch.Paroissin, L.Rabehasaina, First and last passage times of spectrally positive Lévy processes with application to reliability. Methodology and Computing in Applied Probability, 17, 351–372, 2015.

[6] N. Karagodin, M. Lifshits, On the distribution of the last exit time over a slowly growing linear boundary for a Gaussian process. https://arxiv.org/abs/2012.03222

Bojana Milošević: **Characterization based approach for construction of goodness-of-fit tests: randomly censored data case**

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Based on the joint work with Marija Cuparić (University of Belgrade, Faculty of Mathematics)
Recently, the characterization based approach for the construction of goodness-of-fit tests has become very popular. Most of the proposed tests have been designed for complete i.i.d. samples. However, in practice, we are often facing incomplete data sets. For example, a common problem in clinical trials is the missing data caused by patients who do not complete the study in a full schedule and drop out of the study without further measurements. This usually results in randomly right-censored data. Here we present an adaptation of the recently proposed tests based on equidistribution-type characterizations for such data. Their asymptotic properties are shown. We also present a wide empirical power study including several recent competitors. Additionally, we also consider an alternative approach to goodness-of-fit tests adaptation and explore its properties. The final part of the talk will be dedicated to several potential directions for future research.

Tatiana Moseeva: **Random sections of convex bodies and distribution of the volume of weighted Gaussian simplex**

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Based on the joint work with Dmitry Zaporozhets (St. Petersburg Department of Steklov Institute of Mathematics) and Alexander Tarasov (Saint Petersburg State University)
Let K be a convex body in the Euclidean space or on the unit sphere. We express the distribution of the distance between two random points in K via the distribution of the length of a random chord of K. As a corollary, in a spherical case we find the density of the distribution of the distance between two random points in a spherical cap.

In the second part of the talk we derive an explicit formula for the distribution of the volume of weighted Gaussian simplex.

References

[1] N. Aharonyan and V. Ohanyan. Moments of the distance between two random points. *Model. Artif. Intell.*, 10(2):64–70, 2016.

[2] E. Arbeiter and M. Zähle. Kinematic relations for Hausdorff moment measures in spherical spaces. *Math. Nachr.*, 153:333–348, 1991.

[3] G.D. Chakerian. Inequalities for the difference body of a convex body. *Proc. Amer. Math. Soc.*, 18:879–884, 1967.

[4] J.F.C. Kingman. Random secants of a convex body. *J. Appl. Probability*, 6:660–672, 1969.

[5] Z. Kabluchko and D. Zaporozhets. Expected volumes of Gaussian polytopes, external angles, and multiple order statistics. *Trans. Amer. Math. Soc.*, 372(3):1709–1733, 2019.

[6] R.E. Miles Isotropic random simplices. *Advances in Applied Probability*, 3(2):353–382, 1971.

[7] J. Randon-Furling and D. Zaporozhets. Convex hulls of several multidimensional Gaussian random walks. *arXiv preprint arXiv:2007.02768*, 2020.

Alexey Naumov: Tight High Probability Bounds for Linear Stochastic Approximation with Fixed Stepsize

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Based on the joint work with Alain Durmus (ENS Paris-Saclay), Eric Moulines (Ecole Polytechnique; HSE University), Sergey Samsonov (HSE University), Kevin Scaman (INRIA, DI/ENS, PSL Research University) and Hoi-To Wai (The Chinese University of Hong Kong) This paper provides a non-asymptotic analysis of linear stochastic approximation (LSA) algorithms with fixed stepsize. This family of methods arises in many machine learning tasks and is used to obtain approximate solutions of a linear system $\bar{A}\theta = \bar{b}$ for which $\bar{A}$ and $\bar{b}$ can only be accessed through random estimates $\{(\mathbf{A}_n, \mathbf{b}_n) : n \in \mathbb{N}^*\}$. Our analysis is based on new results regarding moments and high probability bounds for products of matrices which are shown to be tight. We derive high probability bounds on the performance of LSA under weaker conditions on the sequence $\{(\mathbf{A}_n, \mathbf{b}_n) : n \in \mathbb{N}^*\}$ than previous works. However, in contrast, we establish polynomial concentration bounds with order depending on the stepsize. We show that our conclusions cannot be improved without additional assumptions on the sequence of random matrices $\{\mathbf{A}_n : n \in \mathbb{N}^*\}$, and in particular that no Gaussian or exponential high probability bounds can hold. Finally, we pay a particular attention to establishing bounds with sharp order with respect to the number of iterations and the stepsize and whose leading terms contain the covariance matrices appearing in the central limit theorems.

Alexander Nazarov: Some new results on spectral equivalence of Gaussian random functions

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The talk is based on the joint works with Ya.Yu. Nikitin, see [1] and [2]. We introduce a new approach to the spectral equivalence of Gaussian processes and fields, based on the methods of operator theory in Hilbert space. Besides several new results including identities in law of quadratic norms for integrated and multiply integrated Gaussian random functions we give an application to goodness-of-fit testing.
The author's work was supported by joint RFBR–DFG grant 20-51-12004.

References

[1] A.I. Nazarov and Ya.Yu. Nikitin,
Spectral equivalence of Gaussian random functions: operator approach.
Preprint available at https://arxiv.org/abs/2005.05771, 14p. To appear in Bernoulli.

[2] A.I. Nazarov and Ya.Yu. Nikitin,
Gaussian processes centered at their online average, and applications.
Stat. Probab. Letters, V. 170 (2021), https://doi.org/10.1016/j.spl.2020.109013.

Sergey Nikitin: Large Deviations of Telecom Processes

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This is a joint work with M.A.Lifshits. The results are published in the prerprint arXiv:2107.11846
We study large deviation probabilities of Telecom processes appearing as limits in a critical regime of the infinite source Poisson model elaborated by I.Kaj and M.Taqqu. We examine three different regimes of large deviations (LD) depending on the deviation level. A Telecom process $(Y_t)_{t\geq 0}$ itself scales as $t^{1/\gamma}$ where t denotes time and $\gamma \in (1,2)$ is the key parameter of Y. One must distinguish moderate LD $\P(Y_t \geq q)$ with $t^{1/\gamma} \ll q \ll t$, intermediate LD with $q \approx t$, and ultralarge LD with $q \gg t$. The results we obtain essentially depend on another parameter of Y, namely resource distribution. We solve completely the cases of moderate and intermediate LD (the latter being the most technical one), whereas the ultralarge deviation asymptotics is found for the case of regularly varying distribution tails. In all considered cases, the large deviation level is essentially reached by the minimal necessary number of "service processes".

Artem Nikolaev: An analogue of the local time of the complex Brownian motion process

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Let $w(s), s > 0$ be a standard Wiener process and A be an arbitrary measurable subset of R. Denote by $\lambda(U)$, the Lebesgue measure of a set U. The sojourn time of the process $w(s)$ in the set A up to the time $t > 0$ is defined by

$$\mu(t, A) = \lambda \{s : w(s) \in A, 0 \leqslant s \leqslant t\},$$

For a fixed t the set function $\mu(t, \cdot)$ is a random measure, moreover it is known (see [1], ch. 1 ß4) that the measure $\mu(t, \cdot)$ is absolutely continuous with respect to the Lebesgue measure with probability 1. The Radon - Nikodym derivative

$$l(t, x) = \frac{d\mu}{d\lambda}(t, x) \tag{1}$$

is called the local time of the process $w(s)$ at the point x up to the time t.
It is well known that the local time of a Wiener process can be obtained using the Fourier analysis. Namely

$$l(t, x) = \lim_{M\to\infty} \frac{1}{2\pi} \int\limits_{-M}^{M} e^{-ipx} \int\limits_{0}^{t} e^{ipw(s)}\, ds\, dp. \tag{2}$$

The last limit exists in the metric of the space $L_2(\Omega \times \mathbf{R}, P \times \lambda)$ (see [1], ch.1, ß4).
Note that we cannot define the local time of the complex Brownian motion process $\sigma\, w(s),\ s \geqslant 0$ (σ – complex parameter) as the limit of a sequence of functions (2) in the space $L_2(\Omega \times \mathbf{R}, P \times \lambda)$, because the corresponding sequence diverges.
We construct an analogue of local time for the complex Brownian motion process $\sigma\, w(s), s \geqslant 0$, where σ is a complex number satisfying the conditions

$$0 < \arg\, \sigma \leqslant \frac{\pi}{4} \text{ and } |\sigma| = 1.$$

References

1. Borodin, A.N., Ibragimov, I.A.: Limit Theorems for Functional of Random Walks. Proceedings of the Steklov Institute of Mathematics 195, No. 259 (1995)

Svyatoslav Novikov: New results on asymptotic independence

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Based on the joint work with Youri Davydov (St. Petersburg State University, Faculty of Mathematics and Computer Science; Université de Lille, Laboratoire Paul Painlevé)

Following the papers [1], [2], we introduce several natural definitions of asymptotic independence (**AI-0 - AI-4**) of two sequences of random elements (X_n) and (Y_n), where $X_n : \Omega \to E_1$, $Y_n : \Omega \to E_2$ and E_1, E_2 are complete separable metric spaces.

AI-0: *For each bounded* **uniformly continuous** *functions*
$f : E_1 \to \mathbb{R}, g : E_2 \to \mathbb{R}$,

$$\mathbb{E}f(X_n)g(Y_n) - \mathbb{E}f(X_n)\mathbb{E}g(Y_n) \to 0,$$

when $n \to +\infty$.

AI-1: *For each bounded* **uniformly continuous** *function*
$h : E_1 \times E_2 \to \mathbb{R}$,

$$\int h(x,y)P_{(X_n,Y_n)}(dx,dy) - \int h(x,y)(P_{X_n} \times P_{Y_n})(dx,dy) \to 0,$$

when $n \to +\infty$.

AI-2: For all $A \in \mathcal{E}_1$, $B \in \mathcal{E}_2$,

$$|P_{(X_n,Y_n)}(A \times B) - P_{X_n}(A)P_{Y_n}(B)| \to 0, \quad n \to +\infty. \tag{3}$$

AI-3: $\sup_{A\in\mathcal{E}_1, B\in\mathcal{E}_2} |P_{(X_n,Y_n)}(A \times B) - P_{X_n}(A)P_{Y_n}(B)| \to 0, \quad n \to +\infty.$

AI-4: $\quad ||P_{(X_n,Y_n)} - P_{X_n} \times P_{Y_n}||_{var} \to 0, \quad n \to +\infty.$

Firstly, we discuss their basic properties, some simple connections between them and connections with properties of weak dependence. In particular, the case of tight sequences is considered in detail. Secondly, we consider the case when the random elements belong to the space of sequences and the case when the joint distributions are Gaussian. Finally, in order to clarify the relationships between different definitions, we provide some counterexamples.

References

[1] S. Novikov, *New results on asymptotic independence of random elements*, Zapiski Nauchnykh Seminarov POMI, 495(2020), 209-236.

[2] Y. Davydov, S. Novikov, *Remarks on asymptotic independence*, Teor. Veroyatnost. i Primenen., 66:1 (2021), 55-72

Nikita Puchkin: Exponential Savings in Agnostic Active Learning through Abstention

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Based on the joint work with Nikita Zhivotovskiy (ETH, Zurich)

Pool-based *active classification* can be seen as an extension of the classical PAC classification setup, where instead of learning from the labeled sample $(X_1, Y_1), \ldots, (X_n, Y_n)$, one can adaptively request the labels from a large pool $X_1, X_2, \ldots$ of i.i.d. unlabelled instances round by round. Our hope is to request significantly fewer labels Y_i and get the same statistical guarantees as in *passive learning*. We show that in pool-based active classification without assumptions on the underlying distribution, if the learner is given the power to abstain from some predictions by paying the price marginally smaller than the average loss 1/2 of a random guess, exponential savings in the number of label requests are possible whenever they are possible in the corresponding realizable problem. We extend this result to provide a necessary and sufficient condition for exponential savings in pool-based active classification under the model misspecification.

Ilya Ragozin: New Goodness-of-fit Tests for Rayleigh Distribution Family Based on Some Characterization and Some Special Property

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In this talk, new scale-free goodness-of-fit tests for the Rayleigh family will be constructed based on some special properties. The Limit theorems for the corresponding statistics are proved and logarithmic functions of large deviations are found. In this case our statistics are scale invariant, so the considered goodness-of-fit criteria are suitable for testing the composite hypothesis. As a result, the local Bahadur efficiency is calculated for common alternatives and for the specific Rice alternative.

Roman Ragozin: On conditional characteristics of PSI-processes and their sums. Gamma-distributed PSI-processes example

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Based on a joint work with Rusakov O. V. (Saint Petersburg State University, Faculty of Mathematics and Mechanics)

The PSI-process (Poisson stochastic index process) $\psi(s)$, $s \geq 0$, we understand as a model of a flow of i.i.d random variables, in a manner that each previous random variable changes to another one at moments of jumps of a Poisson-type process. PSI-processes generalise Pseudo-Poisson processes which are introduced in the famous Feller's monograph [1]. For details of description and properties of PSI-processes we refer to [2].

Main goal of our research is to study limits of sums of independent PSI-processes $\sum_{i=1}^{[tN]} \psi_i(N, s)$, as $N \to \infty$. The limit random field we denote as $\Psi(t, s),\ t \in [0, 1]$. We focus our interest in conditional characteristics of $\Psi(t, s)$ under condition that $\Psi(1, s_0)$ is fixed. The explicit formula for the conditional mathematical expectation is found for all cases when $\Psi(t, s)$ has an infinitely divisible distribution. Note that $\Psi(t, s_0)$ is a bridge-type process over $t \in [0, 1]$, when $\Psi(1, s_0)$ is fixed. In a special case when the terms of the initial sequence belong to the domain of attraction of the Gamma Law we examine in details Gamma bridges. We are able to obtain some explicit formulas due to a parallel between Gamma bridges and Brownian bridges (see [3] and [4]). This work one can apply to Information Theory and Stochastic Finance. Such kind of the model has found applications for pricing in real estate market in St. Petersburg.
The research was supported by the Russian Foundation for the Basic Research (RFBR), project No. 20-01-00646 (A).

References

[1] W. Feller. An introduction to Probability Theory and its applications. *New York: John Wiley & Sons, Inc.*, 1971.

[2] O. V. Rusakov. Pseudo-Poisson Processes with Stochastic Intensity and a Class of Processes which Generalize the Ornstein-Uhlenbeck Process. *Vestnik SPbSU. Mathematics. Mechanics. Astronomy*, vol. 4(62), issue 2, pp. 247–257, 2017.

[3] R. Mansuy, M. Yor. Harnesses, Lévy bridges and Monsieur Jourdain. *Stochastic Processes and their Applications* vol. 115(2), pp. 329-338, 2005.

[4] M. Emery, M. Yor. A Parallel between Brownian Bridges and Gamma Bridges. *Kyoto University, Publications of the Research Institute for Mathematical Sciences* vol. 40, pp. 669-688, 2004.

Sergey Samsonov: Probability and moment inequalities for additive functionals of geometrically ergodic Markov chains

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The talk is based on the joint work with Alain Durmus (Université Paris-Saclay, ENS Paris-Saclay), Eric Moulines (Ecole polytechnique, Paris, France), and Alexey Naumov (Higher School of Economics, Moscow, Russia).

In this talk we discuss novel moment and Bernstein-type inequalities for additive functionals of geometrically ergodic Markov chains. These inequalities extend the corresponding inequalities for independent random variables. Our conditions cover Markov chains con-verging geometrically to the stationary distribution either in V-norms or in weighted Wasserstein distances. Our inequalities apply to unbounded functions and depend explicitly on constants appearing in the conditions that we consider.

Irina Shevtsova: Convergence rate estimates in the Rényi theorem with no support constraints

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Based on the joint work with Mikhail Tselishchev (Lomonosov Moscow State University)
We generalize the notion of the equilibrium transform (also called the integrated tail, or the stationary renewal distribution) to distributions whose support may also intersect the negative axis. We study its properties and, in particular, we prove an optimal moment-type inequality for the L_1-distance between a distribution and its generalized equilibrium transform. Using the introduced transform and Stein's method we obtain convergence rate estimates of the accuracy of the exponential approximation to distributions of geometric randoms sums of independent random variables with no support constraints in the Kantorovich distance. By presenting some lower bounds we demonstrate that the obtained estimates are sharp. Furthermore, we extend the constructed estimates to generalized negative binomial random sums whose limit distributions are generalized gamma.
The presented results are published in [1] and [2].

References

[1] I. Shevtsova and M. Tselishchev. A generalized equilibrium transform with application to error bounds in the Rényi theorem with no support constraints. *Mathematics*, 8(4):577, 2020.

[2] I. Shevtsova and M. Tselishchev. On the accuracy of the generalized gamma approximation to generalized negative binomial random sums. *Mathematics*, 9(13):1571, 2021.

Ekaterina Simarova: Limit theorems for U-max statistics with kernels defined on a plane

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Based on the joint work with Yakov Nikitin (Saint Petersburg State University)
Let $\xi_1, \ldots, \xi_n, \ldots$ be i.i.d. random elements taking values in some measurable space. U-max statistics of degree $m \geq 1$ with kernel h are defined by the formula

$$H_n = \max_{\{(i_1,\ldots,i_m):1\leq i_1<\ldots<i_m\leq n\}} h(\xi_{i_1}, \ldots, \xi_{i_m}). \tag{4}$$

They were introduced by Lao and Mayer in 2008 in their joint paper [1] as extreme counterparts of U-statistics.
We discuss limiting behaviour of such statistics and obtain a limit relation with Weibull distribution as a limit for wide class of U-max statistics. We also consider the applications of these relations to the limiting behaviour of stochastic geometric objects, such as diameters, perimeters, areas of convex m-gons, e.t.c.

References

[1] W. Lao, M. Mayer . U-max-statistics. *J. Multivariate Anal.*, 99: 2039-2052,2008.

[2] Ya. Nikitin, E. Simarova. Generalized Limit Theorems For U-max Statistics. preprint.

Evgeny Spodarev: Extrapolation of Heavy Tailed Random Fields Via Level Sets

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Based on the joint work with Abhinav Das and Vitalii Makogin (Ulm University, Institute of Stochastics) We use the concept of excursion sets for the extrapolation of (possibly heavy-tailed) stationary infinitely divisible random fields. Doing so, we define excursion sets for a measurable random field and its linear predictor, and then minimize the expected volume of the symmetric difference of these sets under the condition that the univariate distributions of the predictor and of the field itself coincide. We illustrate the new approach on Gaussian random fields. There, our extrapolation problem appears to be a well–known linear programming problem with linear as well as quadratic constraints, a special case of the Second Order Cone Programming. Its complete solution is presented including the issues of existence and uniqueness. The solution is different depending on whether the mean of the field is assumed to be unknown or zero which shows direct parallels to ordinary or simple kiriging. It is shown that the new extrapolation method is exact and weakly (or a.s.) consistent. Moreover, it differs from known kriging methods.
A numerical simulation study shows that the new extrapolation performs well in Gaussian processes' case. These results are published in [1].

References

[1] A. Das, V. Makogin and E. Spodarev. Extrapolation of Stationary Infinitely Divisible Random Fields Via Level Sets. *Preprint*, Ulm University, 2021.

Natalia Stepanova: On sup-functionals of weighted empirical processes with applications to inferential problems in high dimensions

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Based on the joint work with Tatjana Pavlenko (KTH Royal Institute of Technology), Yibo Wang (University of Alberta), and Lee Thompson (Carleton University)
The object of our interest is a family of statistics based on sup-functionals of weighted empirical processes with a certain type of weight functions. The weight functions employed are Erdős–Feller–Kolmogorov–Petrovski (EFKP) upper-class functions of a Brownian bridge; Chibisov–O'Reilly functions are also considered. This family of statistics was introduced and studied in [1]. In high dimensions, the sup-norms of weighted empirical processes standardized by the EFKP upper-class

functions of a Brownian bridge have been seen to be good alternatives to Tukey's "higher criticism" statistic, as introduced in [2]. In this talk we demonstrate how the family of statistics at hand can be effectively used to solve various inferential problems where the higher criticism statistic and its modifications were previously commonly applied.

References

[1] M. Csörgő, S. Csörgő, L. Horváth and D. Mason. Weighted empirical and quantile processes. *Ann Probab.*, 14:31–85, 1986.

[2] D. Donoho and J. Jin. Higher criticism for detecting sparse heterogeneous mixtures. *Ann. Statist.*, 32:962–994, 2004.

Arman Tadevosian: Gaussian Assignment Process

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Based on the joint work with Mikhail Lifshits (St.Petersburg University, Department of Mathematics and Computer Sciences)

Consider the following *random assignment problem*. Let (X_{ij}) be a $n \times n$ random matrix with i.i.d. random entries having a common distribution $\mathcal{P}$. Let $\mathcal{S}_n$ denote the group of permutations $\pi : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$. For every $\pi \in \mathcal{S}_n$ let

$$S(\pi) = \sum_{i=1}^{n} X_{i\pi(i)}.$$

We are interested in the study of $\max_{\pi\in\mathcal{S}_n} S(\pi)$ in case, where $\mathcal{P} = \mathcal{N}(0, 1)$, and in finding of the optimal permutation π^*, such that $\mathbb{E}\, S(\pi^*) = \mathbb{E} \max_{\pi\in\mathcal{S}_n} S(\pi)$. We call $\{S(\pi), \pi \in \mathcal{S}_n\}$ *a Gaussian assignment process*.

The setting with (X_{ij}) uniformly distributed on $[0, 1]$ was studied by Steele [5] and Mézard and Parisi [3], where the authors proved that

$$\mathbb{E} \min_{\pi\in\mathcal{S}_n} S(\pi) = \zeta(2) - \frac{\zeta(2) + 2\zeta(3)}{n} + O\left(\frac{1}{n^2}\right), \quad n \to \infty,$$

$\zeta(\cdot)$ being Riemann's zeta function. Mézard et al. [4] also conjectured that in the exponential case ($\mathcal{P} = \mathrm{Exp}(1)$) it is true that

$$\mathbb{E} \min_{\pi\in\mathcal{S}_n} S(\pi) \to \zeta(2). \tag{5}$$

Aldous [1] gave a rigorous proof of conjecture (5). His approach is based on the assignment analysis of a graph with edges provided with exponentially distributed weights, see [2].

We show that in the Gaussian case

$$\mathbb{E} \max_{\pi\in\mathcal{S}_n} S(\pi) \sim n\sqrt{2 \log n}, \quad n \to \infty.$$

References

[1] D. J. Aldous. The $\zeta(2)$ limit in the random assignment problem. Random Structures & Algorithms, 18, No. 4, 381–418, 2001.

[2] D. J. Aldous. Asymptotics in the random assignment problem. Probability Theory and Related Fields, 93, No. 4, 507–534, 1992.

[3] M. Mézard and G. Parisi. On the solution of the random link matching problem. Journal de Physique, 48 (9), 1451–1459, 1987.

[4] M. Mézard, G. Parisi, and M. A. Virasoro. Spin glass theory and beyond. Ser.: World Scientific Lecture Notes in Physics, Vol.9, 1987.

[5] J. M. Steele. Probability theory and combinatorial optimization. Ser.: CBMS-NSF Regional Conference Series in Applied Mathematics,, Vol.69., 1997.

Alexander Tarasov: Reconstruction of the covariance matrix of a 3D centered Gaussian vector from the distribution of the coordinate-wise maximum

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Let

$$\mathbf{X} = (X_1, \dots, X_d), \quad \mathbf{Y} = (Y_1, \dots, Y_d)$$

be two centered Gaussian vectors in $\mathbb{R}^d$ with the covariance matrices Σ_X, Σ_Y. We are interested in the following question:
Is it true that if

$$\max(X_1, \dots, X_d) \stackrel{d}{=} \max(Y_1, \dots, Y_d) \tag{6}$$

then up to a permutation of indices

$$\Sigma_X = \Sigma_Y? \tag{7}$$

A slightly modified problem has been firstly formulated by Nádas [1] who considered the case $d = 2$ and imposed an additional assumption to (6): if

$$(\max(X_1, X_2), \operatorname{argmax}(X_1, X_2)) \stackrel{d}{=} (\max(Y_1, Y_2), \operatorname{argmax}(Y_1, Y_2)),$$

then Σ_X and Σ_Y coincide up to the permutation.
In a setting (6) the problem was originated in the works of Anderson and Ghurye [2], [3]. They proved (7) for $d = 2$ and also considered the case of non-centered $\mathbf{X}, \mathbf{Y}$ under the additional assumption

$$\operatorname{cov}(X_1, X_2), \operatorname{cov}(Y_1, Y_2) < 0,$$

where cov denoted the covariance. Very soon this assumption has been removed by Basu and Ghosh [4] who also considered the case $d = 3$ with the centered $\mathbf{X}, \mathbf{Y}$ under some assumption on correlation between vector's coordinates. Finally, in full generality this problem was open even in $\mathbb{R}^3$. Out result is (7) for $d = 3$ under the only assumption that covariance matrices are nondegenerate.

References

[1] Arthur Nádas. The Distribution of the Identified Minimum of a Normal Pair Determines' the Distribution of the Pair. *Technometrics*, 13(1):201-202, 1971.

[2] Anderson, T. W. and Ghurye, S. G. Identification of parameters by the distribution of a maximum random variable. *J. Roy. Statist. Soc. Ser. B*, 39(3):337-342, 1977

[3] Anderson, T. W. and Ghurye, S. G. Unique factorization of products of bivariate normal cumulative distribution functions. *Ann. Inst. Statist. Math.*, 30(1):63-69, 1978

[4] Basu, A. P. and Ghosh, J. K. Identifiability of the multinormal and other distributions under competing risks model. *J. Multivariate Anal.*, 8(3):413-429, 1978

Anna Tchirina: A modified problem by Ya. Yu. Nikitin

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Based on the joint work with A. I. Nazarov (St. Petersburg Department of Steklov Mathematical Institute of Russian Academy of Sciences)
The concept of local asymptotic Bahadur optimality for a sequence of statistics is well known. The following problem was posed and in many cases solved by Ya. Yu. Nikitin: to describe the class of distributions for which a given sequence of statistics is locally asymptotically optimal against specific alternatives (shift, scale, etc.). However, in some cases this problem has no solution. We propose to consider a modified problem. Its solution gives the value of the available asymptotic efficiency of the corresponding statistics.

References

[1] Ya. Yu. Nikitin. Asymptotic Efficiency of Nonparametric Tests. *Cambridge University Press*, 1995.

[2] N. Henze and Ya. Yu. Nikitin. Watson-Type Goodness-of-Fit Tests Based on the Integrated Empirical Process. *Math. Meth. Stat..*, 11(2):183–202, 2002.

[3] A. I. Nazarov and A. V. Tchirina. On the available local asymptotic efficiency of some goodness-of-fit criteria. *Preprints of the St. Petersburg Mathematical Society*, http://mathsoc.spb.ru/preprint/2021/index.html#01A

Daniil Tiapkin: Shattering Threshold for the Coloring Problem of a Random Hypergraph

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Based on the joint work with Dmitry Shabanov (HSE University, Moscow Institute of Physics and Technology, Lomonosov Moscow State University)

One of the most interesting and challenging problems in the graph theory is to determine the chromatic number of a graph and find the corresponding coloring. This problem is rather hard from the computational point of view since the problem of determining whether a graph is r-colorable or not is NP-complete for any given $r > 2$. However, in the setting of a *random* graph, or, equivalently, a typical graph with a very big number of vertices, the situation is much more optimistic: there is a sharp threshold for the property of r-colorability and there are very tight bounds for this threshold (see [1], [2]). Any such threshold corresponds to the sparse case when the number of edges is a linear function of n (see [3]). So, we can always believe that there is a proper coloring below the threshold. However, the known algorithms that quickly provide the required coloring with r colors work only when the number edges is significantly less than the r-colorability threshold (see [4]). The attempt to understand this effect was initiated by Achlioptas and Coja-Oghlan in [5], where they have proven the *shattering* effect: the space of proper colorings of a random graph with an edge-density greater than the some estimate starts to behave very poorly with probability tending to 1. For random hypergraphs, Achlioptas and Coja-Oghlan considered only the case of 2 colors and showed that the *shattering* effect also takes place. Our aim was to extend their result to an arbitrary number of colors and determine the shattering threshold for the r-coloring problem of a random k-uniform hypergraph for $r \geq 2, k \geq 3$. From the purely combinatorial point of view, this problem might be considered as a determining the structure of the set of proper r-colorings of the typical hypergraph with given number of edges. Also, our result unites the results concerning graph colorings and hypergraph 2-colorings into one theorem.

References

[1] Coja-Oghlan, A., Vilenchik, D.: The chromatic number of random graphs for most average degrees. International Mathematics Research Notices. 2016, 5801–5859 (2015). doi:10.1093/imrn/rnv333

[2] Coja-Oghlan, A.: Upper-bounding the k-colorability threshold by counting covers. Electronic Journal of Combinatorics. 20, #P32 (2013). doi:10.37236/3337

[3] Achlioptas, D., Friedgut E.: A sharp threshold for k-colorability. Random Structures and Algorithms. 14, 63–70 (1999). doi:10.1002/(SICI)1098-2418(1999010)14:1<63::AID-RSA3>3.0.CO;2-7

[4] Achlioptas D., Molloy M.: The analysis of a list-coloring algorithm on a random graph. In: Proceedings 38th Annual Symposium on Foundations of Computer Science, pp. 204–212, Miami Beach, FL, USA (1997). doi:10.1109/SFCS.1997.646109

[5] Achlioptas D., Coja-Oghlan A.: Algorithmic Barriers from Phase Transitions. In: 2008 49th Annual IEEE Symposium on Foundations of Computer Science, pp. 793–802, Philadelphia, PA, USA (2008). doi:10.1109/FOCS.2008.11

Alexandr Tikhomirov: Extreme singular values of sparse rectangular random matrices

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Based on the joint work with Professor F. Götze (Bielefeld University)
In the last five to ten years, significant progress has been made in the study of the asymptotic behavior of the spectrum of sparse random matrices. A typical example of such matrices is the incidence matrices of random graphs. Thus, for Bernoulli matrices, K. Tikhomirov obtained the exact asymptotics of the probability of singularity. See [5]. See as well the paper [1]. For the adjacency matrix of Erdös - Renyi random graphs, in the papers of Yau and L. Erdös and Co, a local semicircular law was proved, and the behavior of the largest and smallest singular numbers, eigenvector statistic was investigated. See paper [4] and literature to them. Lee and Hwang considered spectral properties of sparse sample covariance matrices, which includes biadjacency matrices of the bipartite Erdös–Renyi graph model. See [6]. In this paper authors proved local law for the eigenvalue density up to the upper spectral edge assuming that sparsity probability p has order $N^{-\varepsilon}$ for some $\varepsilon > 0$ and enries of msatrix X_{ij} are i.i.d. r.v.'s such that

$$\mathbb{E}\,|X_{11}|^2 = 1, \text{ and } \mathbb{E}\,|X_{11}|^q \leq \frac{Cq)^{cq}}{(Np)^{q-2}} \text{ for every } q \geq 1. \tag{8}$$

Here N denotes the main parameter of growing the dimension of matrices. More precise see below. We consider the bounds for the smallesest and for the largest singular values of sparse rectuangular random matrices assming that probability of sparsity p may decrease such that $Np \leq \log^{\frac{2}{\varkappa}} N$ and moment conditions more weakener as (8) (see condition (12) and (13)). Let $n = n(N)$, $N \geq n$. Consider independent identically distributed zero mean random variables X_{jk}, $1 \leq j \leq N$, $1 \leq k \leq n$ with $\mathbb{E}\,X_{jk}^2 = 1$ and independent of that set independent Bernoulli random variables ξ_{jk}, $1 \leq j \leq n$, $1 \leq k \leq m$ with $\mathbb{E}\,\xi_{jk} = p_N$. In addition suppose that $Np_N \to \infty$ as $N \to \infty$. In what follows we put $p = p_N$. Observe the sequence of random matrices

$$\mathbf{X} = (\xi_{jk} X_{jk})_{1 \leq j \leq N, 1 \leq k \leq n}. \tag{9}$$

Denote by $s_1 \geq \cdots \geq s_n$ the singular values of $\mathbf{X}$ and define the sample covariance matrix $\mathbf{Y} = \mathbf{X}^*\mathbf{X}$. Let $y = y(N) = \frac{n}{N}$. We shall assume that $y(N) \leq y_0 < 1$. The main results are the following

Theorem 1. Let $\mathbb{E}\,X_{jk} = 0$ and $\mathbb{E}\,|X_{jk}|^2 = 1$. Assume that

$$\mathbb{E}\,|X_{jk}|^{4+\delta} \leq C < \infty,$$

for any $j, k \geq 1$ and for some $\delta > 0$. Suppose that there exists a positive constant B, such that

$$Np_N \geq B \log^{\frac{2}{\kappa}} N, \tag{10}$$

where $\kappa = \frac{\delta}{2(4+\delta)}$. Assume additionally that

$$|X_{jk}| \le C(Np_N)^{\frac{1}{2}-\kappa}. \tag{11}$$

Then for every $q \ge 1$ there exists a constant $K = C(q, \delta, \mu_{4+\delta})$ such that

$$\Pr\{\, s_1 \ge K\sqrt{Np}\} \le CN^{-Q}$$

Theorem 2. Let $\mathbb{E}\, X_{jk} = 0$ and $\mathbb{E}\,|X_{jk}|^2 = 1$. Assume that

$$\mathbb{E}\,|X_{jk}|^{4+\delta} \le C < \infty,$$

for any $j, k \ge 1$ and for some $\delta > 0$. Suppose that there exists a positive constant B, such that

$$Np_N \ge B \log^{\frac{2}{\kappa}} N, \tag{12}$$

where $\kappa = \frac{\delta}{2(4+\delta)}$. Assume additionally that

$$|X_{jk}| \le C(Np_N)^{\frac{1}{2}-\kappa}. \tag{13}$$

Then for every $q \ge 1$ there exists a constant $C = C(q, \delta, \mu_{4+\delta})$

$$\Pr\{\, s_n \le \tau\sqrt{Np}\} \le CN^{-q},$$

References

[1] A. [Litvak], K.E. [Tikhomirov] Singularity of sparse bernoulli matrices. Preprint, arXiv:2004.03131

[2] M [Rudelson], R [Vershynin], The smallest singular value of a random rectangular matrix. *Pure and Applied Mathematics* 62 (2009), 1707 – 1739.

[3] M. [Rudelson], R. [Vershynin]. The Littlwood -Offord problem and invertibility of random matrices. *Advances in Mathematics*, 218(2008), 600–633.

[4] P. [Bourgate], J. [Huang] and H.-T. [Yau]. Eigenvector statistics of sparse random matrices *Preprint arXiv:1609.09022*

[5] Konstantin [Tikhomirov] Singularity of random Bernoulli matrices Annals of Mathematics, vol. 191, pp 592–639.

[6] Jong Yun [Hwang] anf Ji Oon [Lee]. Local Law and Tracy – Widom Limit fpr Sparse Sam[le Covariance Matrices *Bernoulli* 26(3):2400-2435 (2020)

Vladislav Vysotsky: Contraction principle for trajectories of random walks

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A large deviations principle (LPD) for trajectories of random walks with finite Laplace transform of their increments was proved by Mogulskii [2] in 1976. For the increments whose Laplace transform is finite only in a neighbourhood of zero, there were no tractable results until 2013, when Borovkov and Mogulskii [1] proved a weaker-than-standard result using a new concept of "metric" LDPs. We establish a contraction principle for general "metric" LDPs, showing that they are preserved under uniformly continuous mappings. This allows us to transform the result of Borovkov and Mogulskii [1] on random walks trajectories into standard LDPs. As an application, we extend the classical Cramér theorem by proving an LPD for kernel-weighted sums of i.i.d. random vectors in $\mathbb{R}^d$.

References

[1] A. A. Borovkov, and A. A. Mogulskii. Large deviation principles for random walk trajectories. II. *Theory Probab. Appl.*, 57:1–27, 2013.

[2] A. A. Mogulskii. Large deviations for the trajectories of multidimensional random walks. *Theor. Probab. Appl.*, 21(2):300–315, 1976.

Elena Yarovaya: On Spatial Distribution of Particle Field for Branching Random Walks

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The limit structure of a particle field for branching random walks (BRWs) depends on the properties of a random walk and a branching environment as well as on the dimension of a space in which the process is considered. Differences in these properties significantly affect the asymptotic behavior in time of the moments for the number of particles on a multidimensional lattice and the rate of their growth. For the BRWs with a finite number of particle generation centers it is shown how the growth of the limiting moments of the number of particles at each point of the multidimensional lattice corresponds to the limiting structure of the particle field, under various assumptions on intensities of generation and transport of particles. The key question is in which cases the normalized limiting (in time) moments of the number of particles guarantees the uniqueness of the probability distribution and, as a consequence, the convergence in distribution to some limiting random variable. The question about the explicit form of the distribution of the limiting random variable remains open. We show that in some cases it can be answered. Under the assumption that an underlying random walk is recurrent, we prove, for BRWs with a critical Markov branching process at each lattice point and one initial particle, the convergence of the distribution of the particle field to the limit stationary distribution. Some of the results are based on the publications [1, 2, 3, 4, 5].
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References

[1] A. Aparin, G. Popov, and E. Yarovaya. On the distribution of the sojourn time of a random walk at a point of a multidimensional lattice. *Theory Probab. Appl.*, 66:4, 2021 (in print).

[2] D. Balashova and E. Yarovaya. Practical simulation and estimation for Gibbs Delaunay-Voronoi tessellations with geometric hardcore interaction. *Proc. Steklov Inst. Math.*, 2022 (in print).

[3] D. Balashova, S. Molchanov, and E. Yarovaya. Structure of the Particle Population for a Branching Random Walk with a Critical Reproduction Law. *Methodol. Comput. Appl. Probab.*, 1–18, 2020, https://doi.org/10.1007/s11009-020-09773-2.

[4] I. Khristolyubov,and E. Yarovaya. A limit theorem for supercritical random branching walks with branching sources of varying intensity. *Theory Probab. Appl.*, 64(3):365–384, 2019.

[5] E. Yarovaya, J. Stoyanov, and K. Kostyashin. A limit theorem for supercritical random branching walks with branching sources of varying intensity. *Theory Probab. Appl.*, 64:4, 579–594, 2019.